6.2 Kinetic Energy and the Work-Energy Theorem

SECTION OUTCOMES

- Use algebraic deductions to derive the work-energy theorem.
- Define kinetic energy.
- Compare empirical and theoretcial values of total energy and account for discrepancies.

KEY TERMS

- work-kinetic energy theorem
- work-energy theorem

Simon Whitfield won a gold medal in the triathlon at the Sydney Olympics. Chemical reactions in his muscles caused them to shorten. This shortening of his muscles did work on the bones of his skeleton by exerting a force that caused them to move. The resulting motion of his bones allowed him to run faster than all of his competitors. How can you mathematically describe the motion resulting from the work his muscles did? In this section, you will investigate the relationship between doing work on an object and the resulting motion of the object.

Figure 6.15 Olympic triathlon winner Simon Whitfield crossing the finish line ahead of all of the other competitors.

Kinetic Energy

A baseball moves when you throw it. A stalled car moves when you push it. The subsequent motion of each object is a result of the work done on it. The energy of motion is called kinetic energy. By doing some simple "thought experiments" you can begin to develop a method to quantify kinetic energy. First, imagine a bowling ball and a golf ball rolling toward you with the same velocity. Which ball would you try hardest to avoid? The bowling ball would, of course, do more "work" on you, such as crushing your toe. Since both balls have the same velocity, the mass must be contributing to the kinetic energy of the balls. Now, imagine two golf balls flying toward you, one coming slowly and one rapidly. Which one would you try hardest to avoid? The faster one, of course. Using the same reasoning, an object's velocity must contribute to its kinetic energy. Could kinetic energy be a mathematical combination of an object's mass and velocity?

Dutch mathematician and physicist Christian Huygens (1629–1695) looked for a quantity involving mass and velocity that was characteristic of an object's motion. Huygens experimented with collisions of rigid balls (similar to billiard balls). He discovered that if he calculated the product of the mass and the square of the velocity (i.e. mv^2) for each ball and then added those products together, the totals were the same before and after the collisions. German mathematician Gottfried Wilhelm Leibniz (1646–1716), Huygens' student, called the quantity *vis viva* for "living force." It was many years later, after numerous, detailed observations and calculations, that physicists realized that the correct expression for the kinetic energy of an object, resulting from the work done on it, is actually *one half* of the quantity that Leibniz came up with.

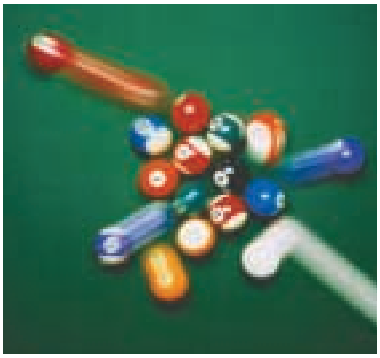


Figure 6.16 When billiard balls collide, the work each ball does on another gives the other ball kinetic energy only.

KINETIC ENERGY

Kinetic energy is one half the product of an object's mass and the square of its velocity.

$$E_k = \frac{1}{2}mv^2$$

Quantity	Symbol	SI unit
kinetic energy	E_k	J (joule)
mass	m	kg (kilogram)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

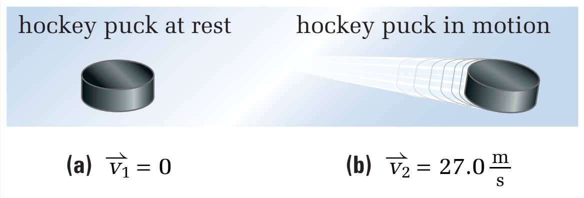
$(\text{mass})(\text{velocity})^2 = \text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2 = \text{kg}\frac{\text{m}^2}{\text{s}^2} = \text{kg}\frac{\text{m}}{\text{s}^2}\text{m} = \text{N} \cdot \text{m} = \text{J}$

Note: When velocity is squared, it is no longer a vector. Therefore, vector notation is not used in the expression for kinetic energy.

MODEL PROBLEM

Calculating Kinetic Energy

A 0.200 kg hockey puck, initially at rest, is accelerated to 27.0 m/s. Calculate the kinetic energy of the hockey puck (a) at rest and (b) in motion.



Frame the Problem

- A hockey puck was *at rest* and was then *accelerated*.
- A moving object has *kinetic energy*.
- The amount of *kinetic energy* possessed by an object is related to its *mass* and *velocity*.

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Identify the Goal

The kinetic energy, E_k, of the hockey puck
at rest
at a velocity of 27.0 $\frac{m}{s}$

PROBLEM TIP

It is very important to carry units through all calculations, as they will provide both a check to see that your work is correct and a hint as to what to do next.

Variables and Constants

Known	Implied	Unknown
$m = 0.200$ kg	$v_1 = 0.0\ \frac{m}{s}$ (at rest)	E_k
$v_2 = 27.0\ \frac{m}{s}$ (moving)		

Strategy / Calculations

Strategy	At rest	Moving
Use the equation for kinetic energy.	$E_k = \frac{1}{2}mv^2$	
All of the needed values are known, so substitute into the formula.	$E_k = \frac{1}{2}(0.200\ \text{kg})\left(0\ \frac{\text{m}}{\text{s}}\right)^2$	$E_k = \frac{1}{2}(0.200\ \text{kg})\left(27.0\ \frac{\text{m}}{\text{s}}\right)^2$
Multiply.	$E_k = 0\ \text{kg}\ \frac{\text{m}^2}{\text{s}^2}$	$E_k = 72.9\ \text{kg}\ \frac{\text{m}^2}{\text{s}^2}$
1 kg $\frac{m}{s^2}$ is equivalent to 1 N.	$E_k = 0\ \text{kg}\ \frac{\text{m}}{\text{s}^2}\ \text{m}$ $E_k = 0\ \text{N} \cdot \text{m}$	$E_k = 72.9\ \text{kg}\ \frac{\text{m}}{\text{s}^2}\ \text{m}$ $E_k = 72.9\ \text{N} \cdot \text{m}$
1 N·m is equivalent to 1 J.	$E_k = 0$ J	$E_k = 72.9$ J

The puck had zero kinetic energy while at rest, and 72.9 J of kinetic energy when moving.

Validate

A moving object has kinetic energy, while an object at rest has none. The units are expressed in joules, which is correct for energy.

PRACTICE PROBLEMS

19. A 0.100 kg tennis ball is travelling at 145 km/h. What is its kinetic energy?

20. A bowling ball, travelling at 0.95 m/s, has 4.5 J of kinetic energy. What is its mass?

21. A 69.0 kg skier reaches the bottom of a ski hill with a velocity of 7.25 m/s. Find the kinetic energy of the skier at the bottom of the hill.

Work and Kinetic Energy

The special relationship between doing work on an object and the resulting kinetic energy of the object is called the **work-kinetic energy theorem**. Everyday experience supports this theorem. If you saw a hockey puck at rest on the ice and a moment later saw it hurtling through the air, you would conclude that someone did work on the puck, by exerting a large force over a short distance, to make it move. This correct conclusion illustrates how doing work on an object gives the object increased velocity or kinetic energy.

Figure 6.17 Find out how the work done on the hockey puck is related to the puck's kinetic energy.

To develop a mathematical expression that relates work to the energy of motion, assume that all of the work done on a system gives the system kinetic energy only. Start with the definition of work and then apply Newton's second law. To avoid dealing with advanced mathematics, assume that a constant force gives the system a constant acceleration so that you can use the equations of motion from Unit 1. Since work and kinetic energy are scalar quantities, vector notation will be omitted from the derivation. This is valid as long as the directions of the force and displacement are parallel and the object is moving in a straight line.

PHYSICS FILE

Deriving or generating a mathematical equation to describe what you observe in the world is one of the things that theoretical physicists do. The challenge is to make appropriate substitutions from information that you already know and develop a useful relationship.

- Assume that the force is constant and write the equation for work. $W = F_{\parallel}\Delta d$
- Recall Newton's second law, $\vec{F} = m\vec{a}$. Assume the force and the acceleration are parallel to the direction of the displacement and motion is in one direction. Then omit vector symbols and use $F = ma$. $F = ma$
- Substitute ma for F in the equation for work. $W = ma\Delta d$
- Recall, from Chapter 3, the definition of acceleration for uniformly accelerated motion. $a = \frac{v_2 - v_1}{\Delta t}$
- Rewrite the equation for work in terms of initial and final velocities by substituting the definition for acceleration into a. $W = m\frac{(v_2 - v_1)}{\Delta t}\Delta d$
- Also, from Chapter 3, recall the equation for displacement for uniformly accelerated motion. $\Delta d = \frac{(v_1 + v_2)}{2}\Delta t$
- Divide both sides of the equation by Δt to obtain an equation for $\frac{\Delta d}{\Delta t}$. $\frac{\Delta d}{\Delta t} = \frac{(v_1 + v_2)}{2}$
- Rewrite the equation for work by substituting the value for $\frac{\Delta d}{\Delta t}$. $W = m\frac{(v_2 - v_1)(v_1 + v_2)}{2}$
- Expand the brackets (FOIL). $W = \frac{1}{2}m(\cancel{v_1v_2} + v_2^2 - v_1^2 - \cancel{v_1v_2})$
- Simplify by combining like terms. $W = \frac{1}{2}m(v_2^2 - v_1^2)$
- Expand. Notice that the result is in the form of initial and final kinetic energies. $W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$
- Conclude that work done on an object results in a change in the kinetic energy of the object.

$$W = E_{k2} - E_{k1}$$
$$W = \Delta E_k$$

Math Link

Try to derive the work-kinetic energy theorem using the work equation, $W = F_{\parallel}\Delta d$, and the equation of motion, $v_2^2 = v_1^2 + 2a\Delta d$, from Chapter 3. You may prefer this derivation rather than the one illustrated. (Hint: Solve the motion equation for acceleration.)

The delta symbol, Δ, denotes change. The expressions

$$W = \Delta E_k$$
$$W = E_{k2} - E_{k1}$$

are mathematical representations of the work-kinetic energy theorem which describes how doing work on an object can change the object's kinetic energy (energy of motion).

The work-kinetic energy theorem is part of the broader **work-energy theorem**. The work-energy theorem includes the concept that work can change an object's potential energy, thermal energy, or other forms of energy.

Conceptual Problem

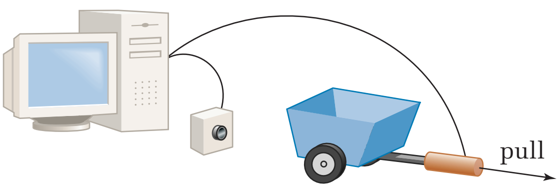
- Raj cannot understand why his answer to a problem is wrong. He was trying to calculate the amount of work done by a hockey stick on a hockey puck. The puck was moving slowly when the stick hit it, making it move faster. First, Raj found the difference of the final and initial velocities, and squared the value. He then multiplied his value by the mass of the puck. Finally, he divided by two. Explain to Raj why his answer was wrong. Tell him how to solve the problem correctly.

QUICK LAB

Pulling a Cart

TARGET SKILLS

- **Performing and recording**
- **Analyzing and interpreting**
- **Communicating results**



A baseball bat does a lot of work on a baseball in a very short period of time. If all goes as the batter plans, the result may be a long fly ball. The work-kinetic energy theorem can predict and quantify the work done on the baseball and its resulting motion. In this investigation, you will test the work-kinetic energy theorem using a slightly more controlled environment than a baseball diamond. Set up the force and motion sensors using one interface, as shown in the illustration. (Alternatively, use a Newton scale and ticker tape.) Pull on the cart with the force meter and collect data that will allow you to

- generate a force-versus-position graph
- determine the final velocity of the cart

Analyze and Conclude

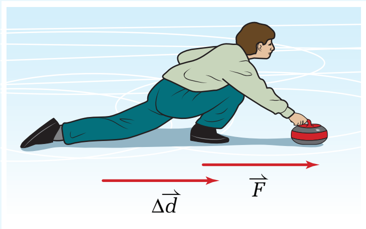
1. Determine the total work that you did on the cart.
2. Using your value for work, predict the final speed of the cart.
3. Determine the actual final speed of the cart.
4. Discuss whether your data support the work-kinetic energy theorem.
5. Give possible reasons for any observed discrepancies.

Applying the Work-Kinetic Energy Theorem

1. A physics student does work on a 2.5 kg curling stone by exerting 4.0×10^1 N of force horizontally over a distance of 1.5 m.

(a) Calculate the work done by the student on the curling stone.

(b) Assuming that the stone started from rest, calculate the velocity of the stone at the point of release. (Consider the ice surface to be effectively frictionless.)



Frame the Problem

- The curling stone was *initially at rest*; therefore, it had *no kinetic energy*.
- The student did *work* on the stone, giving it *kinetic energy*.
- The *force* exerted by the student was in the *same direction* as the *displacement* of the stone; thus, the equation for work applies.
- Since an ice surface has so little friction, we can *ignore* any effects of *friction*.
- The ice surface is level; therefore, there was no change in the height of the stone.
- The *work-kinetic energy theorem* applies to the problem.

Identify the Goal

Work done by the student on the stone
Velocity of the stone at release

Variables and Constants

Known	Implied	Unknown
$F = 4.0 \times 10^1$ N	$v_{\text{initial}} = 0\ \frac{\text{m}}{\text{s}}$	W
$\Delta d = 1.5$ m	$E_{\text{k(initial)}} = 0$ J	$E_{\text{k(final)}}$
$m = 2.5$ kg		v_{final}

Strategy	Calculations
The formula for work done by a force parallel to the displacement applies.	$W = F_{\parallel}\Delta d$
The values are known, so substitute.	$W = (4.0 \times 10^1\ \text{N})(1.5\ \text{m})$
Multiply.	$W = 6.0 \times 10^1\ \text{N} \cdot \text{m}$
An N·m is equivalent to a J.	$W = 6.0 \times 10^1$ J

The student did 6.0×10^1 J of work on the curling stone.

Strategy

Knowing the work, you can use the work-kinetic energy theorem to find final kinetic energy.

Calculations

$W = E_{k(final)} - E_{k(initial)}$

Substitute first

$W = E_{k(final)} - E_{k(initial)}$

$6.0 \times 10^1\ J = E_{k(final)} - 0\ J$

$6.0 \times 10^1\ J + 0\ J = E_{k(final)}$

$E_{k(final)} = 6.0 \times 10^1\ J$

Solve for $E_{k(final)}$ first

$W = E_{k(final)} - E_{k(initial)}$

$W + E_{k(initial)} = E_{k(final)}$

$6.0 \times 10^1\ J + 0\ J = E_{k(final)}$

$E_{k(final)} = 6.0 \times 10^1\ J$

Knowing the final kinetic energy, you can use the equation for kinetic energy to find the final velocity.

$E_k = \frac{1}{2}mv^2$

Substitute first

Divide both sides of the equation by the terms beside v^2.

$6.0 \times 10^1\ J = \frac{1}{2}(2.5\ kg)v^2$

$\frac{6.0 \times 10^1\ J}{\frac{1}{2}(2.5\ kg)} = \frac{\cancel{\frac{1}{2}(2.5\ kg)}}{\cancel{\frac{1}{2}(2.5\ kg)}}v^2$

$48\frac{\cancel{kg}\ m^2}{\cancel{kg}\ s^2} = v^2$

Take the square root of both sides of the equation.

$\sqrt{48\frac{m^2}{s^2}} = v$

Simplify.

$v = \pm 6.928\ \frac{m}{s}$

Solve for v_{final} first

$E = \frac{1}{2}mv^2$

$\frac{E}{\frac{1}{2}m} = v^2$

$\frac{2E}{m} = v^2$

$\sqrt{\frac{2E}{m}} = v$

$\sqrt{\frac{2(6.0 \times 10^1\ J)}{2.5\ kg}} = v$

$\sqrt{48\frac{kg\ m^2}{kg\ s^2}} = v$

$v = \pm 6.928\ \frac{m}{s}$

The velocity of the stone, on release, was 6.9 m/s.

Validate

The student did work on the stationary stone, transferring energy to the stone. The stone's kinetic energy increased and, therefore, so did its velocity, as a result of the work done on it.

Notice that *directional information* (±) is *lost* when the *square root* is taken. You must determine which answer (positive or negative) is correct by choosing the one that logically agrees with the situation. In this case, the direction of the applied force was considered positive; therefore, any motion resulting from the application of this force will also be positive.

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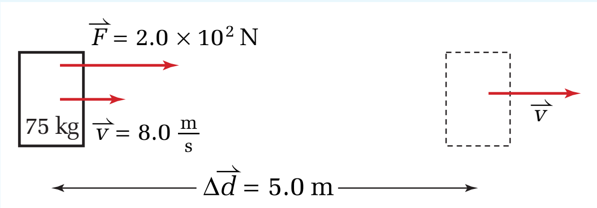
2. A 75 kg skateboarder (including the board), initially moving at 8.0 m/s, exerts an average force of 2.0×10^2 N by pushing on the ground, over a distance of 5.0 m. Find the new kinetic energy of the skateboarder if the trip is completely horizontal.

Frame the Problem

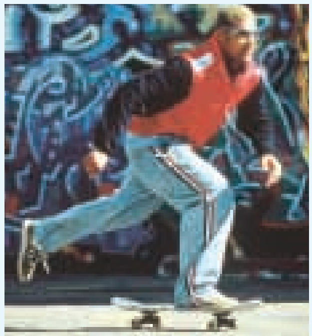
Make a sketch of the force and motion vectors.



- The skateboarder has an *initial velocity* and, therefore, an *initial kinetic energy*.
- He exerts a force by pushing along the ground. As a result, the *ground exerts a force* on him in the *direction of his motion*.
- The ground *does work* on the skateboarder, thus changing his *kinetic energy*.
- Assuming that the friction in the wheels is negligible, *all of the work* goes into *kinetic energy*. Therefore, the *work-kinetic energy theorem* applies to the problem.

Identify the Goal

The final kinetic energy, E_{k2}, of the skateboarder



A skateboarder does work by pushing on the ground. The work gives the skateboarder kinetic energy.

Variables and Constants

Known	Unknown
$v = 8.0\ \frac{m}{s}$	E_{k1}
$m = 75$ kg	E_{k2}
$\Delta d = 5.0$ m	W
$F = 2.0 \times 10^2$ N	

Strategy

A tree diagram showing relationships among the variables is often helpful when several steps are involved in obtaining a solution.

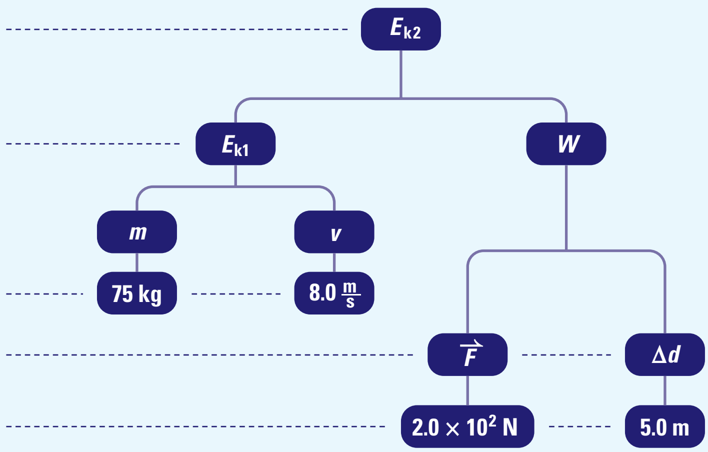
You can calculate final kinetic energy if you can find the initial kinetic energy and the work done.

You can calculate the initial kinetic energy if you know the mass and the initial velocity.

The mass and initial velocity are known.

You can calculate the work done if you can find the force and the displacement.

The force and displacement are known.



The problem is essentially solved. All that remains is the math.

Strategy	Calculations	
Find the initial kinetic energy by using the equation.	$E_k = \frac{1}{2}mv^2$	
Substitute.	$E_{k1} = \frac{1}{2}(75\ \text{kg})(8.0\ \frac{\text{m}}{\text{s}})^2$	
Multiply.	$E_{k1} = 2.4 \times 10^3\ \text{kg}\ \frac{\text{m}^2}{\text{s}^2}$	
Simplify.	$E_{k1} = 2.4 \times 10^3\ \text{kg}\ \frac{\text{m}}{\text{s}^2}\ \text{m}$	
1 N·m is equivalent to 1 J.	$E_{k1} = 2.4 \times 10^3\ \text{N} \cdot \text{m}$ $E_{k1} = 2.4 \times 10^3\ \text{J}$	
Find work by using the equation.	$W = F_{\parallel}\Delta d$	
Substitute.	$W = (2.0 \times 10^2\ \text{N})(5.0\ \text{m})$	
Multiply.	$W = 1.0 \times 10^3\ \text{N} \cdot \text{m}$	
1 N·m is equivalent to 1 J.	$W = 1.0 \times 10^3\ \text{J}$	
Find the final kinetic energy by using the work-kinetic energy theorem.	$W = E_{k2} - E_{k1}$	
	Substitute first	**Solve for E_{k2} first**
Add E_{k1} or its value to both sides of the equation.	$1.0 \times 10^3\ \text{J} = E_{k2} - 2.4 \times 10^3\ \text{J}$ $1.0 \times 10^3\ \text{J} + 2.4 \times 10^3\ \text{J} = E_{k2}$	$W + E_{k1} = E_{k2}$ $1.0 \times 10^3\ \text{J} + 2.4 \times 10^3\ \text{J} = E_{k2}$
Simplify.	$E_{k2} = 3.4 \times 10^3\ \text{J}$	$E_{k2} = 3.4 \times 10^3\ \text{J}$

The skateboarder's final kinetic energy was 3.4×10^3 J.

Validate

The ground did positive work on the skateboarder, exerting a force in the direction of the motion. Intuitively, this work should increase the velocity and by definition, the kinetic energy, which it did.

PRACTICE PROBLEMS

22. A 6.30 kg rock is pushed horizontally across a 20.0 m frozen pond with a force of 30.0 N. Find the velocity of the rock once it has travelled 13.9 m. (Assume there is no friction.)

23. The mass of an electron is 9.1×10^{-31} kg. At what speed does the electron travel if it possesses 7.6×10^{-18} J of kinetic energy?

24. A small cart with a mass of 500 g is accelerated, uniformly, from rest to a velocity of 1.2 m/s along a level, frictionless track. Find the kinetic energy of the cart once it has reached a velocity of 1.2 m/s. Calculate the force that was exerted on the cart over a distance of 0.1 m in order to cause this change in kinetic energy.

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25. A child's toy race car travels across the floor with a constant velocity of 2.10 m/s. If the car possesses 14.0 J of kinetic energy, find the mass of the car.

26. A 1250 kg car is travelling 25 km/h when the driver puts on the brakes. The car comes to a stop after going another 10 m. What was the average frictional force that caused the car to stop? If the same car was travelling at 50 km/h when the driver put on the brakes and the car experienced the same average stopping force, how far would it go before coming to a complete stop? Repeat the calculation for 100 km/h. Make a graph of stopping distance versus speed. Write a statement that describes the relationship between speed and stopping distance.

6.2 Section Review

1. K/U State the work-kinetic energy theorem and list three common examples that effectively support the theorem.

2. C Discuss how the following supports the work-kinetic energy theorem. A cue ball is at rest on a pool table, and then moves after being struck by a pool cue.

3. K/U A pitcher does work, *W*, on a baseball when he pitches it. How much more work would he have to do to pitch the ball three times as fast?

4. K/U Two identical cars are moving down a highway. Car X is travelling twice as fast as car Y. Both drivers see deer on the road ahead and apply the brakes. The forces of friction that are stopping the cars are the same. What is the ratio of the stopping distance of car X compared to car Y?

5. K/U Two cars, A and B, are moving. B's mass is half that of A and B is moving with twice the velocity of A. Is B's kinetic energy four times as great, twice as great, the same, or half as great as A's kinetic energy?

6. I How much force do the tires of a bicycle apply to the pavement when they are braking and/or skidding to a stop? Design and perform an experiment that will help you determine the average braking force indirectly. (It would be very difficult to measure the braking force of skidding tires directly.) Use the work-kinetic energy theorem. It implies that the initial kinetic energy of the bicycle and rider, before the brakes are applied, will be equal to the work done by the brakes and tires in stopping the bicycle.

(a) What equipment, materials, and tools will you need to determine the initial kinetic energy of the bicycle and rider before applying the brakes? What will you need to determine the stopping distance?

(b) Develop a procedure that lists all the steps you will follow.

(c) Repeat the experiment several times with the same rider. Next, repeat it with different riders. What do you conclude?

UNIT INVESTIGATION PREP

Sporting activities and equipment involve a series of energy transformations governed by the work-kinetic energy theorem.

- Identify specific energy transformations related to your project topic.
- Analyze the energy transformations both qualitatively and quantitatively by making appropriate assumptions.