5.4 Momentum and Newton's Laws

Figure 5.18 The crash in the photograph occurred at the start of the race. The race was temporarily suspended while the debris was cleared from the track. When the race was restarted, the driver of the car that is "airborne" in the photograph was in his backup car, ready to go.

SECTION OUTCOMES

- Apply Newton's laws of motion to explain momentum.
- Design an experiment identifying and controlling major variables.
- Describe the functioning of technology devices based on principles of momentum.
- Analyze the influence of society on scientific and technological endeavours in dynamics.

KEY TERMS

- momentum
- impulse
- impulse-momentum theorem

The driver of the race car in the above photograph walked away from the crash without a scratch. Luck had little to do with this fortunate outcome, though — a practical application of Newton's laws of motion by the engineers who designed the car and its safety equipment protected the driver from injury.

You have learned that Newton's laws can explain and predict a wide variety of patterns of motion. How can some of the same laws that guide the stars and planets protect a race car driver who is in a crash?

When Newton originally formulated his laws of motion, he expressed them in a somewhat different form than you see in most textbooks today. Newton emphasized a concept called a "quantity of motion," which is defined as the product of an object's mass and its velocity. Today, we call this quantity "momentum." In this section, you will see how the use of momentum allows you to analyze and predict the motion of objects in countless situations that you have not yet encountered.

There are a few types of interactions for which it is difficult to determine or describe the forces acting on an object or on a group of objects. These interactions include collisions, explosions, and recoil. For these more complex scenarios, it is easier to observe the motion of the objects before and after the interaction and then analyze the interaction by using Newton's concept of a quantity of motion.

Defining Momentum

Although you have not used the mathematical expression for momentum, you probably have a qualitative sense of its meaning. For example, when you look at the photographs in Figure 5.19, you could easily list the objects in order of their momentum. Becoming familiar with the mathematical expression for momentum will help you to analyze interactions between objects.

Momentum is the product of an object's mass and its velocity, and is symbolized by $\vec{p}$. Since it is the product of a vector and a scalar, momentum is a vector quantity. The direction of the momentum is the same as the direction of the velocity.

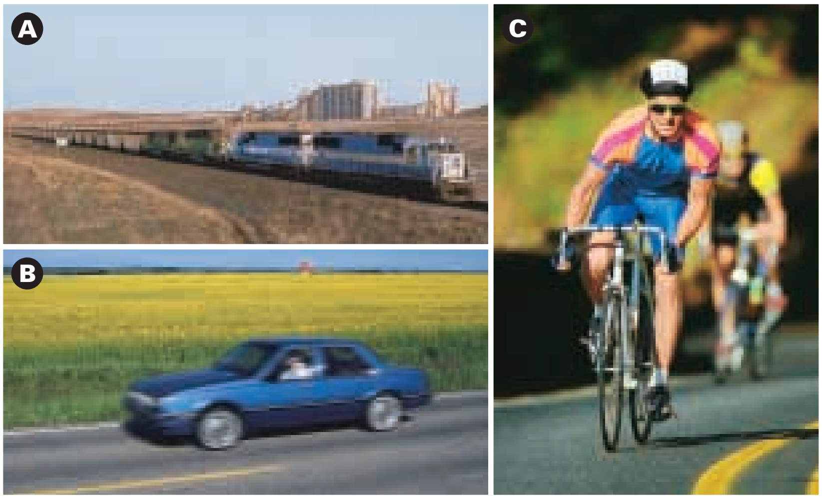


Figure 5.19 If the operator of each of these vehicles was suddenly to slam on the brakes, which vehicle would take the longest time to stop?

DEFINITION OF MOMENTUM

Momentum is the product of an object's mass and its velocity.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI unit
momentum	$\vec{p}$	$\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$ (kilogram metres per second)
mass	m	kg (kilograms)
velocity	$\vec{v}$	$\frac{\mathrm{m}}{\mathrm{s}}$ (metres per second)

Unit Analysis

$$(\text{mass})(\text{velocity}) = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}} = \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

Note: Momentum does not have a unique unit of its own.

MODEL PROBLEM

Momentum of a Hockey Puck

Determine the momentum of a 0.300 kg hockey puck travelling across the ice at a velocity of 5.55 m/s[N].

Frame the Problem

- The *mass* is *moving*; therefore, it has *momentum*.
- The *direction* of an object's *momentum* is the same as the *direction* of its *velocity*.

Identify the Goal

The momentum, $\vec{p}$, of the hockey puck

Variables and Constants

Known

$m = 0.300$ kg

$\vec{v} = 5.55\ \frac{\text{m}}{\text{s}}[\text{N}]$

Unknown

$\vec{p}$

Strategy

Use the equation that defines momentum.

Calculations

$$\vec{p} = m\vec{v}$$

$$\vec{p} = (0.300\ \text{kg})\left(5.55\ \frac{\text{m}}{\text{s}}[\text{N}]\right)$$

$$\vec{p} = 1.665\ \frac{\text{kg}\cdot\text{m}}{\text{s}}[\text{N}]$$

$$\vec{p} \cong 1.67\ \frac{\text{kg}\cdot\text{m}}{\text{s}}[\text{N}]$$

The momentum of the hockey puck was $1.67\ \frac{\text{kg}\cdot\text{m}}{\text{s}}[\text{N}]$.

Validate

Approximate the solution by multiplying 0.3 kg times 6 m/s. The magnitude of the momentum should be slightly less than this product, which is 1.8 kg · m/s. The value, 1.67 kg · m/s, fits the approximation very well. The direction of the momentum is always the same as the velocity of the object.

PRACTICE PROBLEM

29. Determine the momentum of the following objects.

(a) 0.250 kg baseball travelling at 46.1 m/s[E]

(b) 7.5×10^6 kg train travelling west at 125 km/h

(c) 4.00×10^5 kg jet travelling south at 755 km/h

(d) electron (9.11×10^{-31} kg) travelling north at 6.45×10^6 m/s

Math Link

In reality, Newton expressed his second law using the calculus that he invented. The procedure involves allowing the time interval to become smaller and smaller, until it becomes "infinitesimally small." The result allows you to find the instantaneous change in momentum at each instant in time. The formulation of Newton's second law using calculus looks like this.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Defining Impulse

Originally, Newton expressed his second law by stating that the change in an object's motion (rate of change of momentum) is proportional to the force impressed on it. Expressed mathematically, his second law can be written as follows.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

To show that this expression is fundamentally equivalent to the equation that you have learned in the past, take the following steps.

- Write the change in momentum as the difference of the final and initial momenta. $$\vec{F} = \frac{\vec{p}_{\mathrm{f}} - \vec{p}_{\mathrm{i}}}{\Delta t}$$
- Write momentum in terms of mass and velocity. $$\vec{F} = \frac{m\vec{v}_{\mathrm{f}} - m\vec{v}_{\mathrm{i}}}{\Delta t}$$
- If you assume that m is constant (that is, does not change for the duration of the time interval), you can factor out the mass, m. $$\vec{F} = \frac{m(\vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{i}})}{\Delta t}$$ $$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$
- Recall that the definition of average acceleration is the rate of change of velocity, and substitute an $\vec{a}$ into the above expression. $$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$ $$\vec{F} = m\vec{a}$$

Knowing that $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ is a valid expression of Newton's second law, you can mathematically rearrange the expression to demonstrate some very useful relationships involving momentum. When you multiply both sides of the equation by the time interval, you derive a new quantity, $\vec{F}\Delta t$, called "impulse."

$$\vec{F}\Delta t = \Delta\vec{p}$$

Impulse is the product of the force exerted on an object and the time interval over which the force acts, and is often given the symbol $\vec{J}$. Impulse is a vector quantity, and the direction of the impulse is the same as the direction of the force that causes it.

DEFINITION OF IMPULSE

Impulse is the product of force and the time interval.

$$\vec{J} = \vec{F}\Delta t$$

Quantity	Symbol	SI unit
impulse	$\vec{J}$	N · s (newton seconds)
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)

Unit Analysis

(impulse) = (force)(time interval) = N · s

Note: Impulse is equal to the change in momentum, which has units of $\frac{\text{kg} \cdot \text{m}}{\text{s}}$. To show that these units are equivalent to the N · s, express N in terms of the base units.

$$\text{N} \cdot \text{s} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{s} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

MISCONCEPTION

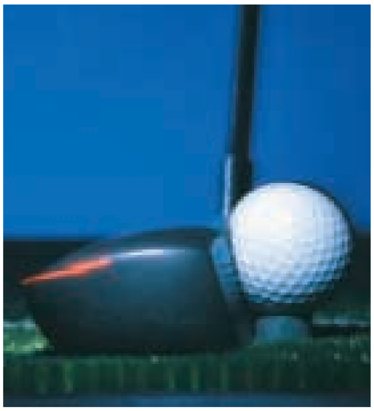
$\vec{F} = m\vec{a}$ Is Correct!

When students read the sentence "If you assume that *m* is constant (that is, does not change for the duration of the time interval), you can factor out the mass, *m*," they sometimes think that the result of the derivation, $\vec{F} = m\vec{a}$, is wrong. However, this equation is a special case of Newton's second law that is correct for all cases in which the mass, *m*, is constant. Since the mass is constant in a very large number of situations, it is acceptable to consider $\vec{F} = m\vec{a}$ as a valid statement of Newton's second law.

MODEL PROBLEM

Impulse on a Golf Ball

If a golf club exerts an average force of 5.25×10^3 N[W] on a golf ball over a time interval of 5.45×10^{-4} s, what is the impulse of the interaction?



Frame the Problem

- The golf club exerts an *average force* on the golf ball for a period of *time*. The product of these quantities is defined as *impulse*.
- Impulse is a *vector* quantity.
- The *direction* of the impulse is the same as the *direction of its average force*.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

Variables and Constants

Known	Unknown
$\vec{F} = 5.25 \times 10^3$ N[W]	$\vec{J}$
$\Delta t = 5.45 \times 10^{-4}$ s	

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Strategy

Apply the equation that defines impulse.

Calculations

$$\vec{J} = \vec{F}\Delta t$$
$$\vec{J} = (5.25 \times 10^3 \text{ N[W]})(5.45 \times 10^{-4} \text{ s})$$
$$\vec{J} = 2.8612 \text{ N} \cdot \text{s[W]}$$
$$\vec{J} \cong 2.86 \text{ N} \cdot \text{s[W]}$$

When the golf club strikes the golf ball, the impulse to drive the ball down the fairway is $2.86 \text{ N} \cdot \text{s[W]}$.

Validate

Round the values in the data to 5000 N[W] and 0.0006 s and do mental multiplication. The product is $3 \text{ N} \cdot \text{s[W]}$. The answer, $2.86 \text{ N} \cdot \text{s[W]}$, is very close to the estimate.

PROBLEM TIP

When you want to use mental math to approximate an answer to validate your calculations, you can usually find the best approximation by rounding one value up and the other value down before multiplying.

PRACTICE PROBLEMS

30. A sledgehammer strikes a spike with an average force of 2125 N[down] over a time interval of 0.0205 s. Calculate the impulse of the interaction.

31. In a crash test, a car strikes a wall with an average force of 1.23×10^7 N[S] over an interval of 21.0 ms. Calculate the impulse.

32. In a crash test similar to the one described in problem 31, another car, with the same mass and velocity as the first car, experiences an impulse identical to the value you calculated in problem 31. However, the second car was designed to crumple more slowly than the first. As a result, the duration of the interaction was 57.1 ms. Determine the average force exerted on the second car.

The Impulse-Momentum Theorem

You probably noticed that the sample and practice problems above always referred to "average force" and not simply to "force." Average force must be used to calculate impulse in these short, intense interactions, because the force changes continually throughout the few milliseconds of contact of the two objects. For example, when a golf club first contacts a golf ball, the force is very small. Within milliseconds, the force is great enough to deform the ball. The ball then begins to move and return to its original shape and the force soon drops back to zero. Figure 5.20 shows how the force changes with time. You could find the impulse by determining the area under the curve of force versus time.

In many collisions, it is exceedingly difficult to make the precise measurements of force and time that you need in order to calculate the impulse. The relationship between impulse and momentum provides an alternative approach to analyzing such collisions, as well as other interactions. By analyzing the momentum before and after an interaction between two objects, you can determine the impulse.

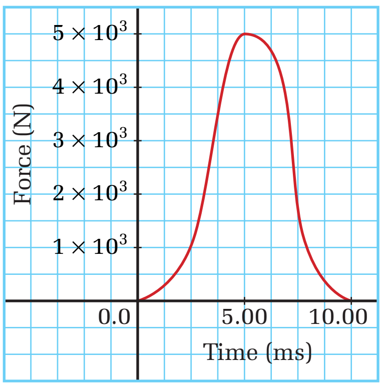

Figure 5.20 You can find the impulse of an interaction (area under the curve) by using the same mathematical methods that you used to find displacement from velocity-versus-time curves.

When you first rearranged the expression for Newton's second law, you focussed only on the concept of impulse, $\vec{F}\Delta t$. By taking another look at the equation $\vec{F}\Delta t = \Delta\vec{p}$, you can see that impulse is equal to the *change* in the momentum of an object. This relationship is called the **impulse-momentum theorem** and is often expressed as shown in the box below.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of momentum.

IMPULSE-MOMENTUM THEOREM

Impulse is the difference of the final momentum and initial momentum of an object involved in an interaction.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

Quantity	Symbol	SI unit
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)
mass	m	kg (kilograms)
initial velocity	$\vec{v}_1$	$\frac{\text{m}}{\text{s}}$ (metres per second)
final velocity	$\vec{v}_2$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

(force)(time interval) = (mass)(velocity)

$\text{N}\cdot\text{s} = \text{kg}\frac{\text{m}}{\text{s}}$ $\qquad \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\,\text{s} = \frac{\text{kg}\cdot\text{m}}{\text{s}}$

Note: Impulse is a vector quantity. The direction of the impulse is the same as the direction of the *change* in the momentum.

MODEL PROBLEM

Impulse and Average Force of a Tennis Ball

A student practises her tennis volleys by hitting a tennis ball against a wall.

(a) If the 0.060 kg ball travels 48 m/s before hitting the wall and then bounces directly backward at 35 m/s, what is the impulse of the interaction?

(b) If the duration of the interaction is 25 ms, what is the average force exerted on the ball by the wall?

Frame the Problem

- The *mass* and *velocities* before and after the interaction are known, so it is possible to calculate the *momentum* before and after the interaction.
- Momentum is a *vector* quantity, so all calculations must include *directions*.

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- Since the motion is all in *one dimension*, use plus and minus to denote direction. Let the *initial direction* be the *positive direction*.
- You can find the impulse from the *change* in momentum.

PROBLEM TIP

Whenever you use a result from one step in a problem as data for the next step, use the unrounded form of the data.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

The average force, $\vec{F}$, on the tennis ball

Variables and Constants

Known

$m = 0.060 \text{ kg}$

$\Delta t = 25 \text{ ms} = 0.025 \text{ s}$

$\vec{v}_1 = 48 \dfrac{\text{m}}{\text{s}}$

$\vec{v}_2 = -35 \dfrac{\text{m}}{\text{s}}$

Unknown

$\vec{J}$

$\vec{F}$

Strategy

Use the impulse-momentum theorem to calculate the impulse.

Calculations

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

$$\vec{F}\Delta t = 0.060 \text{ kg}\left(-35 \frac{\text{m}}{\text{s}}\right) - 0.060 \text{ kg}\left(48 \frac{\text{m}}{\text{s}}\right)$$

$$\vec{F}\Delta t = -2.1 \frac{\text{kg}\cdot\text{m}}{\text{s}} - 2.88 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\vec{F}\Delta t = -4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\vec{F}\Delta t \cong -5.0 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

(a) The impulse was 5.0 kg · m/s in a direction opposite to the initial direction of the motion of the ball.

Use the definition of impulse to find the average force.

$$\vec{F}\Delta t = -4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\vec{F} = \frac{-4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{\Delta t}$$

$$\vec{F} = \frac{-4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{0.025 \text{ s}}$$

$$\vec{F} = -199.2 \text{ N}$$

$$\vec{F} \cong -2.0 \times 10^2 \text{ N}$$

(b) The average force of the wall on the tennis ball was 2.0×10^2 N in the direction opposite to the initial direction of the ball.

Validate

Use an alternative mathematical technique for the impulse calculation by factoring out the mass, subtracting the velocities, then multiplying to see if you get the same answer.

$$\vec{F}\Delta t = m(\vec{v}_2 - \vec{v}_1)$$

$$\vec{F}\Delta t = 0.060 \text{ kg}\left(-35\frac{\text{m}}{\text{s}} - 48\frac{\text{m}}{\text{s}}\right)$$

$$\vec{F}\Delta t = (0.060 \text{ kg})\left(-83 \frac{\text{m}}{\text{s}}\right)$$

$$\vec{F}\Delta t = -4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}} \cong -5.0 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Check the units for the second part of the problem.

$$\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}}}{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

PRACTICE PROBLEMS

33. The velocity of the serve of some professional tennis players has been clocked at 43 m/s horizontally. (Hint: Assume that any vertical motion of the ball is negligible and consider only the horizontal direction of the ball after it was struck by the racquet.) If the mass of the ball was 0.060 kg, what was the impulse of the racquet on the ball?

34. A 0.35 kg baseball is travelling at 46 m/s toward the batter. After the batter hits the ball, it is travelling 62 m/s in the opposite direction. Calculate the impulse of the bat on the ball.

35. A student dropped a 1.5 kg book from a height of 1.75 m. Determine the impulse that the floor exerted on the book when the book hit the floor.

Impulse and Auto Safety

One of the most practical and important applications of impulse is in the design of automobiles and their safety equipment. When a car hits another car or a solid wall, little can be done to reduce the change in momentum. The mass of the car certainly does not change, while the velocity changes to zero at the moment of impact. Since you cannot reduce the change in momentum, you cannot reduce the impulse. However, since impulse ($\vec{F}\Delta t$) depends on both force and time, engineers have found ways to reduce the force exerted on car occupants by extending the time interval of the interaction. Think about how the design of a car can expand the duration of a crash.

In the early days of auto manufacturing, engineers and designers thought that a very strong, solid car would be ideal. As the number of cars on the road and the speed of the cars increased, the number and seriousness of accident injuries made it clear that the very sturdy cars were not protecting car occupants. By the late 1950s and early 1960s, engineers were designing cars with very rigid passenger cells that would not collapse onto the passengers, but with less rigid "crumple zones" in the front and rear, as shown in Figure 5.21.

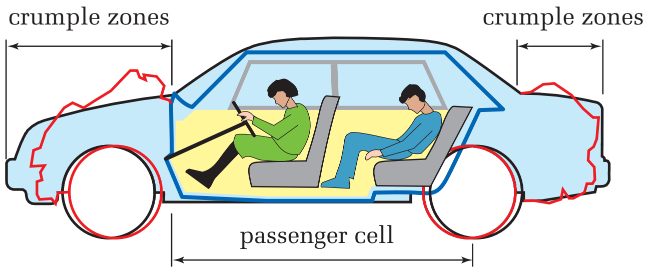

Figure 5.21 Although a car crash seems almost instantaneous, the time taken for the front or rear of the car to "crumple" is great enough to significantly reduce the average force of the impact and, therefore, the average force on the passenger cell and the passengers.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, visit the above Internet site and follow the links for an in-depth activity on impulse and momentum.

ELECTRONIC LEARNING PARTNER

Use the crash test provided by your Electronic Learning Partner to enhance your understanding of momentum.

QUICK LAB

Designing Crumple Zones

TARGET SKILLS

- Hypothesizing
- Performing and recording
- Analyzing and interpreting
- Communicating results

How soft is too soft and how rigid is too rigid for an effective vehicle crumple zone? In this lab, you will design and test several materials to determine the optimum conditions for passengers in a vehicle.

Obtain a rigid (preferably metal) toy vehicle to simulate the passenger cell of an automobile. The vehicle must have an open space in the centre for the "passenger." Make a passenger out of putty, modelling clay, or some material that will easily show "injuries" in the form of dents and deformations.

Design and build some type of device that will propel your vehicle rapidly into a solid wall (or stack of bricks) with nearly the same speed in all trials. The wall must be solid, but you will need to ensure that you do not damage the wall. Perform several crash tests with your vehicle and passenger and observe the types of injuries and the extent of injuries caused by the collision.

Select a variety of materials, from very soft to very hard, from which to build crumple zones. For example, you could use very soft foam rubber for the soft material. The thickness of each crumple zone must be approximately one third the length of your vehicle.

One at a time, attach your various crumple zones to your vehicle and test the effectiveness of the material in reducing the severity of injury to the passenger. Be sure that the vehicle travels at the same speed with the crumple zone attached as it did in the original crash tests without a crumple zone. Also, be sure that the materials you use to attach the crumple zones do not influence the performance of the crumple zones. Formulate an hypothesis about the relative effectiveness of each of the various crumple zones that you designed.

Analyze and Conclude

1. How do the injuries to the passenger that occurred with a very soft crumple zone compare to the injuries in the original crash tests?
2. How do the injuries to the passenger that occurred with a very rigid crumple zone compare to the injuries in the original crash tests?
3. Describe the difference in the passenger's injuries between the original crash tests and the test using the most effective crumple zone material.

Apply and Extend

4. The optimal crumple zone for a very massive car would be much more rigid than one for a small, lightweight car. However, a crash between a large and a small car would result in much greater damage to the small car. Write a paragraph responding to the question "Should car manufacturers consider other cars on the road when they design their own cars, or should they ignore what might happen to other manufacturers' cars?"
5. Crumple zones are just one of many types of safety systems designed for cars. Should the government regulate the incorporation of safety systems into cars? Give a rationale for your answer.
6. Some safety systems are very costly. Who should absorb the extra cost — the buyer, the manufacturer, or the government? For example, should the government provide a tax break or some other monetary incentive for manufacturers to build or consumers to buy cars with highly effective safety systems? Give a rationale for your answer.

When a rigid car hits a wall, a huge force stops the car almost instantaneously. The car might even look as though it was only slightly damaged. However, parts of the car, such as the steering wheel, windshield, or dashboard, exert an equally large force on the passengers, stopping them exceedingly rapidly and possibly causing very serious injuries.

When a car with well-designed crumple zones hits a wall, the force of the wall on the car causes the front of the car to collapse over a slightly longer time interval than it would in the absence of a crumple zone. Since $\vec{F}\Delta t$ is constant and Δt is larger, the average force, $\vec{F}$, is smaller than it would be for a rigid car. Although many other factors must be considered to reduce injury in collisions, the presence of crumple zones has had a significant effect in reducing the severity of injuries in automobile accidents.

The concept of increasing the duration of an impact applies to many forms of safety equipment. For example, the linings of safety helmets are designed to compress relatively slowly. If the lining was extremely soft, it would compress so rapidly that the hard outer layer of the helmet would impact on the head very quickly. If the lining did not compress at all, it would collide with the head over an extremely short time interval and cause serious injury. Each type of sport helmet is designed to compress in a way that compensates for the type of impacts expected in that sport.

Web Link

www.mcgrawhill.ca/links/atlphysics

To learn more about the design and testing of helmets and other safety equipment in sports, go to the above Internet site and click on **Web Links**.

5.4 Section Review

1. K/U Define momentum qualitatively and quantitatively.

2. K/U What assumption do you have to make in order to show that the two forms of Newton's second law ($\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ and $\vec{F} = m\vec{a}$) are equivalent?

3. I Try to imagine a situation in which the form $\vec{F} = m\vec{a}$ would not apply, but the form $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ could be used. Describe that situation. How could you test your prediction?

4. C State the impulse-momentum theorem and give one example of its use.

5. MC A bungee jumper jumps from a very high tower with bungee cords attached to his ankles. As he reaches the end of the bungee cord, it begins to stretch. The cord stretches for a relatively long period of time and then it recoils, pulling him back up. After several bounces, he dangles unhurt from the bungee cord (if he carried out the jump with all of the proper safety precautions). If he jumped from the same point with an ordinary rope attached to his ankles, he would be very severely injured. Use the concept of impulse to explain the difference in the results of a jump using a proper bungee cord and a jump using an ordinary rope.

UNIT PROJECT PREP

In your unit project, you will consider how the impulse-momentum theorem can be applied to collisions and explosions.

- How could you predict the magnitude of impulse that sent the *Mont Blanc's* anchor shaft and its cannon barrel blasting in opposite directions?