6.1 Work and Energy

SECTION OUTCOMES

- Analyze quantitatively the relationships among force, displacement, and work.
- Explain the importance of using appropriate language and conventions.

KEY TERMS

- kinetic energy
- potential energy
- mechanical energy
- work
- joule

Each morning, people throughout Canada perform the same basic activities as they prepare for the day ahead. The ritual might begin by swatting the alarm clock, turning on a light, and heading for a warm shower. Following a quick breakfast of food taken from the refrigerator, they hurry on their way, travelling by family car, bus, subway, train, bicycle, or on foot. This ritual, repeated the world over, demands energy. Electrical energy sounds the alarm, lights the hallways, heats the water, and refrigerates and then cooks the food. Fossil fuels provide the energy for the engines that propel our vehicles. Energy is involved in everything that happens and, in fact, is the reason that everything *can* happen.

Figure 6.1 One thousand cars could be driven about 375 km, the distance from Fredericton to Charlottetown, with the amount of energy it takes to launch a space shuttle into orbit — 2.5×10^{12} J of energy.

Types of Energy

Physicists classify energy into two fundamental types — **kinetic energy** (the energy of motion) and **potential energy** (energy that is stored). The many different forms of energy, such as light energy, electrical energy, and sound energy that you will study in this and other units, all fit into one of these two categories. In this chapter, you will focus on one form of energy called **mechanical energy**.

The mechanical energy of an object is a combination of kinetic energy and potential energy. For example, the football in the photograph on page 216 has kinetic energy because it is moving. It also has potential energy because it is high in the air. The force of gravity acts on the ball, causing it to fall. As it falls, its speed increases and it gains kinetic energy. The best way to begin to understand energy is to study the relationship between energy and work.

Defining Work

If you have ever helped someone to move, you will understand that lifting heavy boxes or sliding furniture along a rough floor or carpet requires a lot of energy and is hard work. You may also feel that solving difficult physics problems requires energy and is also hard work. These two activities require very different types of work and are examples of how, in science, we need to be very precise about the terminology we use.

In physics, a force does work on an object if it causes the object to move. Work is always done *on* an object and results in a change in the object. **Work** is not energy itself, but rather it is a transfer of mechanical energy. A pitcher does work on a softball when she throws it. A bicycle rider does work on the pedals, which then cause the bicycle to move along the road. You do work on your physics textbook each day when you lift it into your locker. Each of these examples demonstrates the two essential elements of work as defined in physics. There is always a *force* acting on an object, causing the object to move a certain *distance*.

You know from experience that it takes more work to move a heavy table than to move a light chair. It also takes more work to move the table to a friend's house than to move it to the other side of the room. In fact, the amount of work depends directly on the magnitude of the *force* and the *displacement* of the object along the line of the force.

WORK

Work is the product of the force and the displacement when the force and displacement vectors are parallel and pointing in the same direction.

$$W = F_{\|}\Delta d$$

Quantity	Symbol	SI unit
magnitude of the force (parallel to displacement)	$F_{\|}$	N (newton)
magnitude of the displacement	Δd	m (metre)
work done	W	J (joule)

Unit Analysis

(force)(displacement) = N · m = J

Note: Both force and displacement are vector quantities, but their product, work, is a scalar quantity. For this reason, vector notations will not be used. Instead, a subscript on the symbol for force will indicate that the force is parallel to the displacement.

Figure 6.2 The joule was chosen as the unit of work in honour of a nineteenth-century physicist, James Prescott Joule.

The derived unit of work, or newton metre (N · m), is called a **joule** (J). One joule of work is accomplished by exerting exactly one newton of force on an object, causing it to move exactly one metre.

The definition for work applies to an individual force, not the net force, acting on an object. As shown in Figure 6.3, two forces are acting on the box. Both forces — the applied force and the force of friction — are doing work. You can calculate the work done by the applied force or the work done by the frictional force.

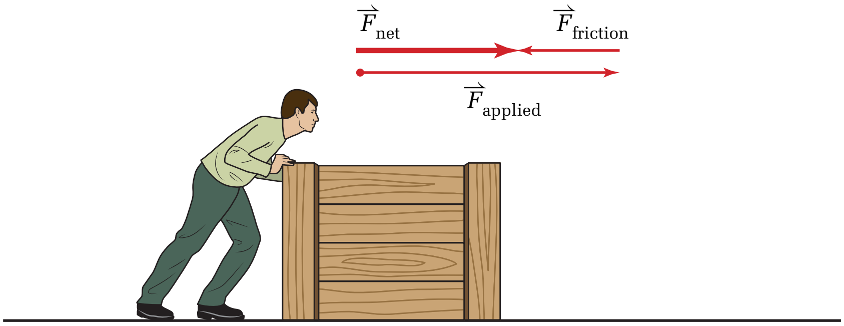


Figure 6.3 When you were determining the motion of objects in Chapter 4, you used the net force acting on the object. The net force is really the vector sum of all of the forces acting on the object. When calculating work, you determine the work done by one specific force, not the net force.

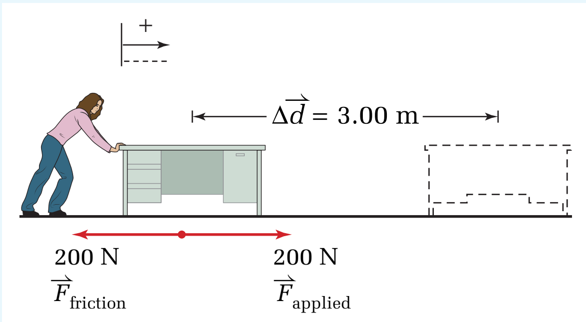
MODEL PROBLEM

Determining the Amount of Work Done

A physics student is rearranging her room. She decides to move her desk across the room, a total distance of 3.00 m. She moves the desk at a constant velocity by exerting a horizontal force of 2.00×10^2 N. Calculate the amount of work the student did on the desk in moving it across the room.

Frame the Problem

- The student *applies a force* to the desk.
- The *applied force* causes the desk to move.
- The *constant velocity* of the desk means that the *acceleration is zero*; thus, the net force on the desk is zero. Therefore, a *frictional force* must be balancing the applied force.
- Since the *force applied* by the student acts in the *same direction* as the *displacement* of the desk, the student is *doing work* on the desk.



Identify the Goal

Amount of work, W, done by the girl on the desk while moving it across the room

Variables and Constants

Known	Unknown
$\Delta d = 3.00$ m	W
$F_{applied} = 2.00 \times 10^2$ N	

> **PROBLEM TIP**
>
> When asked to calculate work done, be sure to identify the force specified by the problem. Then, consider only that force when setting up the calculation. Other forces may be doing work on the object, but you should consider only the work done by the force identified.

Strategy	Calculations
Use the formula for work done by a force acting in the same direction as the motion.	$W = F_{\parallel}\Delta d$
All of the needed variables are given, so substitute into the formula.	$W = (2.00 \times 10^2 \text{ N})(3.00 \text{ m})$
Multiply.	$W = 6.00 \times 10^2 \text{ N} \cdot \text{m}$
An N · m is equivalent to a J, therefore,	$W = 6.00 \times 10^2 \text{ J}$

The student did 6.00×10^2 J of work while moving the desk.

Validate

The applied force was used, because that is the force identified by the problem. The frictional force also did work, but the problem statement did not include work done by friction. Work has units of energy or joules, which is correct.

PRACTICE PROBLEMS

1. A weight lifter, Paul Anderson, used a circular platform attached to a harness to lift a class of 30 children and their teacher. While the children and teacher sat on the platform, Paul lifted them. The total weight of the platform plus people was 1.1×10^4 N. When he lifted them a distance of 52 cm, at a constant velocity, how much work did he do? How high would you have to lift one child, weighing 135 N, in order to do the same amount of work that Paul did?

2. A 75 kg boulder rolled off a cliff and fell to the ground below. If the force of gravity did 6.0×10^4 J of work on the boulder, how far did it fall?

3. A student in physics lab pushed a 0.100 kg cart on an air track over a distance of 10.0 cm, doing 0.0230 J of work. Calculate the acceleration of the cart. (Hint: Since the cart was on an air track, you can assume that there was no friction.)

When Work Done Is Zero

Physicists define work very precisely. Work done on an object is calculated by multiplying the *force* times the *displacement* of the object when the two vectors are *parallel*. This very precise definition of work can be illustrated by considering three cases where intuition suggests that work has been done, but in reality, it has not.

Case 1: Applying a Force That Does Not Cause Motion

Consider the energy that you could expend trying to move a house. Although you are pushing on the house with a great deal of force, it does not move. Therefore, the work done on the house, according to the equation for work, is zero (see Figure 6.4). In this case, your muscles feel as though they did work; however, they did no work on the house. The work equation describes work done by a force that moves the object on which the force is applied. Recall that work is a transfer of energy to an object. In this example, the *condition* of the house has not changed; therefore, no work could have been done on the house.

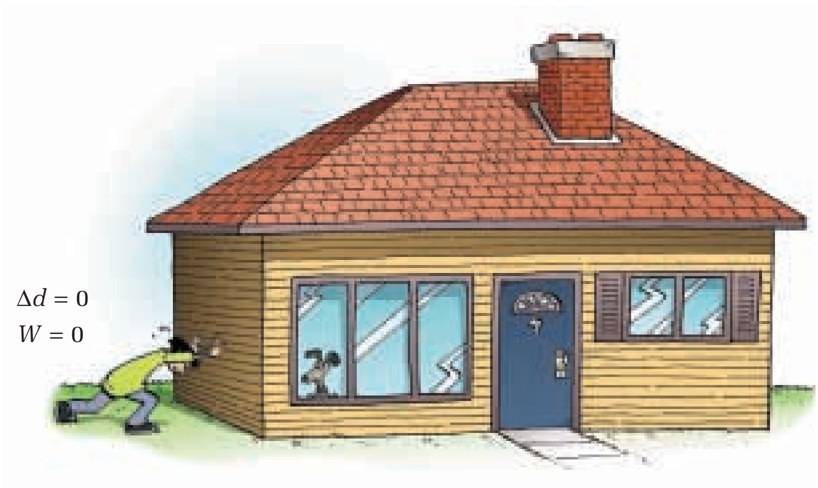


Figure 6.4 $W = F_{\parallel}\Delta d \quad W = F_{\parallel} \times 0 \quad W = 0$

Language Link

"The *condition* of the house has not changed." In this sentence, the term "condition" is used to represent a measurable and obvious change in the total energy of the house. A change in condition would mean that the house gained some form of kinetic or potential energy from the work done by pushing on it. How does kicking a stationary soccer ball change the condition of the ball?

Case 2: Uniform Motion in the Absence of a Force

Recall from Chapter 5 that Newton's first law of motion predicts that an object in motion will continue in motion unless acted on by an *external* force. A hockey puck sliding on a frictionless surface at constant speed is moving and yet the work done is still zero (see Figure 6.5). Work was done to start the puck moving, but because the surface is frictionless, a force is not required to keep it moving; therefore, no work is done on the puck to keep it moving.

Math Link

When the direction of the displacement changes, calculus must be used to properly calculate the work done. The result of the method is the adding of work done on tiny increments of displacement along the entire length (distance) of the path. Consequently, when reporting results, the term distance more accurately describes the results than does displacement.

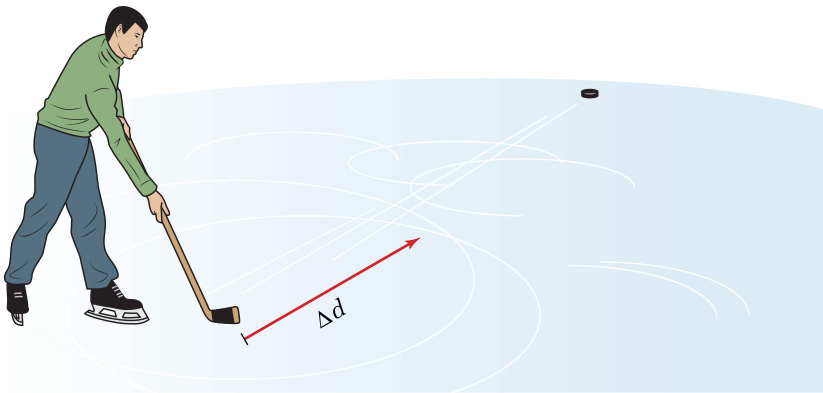


Figure 6.5 $W = F_{\parallel}\Delta d \quad W = 0 \times \Delta d \quad W = 0$

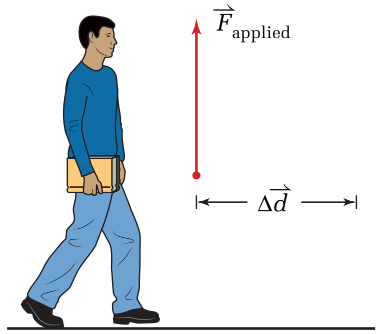
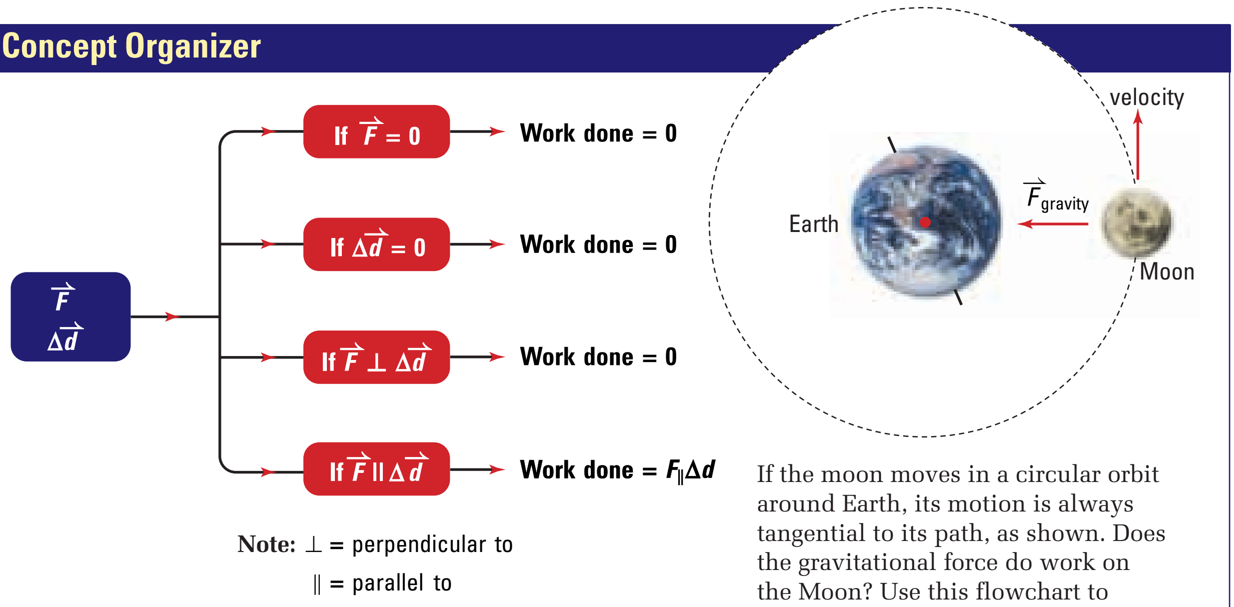


Figure 6.6 You are exerting an upward force (against gravity) on your book to prevent it from falling. However, since this force is perpendicular to the motion of the book, it does no work on the book.

Case 3: Applying a Force That Is Perpendicular to the Motion

Assume that you are carrying your physics textbook down the hall, at constant velocity, on your way to class. Your hand applies a force directly upward to your textbook as you move along the hallway. When considering the work done on the textbook by your hand, you can see that the upward force is perpendicular (i.e., at 90˚) to the displacement. In this case, the work done by your hand on the textbook is zero (see Figure 6.6). It is important to note that your hand does do work on the textbook to accelerate it when you begin to move, but once you and the textbook are moving at a constant velocity, you are no longer doing work on the book.

Concept Organizer



If the moon moves in a circular orbit around Earth, its motion is always tangential to its path, as shown. Does the gravitational force do work on the Moon? Use this flowchart to determine the answer.

Figure 6.7 Making decisions about work done

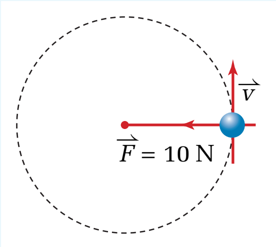
MODEL PROBLEM

Work Done by Swinging a Mass on a String

A child ties a ball to the end of a 1.0 m string and swings the ball in a circle. If the string exerts a 10 N force on the ball, how much work does the string do on the ball during a swing of one complete circle?

Frame the Problem

- The string *applies a force* to the ball.
- The ball is *moving*.
- The direction of the motion of the ball is *perpendicular* to the direction of the force.



Identify the Goal

Amount of work, W, done by the string on the ball

Variables and Constants

Known	Unknown
$\Delta d = 2\pi\ (1.0\ \text{m})$ (circumference of a circle of radius 1.0 m)	W
$F_{\text{applied}} = 10\ \text{N}$ (perpendicular to the direction of the motion)	$F_{\parallel}$

PROBLEM TIP

When solving problems involving work, consider the following questions.

- Is a force acting on the object to be moved?
- Does the force *cause* displacement of the object?
- Is the force acting in the same direction as the displacement?

If the answer to each question is yes, then you can safely apply the equation for work done. If the answer to any of the questions is no, compare the problem to the cases discussed on pages 222 to 223. The work done might be zero.

Strategy

The force that the string applies on the ball is not in the direction of the motion; therefore, it cannot do work on the ball. No information is given about any possible force that is parallel to the direction of the motion. However, the problem did not ask for work done by any force other than that exerted by the string.

The string does no work on the ball as it swings around on the end of the string.

Calculations

$W = 0\ \text{J}$

Validate

The orientations of the force and displacement vectors for each tiny increment of the path fit the conditions described in Case 3. It is not possible for the force to do work. Therefore the work must be zero.

PRACTICE PROBLEMS

4. With a 3.00×10^2 N force, a mover pushes a heavy box down a hall. If the work done on the box by the mover is 1.90×10^3 J, find the length of the hallway.

5. A large piano is moved 12.0 m across a room. Find the average horizontal force that must be exerted on the piano if the amount of work done by this force is 2.70×10^3 J.

6. A crane lifts a 487 kg beam vertically at a constant velocity. If the crane does 5.20×10^4 J of work on the beam, find the vertical distance that it lifted the beam.

7. A teacher carries his briefcase 20.0 m down the hall to the staff room. The teacher's hand exerts a 30.0 N force upward as he moves down the hall at constant velocity.

(a) Calculate the work done by the teacher's hand on the briefcase.

(b) Explain the results obtained in part (a).

8. A 2.00×10^2 N force acts horizontally on a bowling ball over a displacement of 1.50 m. Calculate the work done on the bowling ball by this force.

9. The *Voyager* space probe has left our solar system and is travelling through deep space, which can be considered to be void of all matter. Assume that gravitational effects may be considered negligible when *Voyager* is far from our solar system.

(a) How much work is done on the probe if it covers 1.00×10^6 km travelling at 3.00×10^4 m/s?

(b) Explain the results obtained in part (a).

10. An energetic group of students attempts to remove an old tree stump for use as firewood during a party. The students apply an average upward force of 650 N. The 865 kg tree stump does not move after 15.0 min of continuous effort, and the group gives up.

(a) How much work did the students do on the tree stump?

(b) Explain the results obtained in part (a).

Work Done by Changing Forces

We have restricted our discussion so far to forces that remain constant throughout the motion. However, the definition of work as given by $W = F_{\|}\Delta d$ applies to all cases, including situations where the force changes. Mathematically, solving problems with changing forces goes beyond the scope of this course. However, you can use a graph to approximate a solution without using complex mathematics. A force-versus-position graph allows you to determine the work done, whether or not the force remains constant. Examine Figure 6.8, in which the graph shows a constant force of 10 N acting over a displacement of 4.0 m. The area under the force-versus-position line is given by the shaded rectangle.

$$\begin{aligned}\text{Area under the curve} &= \text{area of the shaded rectangle}\\ &= \text{length} \times \text{width}\\ &= (10\ \text{N})(4.0\ \text{m})\\ &= 40\ \text{N} \cdot \text{m}\\ &= 40\ \text{J}\end{aligned}$$

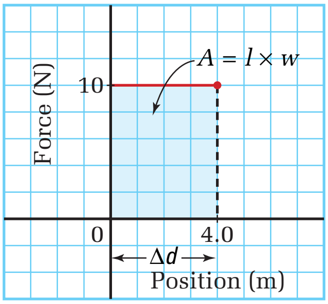


Figure 6.8 The length of the shaded box is $F_{\|} = 10$ N. The width is $\Delta d = 4.0$ m. Therefore, the area is $F_{\|} \times \Delta d = 40$ N · m. The area under the curve is the same as the work done by the force.

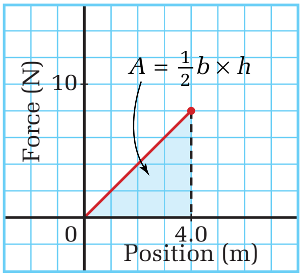


Figure 6.9 When the force increases linearly, the average force can be calculated by using the equation

$$F_{ave} = \frac{1}{2}(F_{initial} + F_{final})$$

The area under the curve is in the shape of a triangle. Thus, the work done can be calculated by calculating the area of the triangle.

This result is identical to the one obtained by applying the equation for the work done: $W = F_{\parallel}\Delta d$.

$$\begin{aligned} W &= F_{\parallel}\Delta d \\ &= (10\ \text{N})(4.0\ \text{m}) \\ &= 40\ \text{N} \cdot \text{m} \\ &= 40\ \text{J} \end{aligned}$$

A force-versus-position graph can be used to determine work done even when the applied force does not remain constant. Consider the force-versus-position graph in Figure 6.9. It shows a force that starts at zero and increases steadily to 10 N over a displacement of 4.0 m. In this case, the area under the curve forms a triangle. Even though the force does not remain constant, it is still possible to calculate the work done by finding the area of the triangle.

$$\begin{aligned} \text{Work done} &= \text{area under the force-versus-position curve} \\ &= \text{area of the triangle} \\ &= \frac{1}{2}\text{base} \times \text{height} \\ &= (0.5)(4.0\ \text{m})(10\ \text{N}) \\ &= 20\ \text{N} \cdot \text{m} \\ &= 20\ \text{J} \end{aligned}$$

$$\begin{aligned} \text{Work done} &= \text{average force multiplied by the displacement} \\ &= \frac{1}{2}(0\ \text{N} + 10\ \text{N})(4.0\ \text{m}) \\ &= (0.5)(10\ \text{N})(4.0\ \text{m}) \\ &= 20\ \text{N} \cdot \text{m} \\ &= 20\ \text{J} \end{aligned}$$

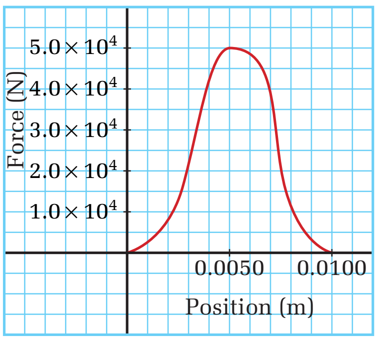


Figure 6.10 Even when a force-versus-position curve is irregular, the area under the curve gives the work done. This curve represents the force of a golf club on the ball when the club strikes the ball.

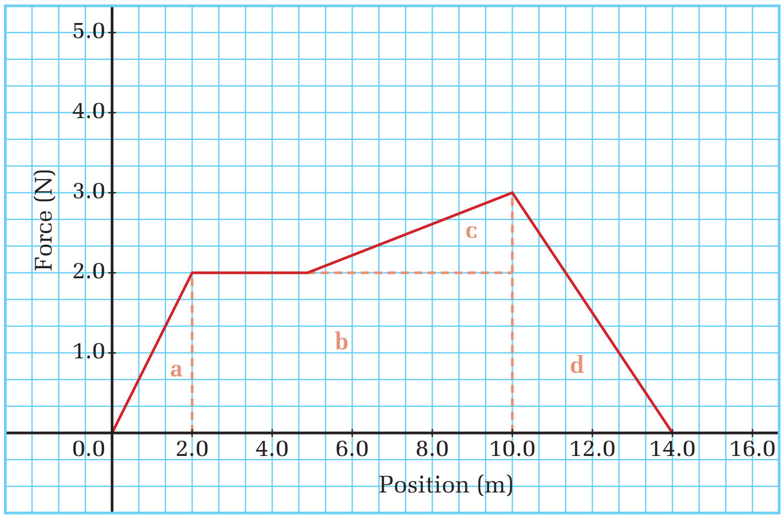
Notice that calculating the area of the triangle yields the same results as the equation for work, if the average force is used. In many situations, however, the applied force changes in a way that makes it difficult to obtain an average force. Solving problems that involve such a changing applied force can be solved using calculus. Since this advanced mathematical technique is not a requirement of this course, you can estimate solutions to problems involving changing forces by estimating the area under a curve, such as the one shown in Figure 6.10.

The force-versus-position curve in Figure 6.10 represents the force exerted on a golf ball by the club when the golfer tees off. Notice how the force changes, reaching a maximum and then falling back to zero. To calculate the area under the curve and, therefore, the work done on the ball by the club, you must count the squares and estimate the area in the partial squares. This method yields a result that is a close approximation to the numerical answer that could be obtained using calculus.

MODEL PROBLEM

Estimating Work from a Graph

Determine the amount of work done by the changing force represented in the force-versus-position plot shown here.



Frame the Problem

- A *changing force* is acting on an object.
- Since the force is not constant, the formula for work does not apply.
- The *area under a force-position curve* is equal to the work done when the units of force and displacement on the graph are used correctly.
- You can divide the area into segments that have simple geometric shapes. Then, use the formulas for the shapes to find the areas. The sum of the areas for each shape gives the total area and, thus, the work done.

Identify the Goal

The work, W, done by the forces represented on the force-versus-position plot

Variables and Constants

Known	Unknown
Scale for force and displacement on graph	A_a
	A_b
	A_c
	A_d
	W

continued ►

continued from previous page

Strategy

Divide the area under the curve into simple triangles and rectangles, as shown on the plot.

Calculations

Calculate A_b, the area of the rectangle b.

Area = base × height

base = 10.0 m – 2.0 m = 8.0 m

height = 2.0 N – 0.0 N = 2 N

$$A = b \times h$$
$$A_b = (8.0\ \text{m})(2.0\ \text{N})$$
$$A_b = 16.0\ \text{N} \cdot \text{m}$$
$$A_b = 16\ \text{J}$$

Calculate A_a, the area of triangle a.

Area = $\frac{1}{2}$base × height

base = 2.0 m – 0.0 m = 2.0 m

height = 2.0 N – 0.0 N = 2 N

$$A = \frac{1}{2} b \times h$$
$$A_a = \frac{1}{2}(2.0\ \text{m})(2.0\ \text{N})$$
$$A_a = 2.0\ \text{N} \cdot \text{m}$$
$$A_a = 2.0\ \text{J}$$

Calculate A_c, the area of triangle c.

Area = $\frac{1}{2}$base × height

base = 10.0 m – 5.0 m = 5.0 m

height = 3.0 N – 2.0 N = 1.0 N

$$A = \frac{1}{2} b \times h$$
$$A_c = \frac{1}{2}(5.0\ \text{m})(1.0\ \text{N})$$
$$A_c = 2.5\ \text{N} \cdot \text{m}$$
$$A_c = 2.5\ \text{J}$$

Calculate A_d, the area of triangle d.

Area = $\frac{1}{2}$base × height

base = 14.0 m – 10.0 m = 4.0 m

height = 3.0 N – 0.0 N = 3.0 N

$$A = \frac{1}{2} b \times h$$
$$A_d = \frac{1}{2}(4.0\ \text{m})(3.0\ \text{N})$$
$$A_d = 6.0\ \text{N} \cdot \text{m}$$
$$A_d = 6.0\ \text{J}$$

Find A_T, the total area.

$$\begin{aligned} A_T &= A_a + A_b + A_c + A_d \\ &= 2.0\ \text{J} + 16.0\ \text{J} + 2.5\ \text{J} + 6.0\ \text{J} \\ &= 26.5\ \text{J} \end{aligned}$$

Work is equal to the total area under the curve.

$$W = 26.5\ \text{J} \cong 27\ \text{J}$$

The force represented by the graph did 27 J of work.

Validate

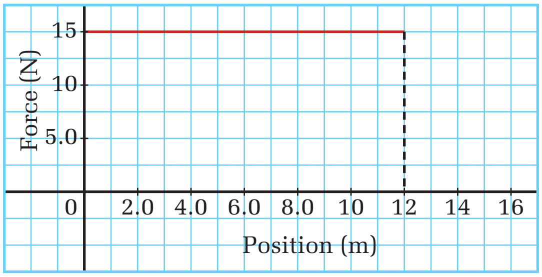
By looking at the graph, you can estimate that the average force is close to 2 N. Therefore, a rough estimate of the work would be

$$\text{Work} \approx 2\ \text{N} \times 14\ \text{m} = 28\ \text{J}$$

This is close enough to give you confidence in the value of 27 J.

11. Determine the amount of work done by the forces represented in the four force-versus-position plots that follow.

A



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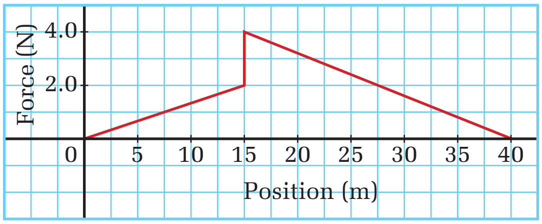

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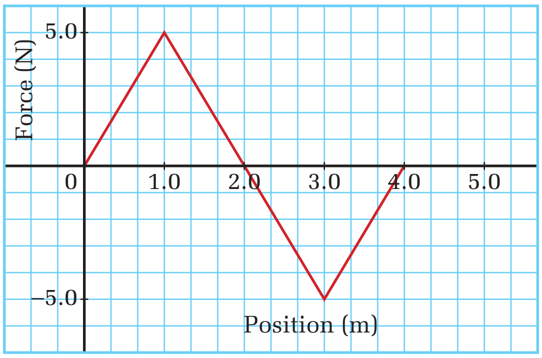

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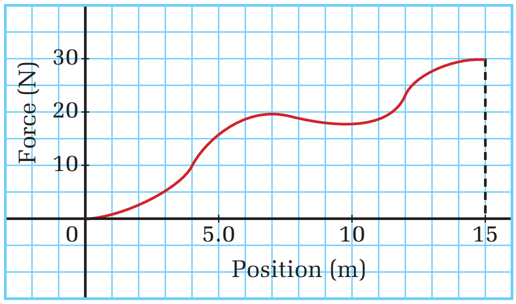


12. Draw a force-versus-position plot that represents a constant force of 60 N exerted on a Frisbee™ over a distance of 80.0 cm. Show the work done on the Frisbee™ by appropriately shading the graph.

13. Stretch a rubber band and estimate the amount of force you are using to stretch it. (Hint: A 100 g mass weighs approximately 1N.) Notice how the force you must exert increases as you stretch the rubber band. Draw a force-versus-position graph of the force you used to stretch the rubber band for a displacement of 15 cm. Use the graph to estimate the amount of work you did on the rubber band.

Constant Force at an Angle

Everyday experience rarely provides situations in which forces act precisely parallel or perpendicular to the motion of an object. For example, in Figure 6.11 on the next page, the applied force exerted by the child pulling the wagon is at an angle relative to the direction of the wagon's displacement. In cases such as this, only part of the force vector or a component of the force does work on the wagon.

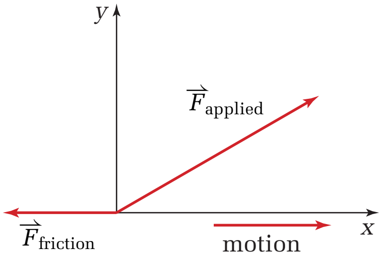


Figure 6.11 To calculate the work done on the wagon, use only the component of the force that is in the direction of the displacement.

To determine the work done by the child on the wagon, you must first find the component of the force that is parallel to the direction of motion of the wagon. The first step in performing this calculation is choosing a coordinate system with one axis along the direction of motion of the wagon. Then, use simple trigonometry to *resolve* the force into its components — one that is parallel to the direction of motion and one that is perpendicular to the

QUICK LAB Cart Bungee

TARGET SKILLS

- Predicting
- Identifying variables
- Performing and recording

Connect one elastic band between a cart and a force sensor on an inclined slope. Set up a motion sensor to track the position of the cart. Have the computer generate a force-versus-position graph as you release the cart and it moves down the incline until the elastic band stops it. Use the computer software to calculate the area under the force-versus-position curve.

Analyze and Conclude

1. What does the area under the curve on your graph represent?
2. Predict how changing the number of elastic bands will affect the area under the curve.
3. Predict how changing the mass of the cart will affect the area under the curve.
4. What other variables could you change? Predict how they would affect the area under the curve.
5. If you have the opportunity, test your predictions.

direction of motion. As shown in Figure 6.12, the parallel component of the force vector is $F_x = |\vec{F}| \cos\theta$. Note that the angle, θ, is always the angle between the force vector and the displacement vector.

To summarize, when the applied force, $\vec{F}$, acts at an angle to the displacement of the object, use the component of the force parallel to the direction of the motion, $F_x = |\vec{F}| \cos\theta$, to calculate the work done. In these cases, the equation for work is derived as follows.

- This equation applies to cases where the force and displacement vectors are in the same direction and, thus, vector notations are not used. $W = F_{\parallel}\Delta d$
- This equation applies to cases where the *x*-component is parallel to the direction of the motion, and thus, vector notations are not used. $W = F_x \Delta d$
- The component of the original force, $\vec{F}$, that is parallel to the displacement $F_x = |\vec{F}| \cos\theta$
- Substitute the expression for F_x into the expression for work. $W = |\vec{F}| \cos\theta \,\Delta d$

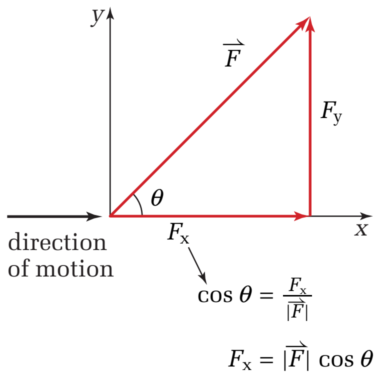


Figure 6.12 To find the horizontal component of the force, start with the definition of the cosine of an angle. Then solve for F_x.

WORK

Work done when the force and displacement are *not* parallel and pointing in the same direction

$$W = F\Delta d \cos\theta$$

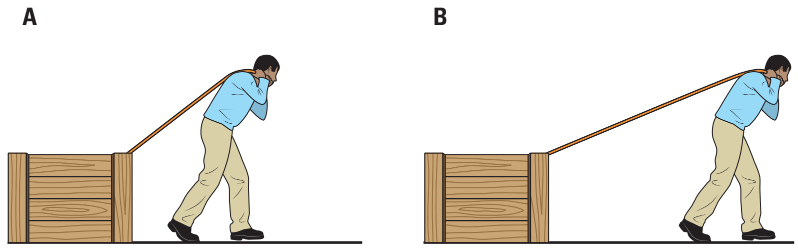
θ is the angle between the force and displacement vectors.

Note: Since work is a scalar quantity and only the magnitudes of the force and displacement affect the value of the work done, vector notations have been omitted.

The example of the child pulling the wagon demonstrates some subtle aspects of the definition of work. Consider the question "Does the force of gravity do work on the wagon?" To answer this, follow the reasoning process outlined in Figure 6.7, the Concept Organizer, on page 223. You can see that the force of gravity is not zero, but that it is perpendicular to the displacement. Therefore, we may conclude that the work done by gravity on the wagon in this case is zero. Note that $F \cos\theta = F \cos 90^\circ = F \times 0 = 0$.

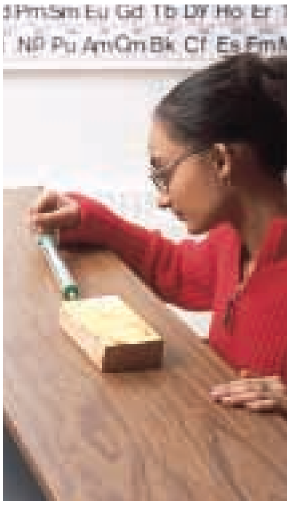
Conceptual Problems

- A child is pulling a wagon up a ramp. The applied force is parallel to the ramp. Is gravity doing any work on the wagon? Explain your reasoning.
- Examine the figure below. Assume that, in both case A and case B, the mass of the crate, the frictional force, and the constant velocity of the crate are the same. In which case, A or B, is the worker exerting a greater force on the crate by pulling on the rope? Explain your reasoning.



QUICK LAB: Elevator Versus a Ramp

TARGET SKILLS
- Predicting
- Performing and recording




What forces are acting on the mass? How will these forces affect its motion?

Set up an inclined plane. Carefully measure both its length and height. Using a Newton spring scale or a force sensor, carefully pull a cart up the incline at a constant velocity. Observe and record the force required. Now lift the cart at a constant velocity through the same height as the incline. Again, record the force required. Predict which method will require more work to accomplish. Calculate the work done in each case.

Analyze and Conclude

1. Compare the work done in each case. How accurate was your prediction?
2. Discuss the experimental variables that most directly affect the work done.
3. Repeat the experiment using a large, rough mass that will slide up the incline rather than roll as the cart did. Predict which method of pulling the large mass will require more work. Was your prediction correct? Why or why not?

Positive and Negative Work

Does the force of friction do work on the mass in Figure 6.13? It is parallel to the displacement, but it is acting in the opposite direction. This means that the angle between the displacement and the force of friction is 180°. Applying the revised equation for work, we find the following results.

Apply the equation for work to this case	$W = F\Delta d \cos 180°$
Because $\cos 180° = (-1)$	$W = F\Delta d(-1)$
Therefore	$W = -F\Delta d$

The work done by the frictional force is non-zero and negative. *Negative work* done by an external force *reduces* the energy of a mass. The energy does not disappear, but is, instead, lost to the surroundings in the form of heat or thermal energy. If the person stopped pulling the mass, its motion would quickly stop, as the frictional force would reduce the energy of motion to zero.

Positive work adds energy to an object; *negative* work removes energy from an object. In many situations, such as the one shown in Figure 6.14, two different forces are doing work on the same object. One force is doing positive work and the other is doing negative work.

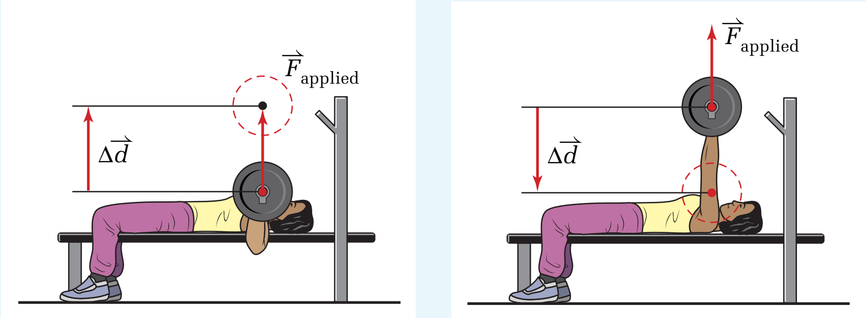
Figure 6.13 In what direction is the force of friction acting?

Figure 6.14 The hammer does positive work on the nail — the applied force and the displacement are in the same direction as the nail moves into the wood. The force of friction does negative work on the nail — the force of friction is opposite to the displacement as the nail moves into the wood. The nail also does negative work on the hammer — the applied force is opposite to the direction of the displacement. The hammer stops moving.

MODEL PROBLEM

Doing Positive and Negative Work

Consider a weight lifter bench-pressing a barbell weighing 6.50×10^2 N through a height of 0.55 m. There are two distinct motions: (1) when the barbell is lifted up and (2) when the barbell is lowered back down. Calculate the work that the weight lifter does on the barbell during each of the two motions.



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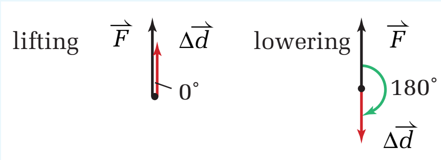
Frame the Problem

- The weight lifter *lifts* and *lowers* the barbell at a *constant velocity*. Therefore, the force *she exerts* is equal to the *weight* of the barbell.
- Use the formula for work when the direction of the force is at an angle with the direction of the displacement.
- Positive work adds energy to the object. Negative work removes energy from the object.
- The angle between the force acting on the barbell and its displacement is
  0° while lifting
  180° while lowering



Identify the Goal

Work, W, done while lifting the barbell

Work, W, done while lowering the barbell

Variables and Constants

Known	Unknown
$F = 6.50 \times 10^2$[up]	W
$\Delta d = 0.55$ m[up] while lifting	
$\Delta d = 0.55$ m[down] while lowering	
$\theta = 0°$ while lifting	
$\theta = 180°$ while lowering	

Strategy / Calculations

Use the equation for work when the force is not the same direction as the displacement.

$$W = F\Delta d\cos\theta$$

Strategy	Lifting	Lowering
Substitute in the values.	$W = (6.5 \times 10^2\ \text{N})(0.55\ \text{m})\cos 0°$	$W = (6.5 \times 10^2\ \text{N})(0.55\ \text{m})\cos 180°$
Multiply.	$W = 3.65 \times 10^2\ \text{N} \cdot \text{m}\ (+1)$ $W = 3.7 \times 10^2\ \text{J}$	$W = 3.65 \times 10^2\ \text{N} \cdot \text{m}\ (-1)$ $W = -3.7 \times 10^2\ \text{J}$

The weight lifter did 3.7×10^2 J of work to lift the barbell and -3.7×10^2 J of work to lower it.

Validate

The barbell gained energy when it was raised. The energy that the barbell gained would become very obvious if it was dropped from the elevated position — it would accelerate downward, onto the weight lifter!

The weight lifter does negative work on the bar to lower it. She is removing the energy that she previously had added to the bar by lifting it. If the weight lifter did not do negative work on the bar, it would accelerate downward.

PRACTICE PROBLEMS

14. A large statue, with a mass of 180 kg, is lifted through a height of 2.33 m onto a display pedestal. It is later lifted from the pedestal back to the ground for cleaning.
    - (a) Calculate the work done by the applied force on the statue when it is being lifted onto the pedestal.
    - (b) Calculate the work done by the applied force on the statue when it is lowered down from the display pedestal.
    - (c) State all of the forces that are doing work on the statue during each motion.
15. A mechanic exerts a force of 45.0 N to raise the hood of a car 2.80 m. After checking the engine, the mechanic lowers the hood. Find the amount of work done by the mechanic on the hood during each of the two motions.
16. A father is pushing a baby carriage down the street. Find the total amount of work done by the father on the baby carriage if he applies a 172.0 N force at an angle of 47° with the horizontal, while pushing the carriage 16.0 m along the level sidewalk.
17. While shopping for her weekly groceries, a woman does 2690 J of work to push her shopping cart 23.0 m down an aisle. Find the magnitude of the force she exerts if she pushes the cart at an angle of 32° with the horizontal.
18. A farmer pushes a wheelbarrow with an applied force of 124 N. If the farmer does 7314 J of work on the wheelbarrow while pushing it a horizontal distance of 77.0 m, find the angle between the direction of the force and the horizontal.

6.1 Section Review

1. K/U In each of the following cases, state whether you are doing work on your textbook. Explain your reasoning.
    - (a) You are walking down the hall in your school, carrying your textbook.
    - (b) Your textbook is in your backpack on your back. You walk down a flight of stairs.
    - (c) You are holding your textbook while riding up an escalator.
2. K/U Your lab partner does the same amount of work on two different objects, A and B. After she stops doing work, object A moves away at a greater velocity than object B. Give two possible reasons for the difference in the velocities of A and B.
3. C Describe two different scenarios in which you are exerting a force on a box but you are doing no work on the box.
4. K/U State all of the conditions necessary for doing positive work on an object.
5. I A student used a force meter to pull a heavy block a total distance of 4.5 m along a floor. Part of the floor was wood, another part was carpeted, and a third part was tiled. In each case, the force required to pull the block was different. The table below lists the distance and the force recorded for the three parts of the trip.

Floor surface	Distance pulled	Force measured
wooden floor	1.5 m	3.5 N
carpet	2.5 m	6.0 N
tiled floor	0.5 m	4.5 N

- (a) Use these data to construct a force-versus-distance graph for the motion.
- (b) Calculate the total work performed on the heavy block throughout the 4.5 m.