5.3 Reaction Forces and Newton's Third Law

The photograph in Figure 5.10 shows the lunar module lifting off the surface of the Moon during the Apollo 16 mission. At first glance, it seems impossible — the module is not aerodynamic. When you think about it, you realize that there is no atmosphere on the moon so there is no air friction. The module does not have to be aerodynamic. If there is no air, what generates lift for the module? Newton's second law states only that in order for an object to accelerate, something must be exerting a force on it. What is exerting a force on the lunar module? The answer lies in Newton's third law which can be stated, "For every action force, there is an equal and opposite reaction force." As the lunar module pushes its exhaust gases down, the exhaust gases push the module up.

Figure 5.10 Astronauts John Young and Charles Duke lift off the moon in the lunar module to dock with the command module and reunite with astronaut Thomas Mattingly for their trip back to Earth. When speaking about the lift off, John Young said, "... Charlie Duke and I are proof that you can hold your breath for 7 minutes and 15 seconds ...."

Newton's Third Law

For Newton's first and second laws, you focussed on individual objects and all of the forces acting on one specific object. The net force determined the change, or lack of change in the motion of that object. In your study of Newton's third law, you will consider the interactions between two objects. You will not even consider all of the forces acting on the two objects but instead concentrate on only the force involved in the interaction between those two objects.

Newton realized that every time an object exerted a force on a second object, that object exerted a force back on the first.

SECTION OUTCOMES

- Apply Newton's laws of motion.
- Use vectors to represent forces.
- Use instruments effectively and accurately for collecting data.

KEY TERMS

- apparent weight
- free fall
- air resistance
- terminal velocity

Web Link

www.mcgrawhill.ca/links/atlphysics

To see a video of the lunar lift off, go to the above Internet site and click on **Web Links**.

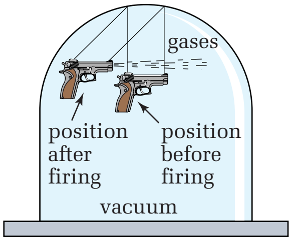
TRY THIS...

Perform each of the following.
- Stretch an elastic band between your hands.
- Gently push a toy across the lab bench.
- Push with all your might on a concrete wall.

Describe what you felt in each situation. Describe the forces that you applied and the forces that you felt applied to you. Draw free body diagrams for each object. Draw free body diagrams of one of your hands for each scenario. Make a general statement about action forces (the forces you applied) and reaction forces (the ones you felt).

PHYSICS FILE

An American physicist, Robert Goddard, published a paper in 1919 that suggested rockets could be used to attain altitudes higher than the atmosphere. Editors of the New York Times ridiculed Goddard, claiming a rocket would not work outside of the atmosphere.



Goddard used the demonstration pictured to show the editors their error.

NEWTON'S THIRD LAW

For every action force on object B due to object A, there is a reaction force, equal in magnitude but opposite in direction, due to object B acting back on object A.

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

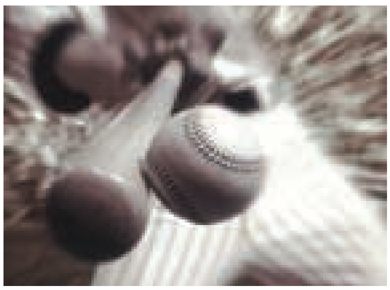
Newton's third law states that forces always act in pairs. An object cannot experience a force without also exerting an equal and opposite force. There are always *two forces* acting and *two objects* involved. To develop an understanding of Newton's third law, consider something as simple as walking across the room.

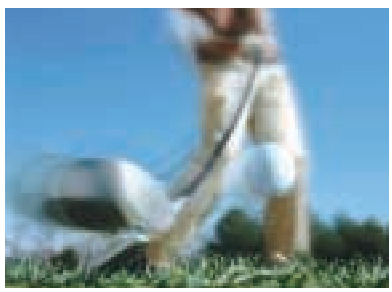
If someone asked you what force caused you to start moving across the room after standing still, you might say, "I push on the floor with my feet." Think about that statement. You *push* on the floor. According to Newton's second law, when you exert a force on an object, that object should move. So, by pushing on the floor, you should cause the floor to move. However, many other objects, such as the walls, the subfloor, and other structures are also pushing on the floor, making the sum of all of the forces equal to zero. Therefore, according to Newton's first law, the floor does not move. You cannot explain why you can walk across the room without calling on Newton's third law. According to Newton's third law, when you exert a frictional force on the floor, it exerts an equal and opposite frictional force on you. In reality, the floor pushes on you, propelling you across the floor as shown in Figure 5.11.

Figure 5.11 Newton's third law explains how the floor pushes you across the room.

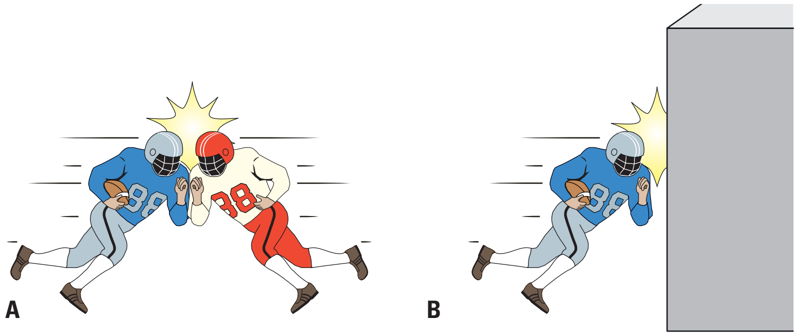
Conceptual Problems

- Draw force diagrams for each of the situations illustrated here.





- Look at the illustration of (A) two identical football players colliding and (B) one of the football players travelling with the exact same speed colliding with a wall. Use Newton's third law and compare the forces exerted on each player in both situations. Explain your answer.



- The famous horse-cart paradox asks, "If the cart is pulling on the horse with a force that is equal in magnitude and opposite in direction to the force that the horse is exerting on the cart, how can the horse make the cart move?" Discuss the answer with a classmate, then write a clear explanation of the paradox.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, go to the above Internet site and follow the links for a laboratory activity on Newton's third law.

Applying Newton's Third Law

Examine the photograph of the tractor-trailer in Figure 5.12 and think about all of the forces exerted on each of the three sections of the vehicle. Automotive engineers must know how much force each trailer hitch needs to withstand. Is the hitch holding the second trailer subjected to as great a force as the hitch that attaches the first trailer to the truck?

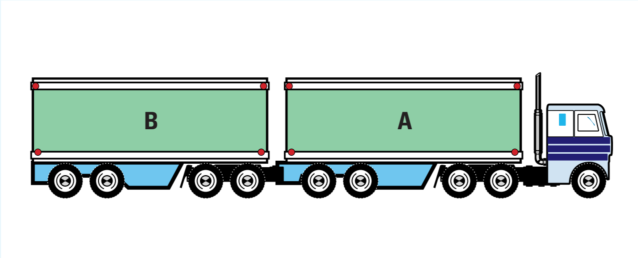
Figure 5.12 This truck and its two trailers move as one unit. The velocity and acceleration of each of the three sections are the same. However, each section is experiencing a different net force.

To analyze the individual forces acting on each part of a train of objects, you need to apply Newton's third law to determine the force that each section exerts on the adjacent section. Study the following model problem to learn how to determine all of the forces on the truck and on each trailer. These techniques will apply to any type of train problem in which the first of several sections of a moving set of objects is pulling all of the sections behind it.

MODEL PROBLEM

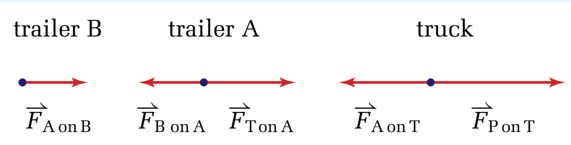
Forces on Connected Objects

A tractor-trailer pulling two trailers starts from rest and accelerates to a speed of 16.2 km/h in 15 s on a straight, level section of highway. The mass of the truck itself (T) is 5450 kg, the mass of the first trailer (A) is 31 500 kg, and the mass of the second trailer (B) is 19 600 kg. What magnitude of force must the truck generate in order to accelerate the entire vehicle? What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating?



Frame the Problem

- The truck engine generates energy to turn the wheels. When the wheels turn, they exert a *frictional force on the pavement*. According to Newton's third law, the *pavement* exerts a reaction *force* that is equal in magnitude and opposite in direction to the force exerted by the tires. The force of the pavement on the truck tires, $\vec{F}_{\text{P on T}}$, *accelerates* the entire system.
- The truck exerts a *force* on trailer A. According to Newton's third law, the trailer exerts a *force of equal magnitude* on the truck.
- Trailer A exerts a *force* on trailer B, and trailer B therefore must exert a force of equal magnitude on trailer A.
- Summarize all of the *forces* by drawing *free-body diagrams* of each section of the vehicle.



- The *kinematic equations* allow you to calculate the *acceleration* of the system.
- Since each section of the system has the *same acceleration*, this value, along with the masses and *Newton's second law*, lead to all of the *forces*.
- Since the motion is in a straight line, let the direction of motion be positive and the opposite direction be negative.

Identify the Goal

The force, $\vec{F}_{\text{P on T}}$, that the pavement exerts on the truck tires; the force, $\vec{F}_{\text{T on A}}$, that the truck exerts on trailer A; the force, $\vec{F}_{\text{A on B}}$, that trailer A exerts on trailer B

Variables and Constants

Known

$\vec{v}_f = 16.2\ \dfrac{km}{h}$ [forward]

$\Delta t = 15\ s$

$m_T = 5450\ kg$

$m_A = 31\ 500\ kg$

$m_B = 19\ 600\ kg$

Implied

$\vec{v}_i = 0\ \dfrac{km}{h}$

Unknown

$\vec{a}$ $\vec{F}_{T\ on\ A}$ $\vec{F}_{A\ on\ T}$ m_{total}

$\vec{F}_{P\ on\ T}$ $\vec{F}_{A\ on\ B}$ $\vec{F}_{B\ on\ A}$

Strategy

Calculations

Use the kinematic equation that relates the initial velocity, final velocity, time interval, and acceleration to find the acceleration.

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\vec{a} = \frac{\left(16.2\ \frac{\cancel{km}}{\cancel{h}} - 0\ \frac{\cancel{km}}{\cancel{h}}\right)\left(\frac{1\ \cancel{h}}{3600\ s}\right)\left(\frac{1000\ m}{1\ \cancel{km}}\right)}{15\ s}$$

$$\vec{a} = 0.30\ \frac{m}{s^2}$$

Find the total mass of the truck plus trailers.

$$m_{total} = m_T + m_A + m_B$$

$$m_{total} = 5450\ kg + 31\ 500\ kg + 19\ 600\ kg$$

$$m_{total} = 56\ 550\ kg$$

Use Newton's second law to find the force required to accelerate the total mass. This will be the force that the pavement must exert on the truck tires.

$$\vec{F} = m\vec{a}$$

$$\vec{F}_{P\ on\ T} = (56\ 550\ kg)\left(0.30\ \frac{m}{s^2}\right)$$

$$\vec{F}_{P\ on\ T} = 16\ 965\ \frac{kg \cdot m}{s^2}$$

$$\vec{F}_{P\ on\ T} \cong 1.7 \times 10^4\ N$$

The pavement exerts 1.7×10^4 N on the truck tires.

Use Newton's second law to find the force necessary to accelerate trailer B at 0.30 m/s^2. This is the force that the second trailer hitch must withstand.

$$\vec{F}_{A\ on\ B} = m_B\vec{a}$$

$$\vec{F}_{A\ on\ B} = (19\ 600\ kg)\left(0.30\ \frac{m}{s^2}\right)$$

$$\vec{F}_{A\ on\ B} = 5.88 \times 10^3\ \frac{kg \cdot m}{s^2}$$

$$\vec{F}_{A\ on\ B} \cong 5.9 \times 10^3\ N$$

The force that the second hitch must withstand is 5.9×10^3 N.

Use Newton's second law to find the total force necessary to accelerate trailer A at 0.30 m/s^2.

$$\vec{F}_{total\ on\ A} = m_A\vec{a}$$

$$\vec{F}_{total\ on\ A} = (31\ 500\ kg)\left(0.30\ \frac{m}{s^2}\right)$$

$$\vec{F}_{total\ on\ A} = 9.45 \times 10^3\ \frac{kg \cdot m}{s^2}$$

$$\vec{F}_{total\ on\ A} \cong 9.5 \times 10^3\ N$$

Use the free-body diagram to help write the expression for total (horizontal) force on trailer A.

$$\vec{F}_{total} = \vec{F}_{T\ on\ A} + \vec{F}_{B\ on\ A}$$

continued ▶

continued from previous page

The force that the first hitch must withstand is the force that the truck exerts on trailer A. Solve the force equation above for $\vec{F}_{\text{T on A}}$ and calculate the value. According to Newton's third law, $\vec{F}_{\text{B on A}} = -\vec{F}_{\text{A on B}}$.

$$\vec{F}_{\text{T on A}} = \vec{F}_{\text{total on A}} - \vec{F}_{\text{B on A}}$$
$$\vec{F}_{\text{T on A}} = 9.45 \times 10^3 \text{ N} - (-5.88 \times 10^3 \text{ N})$$
$$\vec{F}_{\text{T on A}} = 1.533 \times 10^4 \text{ N}$$
$$\vec{F}_{\text{T on A}} \cong 1.5 \times 10^4 \text{ N}$$

The force that the first hitch must withstand is 1.5×10^4 N.

Validate

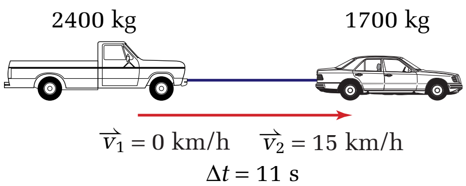
You would expect that $\vec{F}_{\text{P on T}} > \vec{F}_{\text{T on A}} > \vec{F}_{\text{A on B}}$. The calculated forces agree with this relationship. You would also expect that the force exerted by the tractor on trailer A would be the force necessary to accelerate the sum of the masses of trailers A and B at 0.30 m/s^2.

$$\vec{F}_{\text{T on A}} = (31\ 500 \text{ kg} + 19\ 600 \text{ kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right) = 15\ 330 \text{ N} \cong 1.5 \times 10^4 \text{ N}$$

This value agrees with the value above.

PRACTICE PROBLEMS

18. A 1700 kg car is towing a larger vehicle with mass 2400 kg. The two vehicles accelerate uniformly from a stoplight, reaching a speed of 15 km/h in 11 s. Find the force needed to accelerate the connected vehicles, as well as the minimum strength of the rope between them.



19. An ice skater pulls three small children, one behind the other, with masses 25 kg, 31 kg, and 35 kg. Assume that the ice is smooth enough to be considered frictionless.

(a) Find the total force applied to the "train" of children if they reach a speed of 3.5 m/s in 15 s.

(b) If the skater is holding onto the 25 kg child, find the tension in the arms of the next child in line.

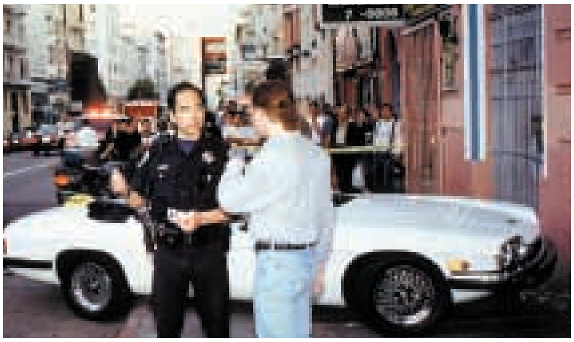
20. A solo Arctic adventurer pulls a string of two toboggans of supplies across level, snowy ground. The toboggans have masses of 95 kg and 55 kg. Applying a force of 165 N causes the toboggans to accelerate at 0.61 m/s^2.

(a) Calculate the frictional force acting on the toboggans.

(b) Find the tension in the rope attached to the second (55 kg) toboggan.

CAREERS IN PHYSICS

The Physics of a Car Crash



A car accident has occurred at a busy intersection. A passenger in one of the cars is seriously injured and both vehicles are extensively damaged.

One of the drivers says she was stopped at a green light, waiting to make a left-hand turn, when an oncoming car swerved and hit her. The driver of the other car says he hit her car because she began to turn as he was passing through the intersection. The police officers investigating the accident also hear conflicting stories from witnesses. Nobody seems to know exactly what happened. It is time to bring in an accident investigator with expertise in physics and motion to determine how the crash actually occurred.

Accident investigators use the principles of physics, such as work and the conservation of energy, to determine the cause of car accidents. The investigators consider a number of factors and make detailed measurements at the scene of an accident. They might consider road conditions, damage to vehicles, the pre- and post-accident positions of vehicles, and vehicle characteristics such as weight and size. An investigator might be asked to determine the speed of each vehicle on impact by considering their masses and the distance they travelled after impact. For example, in the accident described above, the investigator would need to determine if the driver of the car in the left-turn lane was, in fact, stationary at the time of impact.

Several different career options are available for accident investigators. Police officers with specialized training are involved in accident investigation and reconstruction, while other investigators are consultants hired on a contractual basis by police departments, insurance companies, and individual citizens. Still others might be full-time employees of insurance companies or legal firms. Accident investigators are also often called on to serve as expert witnesses in criminal or civil law cases.

The training required to become an accident investigator varies. Some investigators have earned degrees in civil or traffic engineering. An excellent understanding of physics and the ability to perform detailed tasks accurately and without bias are important requirements. A knowledge of computers is becoming increasingly important as collision analysis becomes more computerized.

For information about where and how law enforcement accident investigators receive their training, contact the Community Services branch of a municipal, provincial, or federal law enforcement agency in your area.

Going Further

1. Investigators are not always able to make actual measurements at the scene of an accident. Often they must reconstruct a scene or determine the cause of an accident from photographs and reports alone. List the factors that an investigator would need to know about the accident scenario just described.
2. How could an investigator determine these factors?

Web Link

www.mcgrawhill.ca/links/atlphysics

In addition to motor vehicle accidents, some investigators work on air, marine, and rail accident cases. For example, investigators for the Canadian Transportation Safety Board worked on the Swissair Flight 111 disaster near Peggy's Cove in Nova Scotia. To learn about this investigation, go to the above Internet site and click on **Web Links** to find out where to go next.

Weight versus Apparent Weight

You experience some strange sensations when an elevator begins to rise or descend or when it slows and comes to a stop. For example, if you get on at the first floor and start to go up, you feel heavier for a moment. In fact, if you are carrying a book bag or a suitcase, it feels heavier, too. When the elevator slows and eventually stops, you and anything you are carrying feels lighter. When the elevator is moving at a constant velocity, however, you feel normal. Are these just sensations that living organisms feel or, if you were standing on a scale in the elevator, would the scale indicate that you *were* heavier? You can answer that question by applying Newton's laws of motion to a person riding in an elevator. You will see that Newton's third law is a critical part of your analysis.

Imagine that you are standing on a scale in an elevator, as shown in Figure 5.13. When the elevator is standing still, the reading on the scale is your weight. Recall that your weight is the force of gravity acting on your mass. Your weight can be calculated by using the equation $F_g = mg$, where g is the acceleration due to gravity. Vector notations are sometimes omitted because the force due to gravity is always directed toward the centre of Earth. Find out what happens to the reading on the scale by studying the following model problem.

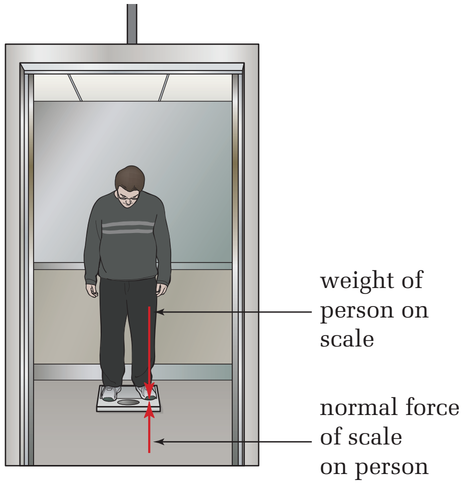


Figure 5.13 When you are standing on a scale, you exert a force on the scale. According to Newton's third law, the scale must exert an equal and opposite force on you. Therefore, the reading on the scale is equal to the force that you exert on it.

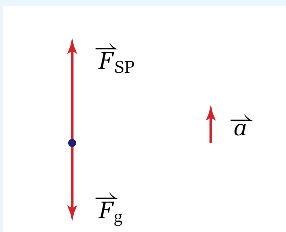
MODEL PROBLEM

Apparent Weight

A 55 kg person is standing on a scale in an elevator. If the scale is calibrated in newtons, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at 0.75 m/s^2, what will be the reading on the scale?

Frame the Problem

- Draw a *free body diagram* of the person on the scale. A free body diagram includes all of the *forces acting on the person*.
- The *forces* acting on the person are *gravity* ($\vec{F}_g$) and the *normal force* of the scale.
- According to *Newton's third law*, when the person exerts a *force* ($\vec{F}_{PS}$) on the scale, it exerts an *equal and opposite force* ($\vec{F}_{SP}$) on the person. Therefore, the reading on the scale is the same as the force that the person exerts on the scale.



- When the elevator is *standing still*, the person's *acceleration is zero*.
- When the elevator begins to *rise*, the person is *accelerating* at the same rate as the elevator.
- Since the motion is in one dimension, use only positive and negative signs to indicate direction. Let "*up*" *be positive* and "*down*" *be negative*.
- Apply *Newton's second law* to find the magnitude of $\vec{F}_{SP}$.
- According to *Newton's third law*, the magnitudes of $\vec{F}_{PS}$ and $\vec{F}_{SP}$ are equal to each other, and therefore to the reading on the scale.

Identify the Goal

The reading on the scale, $|\vec{F}_{SP}|$, when the elevator is standing still and when it is accelerating upward

Variables and Constants

Known	Implied	Unknown
$m = 55\ \text{kg}$	$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$	$\vec{F}_{PS}$ $\quad\vec{F}_{SP}$
$\vec{a} = +0.75\ \dfrac{\text{m}}{\text{s}^2}$		$\vec{F}_g$

Strategy

Calculations

Apply Newton's second law and solve for the force that the scale exerts on the person.
The force in Newton's second law is the vector sum of all of the forces acting *on* the person.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$

In the first part of the problem, the acceleration is zero.

$$\vec{F}_{SP} = (55\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + 0$$
$$\vec{F}_{SP} = 539.55\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_{SP} \cong 5.4 \times 10^2\ \text{N}$$

When the elevator is not moving, the reading on the scale is 5.4×10^2 N, which is the person's weight.

Apply Newton's second law to the case in which the elevator is accelerating upward. The acceleration is positive.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$
$$\vec{F}_{SP} = (55\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (55\ \text{kg})\left(+0.75\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_{SP} = 580.8\ \text{N}$$
$$\vec{F}_{SP} \cong 5.8 \times 10^2\ \text{N[up]}$$

When the elevator is accelerating upward, the reading on the scale is 5.8×10^2 N.

continued ►

continued from previous page

Validate

When an elevator first starts moving upward, it must exert a force that is greater than the person's weight so that, as well as supporting the person, an additional force causes the person to accelerate. The reading on the scale should reflect this larger force. It does. The acceleration of the elevator was small, so you would expect that the reading on the scale would not increase by a large amount. It increased by only about 7%.

PRACTICE PROBLEMS

21. A 64 kg person is standing on a scale in an elevator. The elevator is rising at a constant velocity but then begins to slow, with an acceleration of 0.59 m/s^2. What is the sign of the acceleration? What is the reading on the scale while the elevator is accelerating?

22. A 75 kg man is standing on a scale in an elevator when the elevator begins to descend with an acceleration of 0.66 m/s^2. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

23. A 549 N woman is standing on a scale in an elevator that is going down at a constant velocity. Then, the elevator begins to slow and eventually comes to a stop. The magnitude of the acceleration is 0.73 m/s^2. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

Figure 5.14 When you are on a free-fall amusement park ride, you feel weightless.

As you saw in the problems, when you are standing on a scale in an elevator that is accelerating, the reading on the scale is not the same as your true weight. This reading is called your **apparent weight**.

When the direction of the acceleration of the elevator is positive — it starts to ascend or stops while descending — your apparent weight is greater than your true weight. You feel heavier because the floor of the elevator is pushing on you with a greater force than it is when the elevator is stationary or moving with a constant velocity.

When the direction of the acceleration is negative — when the elevator is rising and slows to a stop or begins to descend — your apparent weight is smaller than your true weight. The floor of the elevator is exerting a force on you that is smaller than your weight, so you feel lighter.

Free Fall

Have you ever dared to take an amusement park ride that lets you fall with almost no support for a short time? A roller coaster as it drops from a high point in its track can bring you close to the same feeling of **free fall**, a condition in which gravity is the only force acting on you. To investigate free fall quantitatively, imagine, once again, that you are standing on a scale in an elevator. If the cable was to break, there were no safety devices, and friction was negligible, what would be your apparent weight?

If gravity is the only force acting on the elevator, it will accelerate downward at the acceleration due to gravity, or g. Substitute this value into Newton's second law and solve for your apparent weight.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Let "up" be positive and "down" be negative. The total force acting on you is the downward force of gravity and the upward normal force of the scale. Your acceleration is g downward. $F_N + F_g = -mg$
- The force of gravity is $-mg$. $F_N - mg = -mg$
- Solve for the normal force. $F_N = mg - mg$
  $F_N = 0$

The reading on the scale is zero. Your apparent weight is zero. This condition is often called "weightlessness." Your mass has not changed, but you feel weightless because nothing is pushing up on you, preventing you from accelerating at the acceleration due to gravity.

Conceptual Problem

- How would a person on a scale in a freely falling elevator analyze the forces that were acting? Make a free-body analysis similar to the one in the sample problem (Apparent Weight) on page 184, using the elevator as your frame of reference. Consider these points.
  (a) To an observer in the elevator, the person on the scale would not appear to be moving.
  (b) The reading on the scale (the normal force) would be zero.

Close to Earth's surface, weightlessness is rarely experienced, due to the resistance of the atmosphere. As an object collides with molecules of the gases and particles in the air, the collisions act as a force opposing the force of gravity. **Air resistance** or air friction is quite different from the surface friction that you have studied. When an object moves through a fluid such as air, the force of friction increases as the velocity of the object increases.

A falling object eventually reaches a velocity at which the force of friction is equal to the force of gravity. At that point, the net force acting on the object is zero and it no longer accelerates but maintains a constant velocity called **terminal velocity**. The shape and orientation of an object affects its terminal velocity.

PHYSICS FILE

In 1942, Soviet air force pilot I. M. Chisov was forced to parachute from a height of almost 6700 m. To escape being shot by enemy fighters, Chisov started to free fall, but soon lost consciousness and never opened his parachute. Air resistance slowed his descent, so he probably hit the ground at about 193 km/h, plowing through a metre of snow as he skidded down the side of a steep ravine. Amazingly, Chisov survived with relatively minor injuries and returned to work in less than four months.

Technology Link

Air resistance is of great concern to vehicle designers, who can increase fuel efficiency by using body shapes that reduce the amount of air friction or drag that is slowing the vehicle. Athletes such as racing cyclists and speed skaters use body position and specially designed clothing to minimize drag and gain a competitive advantage. Advanced computer hardware and modelling software are making computerized simulations of air resistance a practical alternative to traditional experimental studies using scale models in wind tunnels.

For example, skydivers control their velocity by their position, as illustrated in Figure 5.15. Table 5.1 lists the approximate terminal velocities for some common objects.

Figure 5.15 Gravity is not the only force affecting these skydivers, who have become experts at manipulating air friction and controlling their descent.

Table 5.1 Approximate Terminal Velocities

Object	Terminal velocity (m/s downward)
large feather	0.4
fluffy snowflake	1
parachutist (chute open)	7
penny	9
skydiver (spread-eagled)	58

Inclined Planes

When you watch speed skiers, it appears as though there is no limit to the rate at which they can accelerate. In reality, their acceleration is always less that that of a free-falling object, because the skier is being accelerated by only a component of the force of gravity and not by the total force. Using the principles of dynamics and the forces affecting the motion, you can predict details of motion along an inclined plane.

Figure 5.16 Gravitational forces acting on downhill skiers have produced speeds greater than 241 km/h, even though only part of the total gravitational force accelerates a skier.

Choosing a Coordinate System for an Incline

The key to analyzing the dynamics and motion of objects on an inclined plane is choosing a coordinate system that simplifies the procedure. Since all of the motion is along the plane, it is convenient to place the *x*-axis of the coordinate system parallel to the plane, making the *y*-axis perpendicular to the plane, as shown in Figure 5.17.

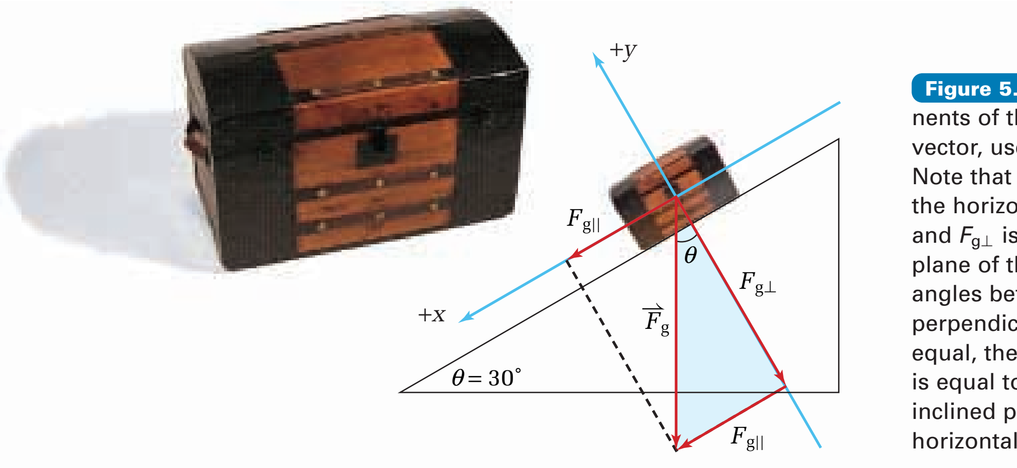


Figure 5.17 To find the components of the gravitational force vector, use the shaded triangle. Note that $\vec{F}_g$ is perpendicular to the horizontal line at the bottom and $F_{g\perp}$ is perpendicular to the plane of the ramp. Since the angles between two sets of perpendicular lines must be equal, the angle (θ) in the triangle is equal to the angle that the inclined plane makes with the horizontal.

The force of gravity affects motion on inclined planes, but the force vector is at an angle to the plane. Therefore, you must resolve the gravitational force vector into components parallel to and perpendicular to the plane, as shown in Figure 5.17. The component of force parallel to the plane influences the acceleration of the object and the perpendicular component affects the magnitude of the friction. Since several forces in addition to the gravitational force can affect the motion on an inclined plane, free-body diagrams are essential in solving problems, as shown in the model problem below.

MODEL PROBLEM

Sliding Down an Inclined Plane

You are holding an 85 kg trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

(a) If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?

(b) If the trunk slides 1.3 m before reaching the bottom of the ramp, for what time interval did it slide?

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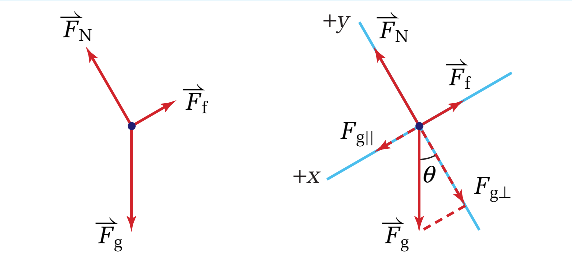
Frame the Problem

- Draw a *free-body diagram*.
- Beside the free-body diagram, draw a coordinate system with the *x-axis parallel* to the ramp. On the coordinate system, draw the forces and *components of forces* acting on the trunk.



- Let the direction pointing *down* the slope be the *positive* direction.
- To find the *normal force* that is needed to determine the magnitude of the *frictional force*, apply *Newton's second law* to the forces or components of forces that are *perpendicular* to the ramp.
- The *acceleration perpendicular* to the ramp is *zero*.
- The *component of gravity parallel* to the trunk causes the trunk to *accelerate* down the ramp.
- *Friction* between the trunk and the ramp *opposes* the motion.
- If the *net force* along the ramp is *positive*, the trunk will *accelerate* down the ramp.
- To find the *acceleration* of the trunk down the ramp, apply *Newton's second law* to the forces or components of forces *parallel* to the ramp.
- Given the *acceleration* of the trunk, you can use the *kinematic equations* to find *other quantities* of motion.

Identify the Goal

(a) The acceleration, $a_{\|}$, of the trunk along the ramp

(b) The time interval, Δt, for the trunk to reach the end of the ramp

Variables and Constants

Known		Implied	Unknown		
$m = 85$ kg	$\theta = 35^\circ$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	$\vec{F}_f$	v_f
$\mu = 0.42$	$\Delta d = 1.3$ m	$v_i = 0$	$F_{g\|}$	$\vec{F}_N$	$a_{\|}$
		$a_{\perp} = 0$	$F_{g\perp}$		

Strategy

Calculations

Apply Newton's second law to the forces perpendicular to the ramp. Refer to the diagram to find all of the forces that are perpendicular to the ramp. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_N + F_{g\perp} = ma_{\perp}$$
$$F_N - mg\cos\theta = ma_{\perp}$$
$$F_N = mg\cos\theta + ma_{\perp}$$

Insert values and solve. Note that the acceleration perpendicular to the ramp ($a_{\perp}$) is zero.

$$F_N = (85\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 35^\circ + 0$$
$$F_N = 683.05\text{ N}$$

Apply Newton's second law to the forces parallel to the ramp. Refer to the diagram to find all of the forces that are parallel to the ramp. Solve for the acceleration parallel to the ramp.

$$\vec{F} = m\vec{a}$$
$$F_{g\|} + F_f = ma_{\|}$$
$$F_f = \mu F_N \text{ in negative direction}$$
$$mg\sin\theta - \mu F_N = ma_{\|}$$
$$a_{\|} = \frac{mg\sin\theta - \mu F_N}{m}$$

Insert values and solve.

$$a_{||} = \frac{(85\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 35^\circ - (0.42)(683.05\text{ N})}{85\text{ kg}}$$

$$a_{||} = 2.251\ 71\ \frac{\text{m}}{\text{s}^2}$$

$$a_{||} \cong 2.3\ \frac{\text{m}}{\text{s}^2}$$

(a) The acceleration of the trunk down the ramp is 2.3 m/s^2.

Apply the kinematic equation that relates displacement, acceleration, initial velocity, and time interval. Given that the initial velocity was zero, solve the equation for the time interval.

$$\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta t^2 = \frac{2\Delta d}{a}$$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Insert values and solve.

$$\Delta t = \sqrt{\frac{2(1.3\text{ m})}{2.251\ 71\ \frac{\text{m}}{\text{s}^2}}}$$

$$\Delta t = \pm 1.075\text{ s}$$

$$\Delta t \cong 1.1\text{ s}$$

(b) The trunk slid for 1.1 s before reaching the end of the ramp. Notice that the positive value for time interval was chosen. In this application, a negative time interval has no meaning.

Validate

(a) Since the ramp is not at an extremely steep slope and since there is a significant amount of friction, you would expect that the acceleration would be much smaller than 9.81 m/s^2, which it is.

(b) The ramp is very short, so you would expect that it would not take long for the trunk to reach the bottom of the ramp. A time of 1.1 s is quite reasonable.

PRACTICE PROBLEMS

24. A 1975 kg car is parked at the top of a steep 42 m long hill inclined at an angle of 15°. If the car starts rolling down the hill, how fast will it be going when it reaches the bottom of the hill? (Neglect friction.)

25. Starting from rest, a cyclist coasts down the starting ramp at a professional biking track. If the ramp has the minimum legal dimensions (1.5 m high and 12 m long), find

(a) the acceleration of the cyclist, ignoring friction

(b) the acceleration of the cyclist if all sources of friction yield an effective coefficient of friction of $\mu = 0.11$

(c) the time taken to reach the bottom of the ramp, if friction acts as in (b)

26. A skier coasts down a 3.5° slope at constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope.

QUICK LAB

The Slippery Slope

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting

You can determine the coefficients of static and kinetic friction experimentally. Use a coin or small block of wood as the object and a textbook as a ramp. Find the mass of the object. Experiment to find the maximum angle of inclination possible before the object begins to slide down the ramp (θ_1). Then, use a slightly greater angle (θ_2), so that the object slides down the ramp. Make appropriate measurements of displacement and time, so that you can calculate the average acceleration. If the distance is too short to make accurate timings, use a longer ramp, such as a length of smooth wood or metal.

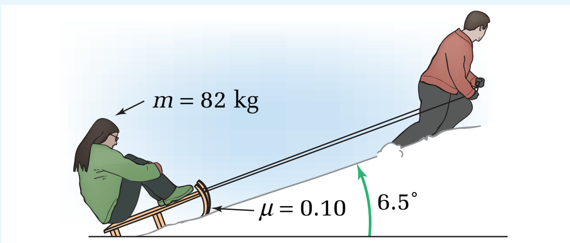
Analyze and Conclude

1. Calculate the gravitational force on the object (weight). Resolve the gravitational force into parallel and perpendicular components, for angles θ_1 and θ_2.
2. Draw a free-body diagram of the forces acting on the object and use it to find the magnitude of all forces acting on the object just before it started to slide (at angle θ_1). **Note:** If the object is not accelerating, no net force is acting on it, so every force must be balanced by an equal and opposite force.
3. Calculate the coefficient of static friction, μ_s, between the object and the ramp, using your answer to question 2.
4. Use the data you collected when the ramp was inclined at θ_2 to calculate the acceleration of the object. Find the net force necessary to cause this acceleration.
5. Use the net force and the parallel component of the object's weight to find the force of friction between the object and the ramp.
6. Calculate the coefficient of kinetic friction, μ_k, between the object and the ramp.
7. Compare μ_s and μ_k. Are they in the expected relationship to each other? How well do your experimental values agree with standard values for the materials that you used for your object and ramp? (Obtain coefficients of friction from reference materials.)

MODEL PROBLEM

Pushing or Pulling an Object Up an Incline

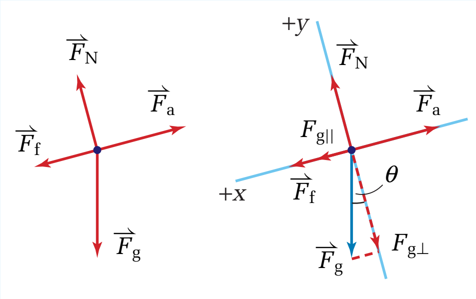
You are pulling a sled and rider with combined mass of 82 kg up a 6.5° slope at a steady speed. If the coefficient of kinetic friction between the sled and snow is 0.10, what is the tension in the rope?



Frame the Problem

- Sketch a *free-body diagram* of the forces acting on the sled. Beside it, sketch the *components of the forces* that are *parallel* and *perpendicular* to the slope.

- Since the sled is moving at a *constant velocity*, the *acceleration is zero*.
- The *parallel component* of the sled's weight and the *force of friction* are acting *down* the slope (*positive* direction).
- The *applied force* of the rope acts up the slope on the sled (*negative* direction).
- The *tension* in the rope is the magnitude of the *force* that the *rope exerts* on the sled.
- *Newton's second law* applies independently to the forces perpendicular and parallel to the slope.



Identify the Goal

The magnitude of the tension, $|\vec{F}_a|$, in the rope

Variables and Constants

Known	Implied	Unknown		
$m = 82$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	$F_{g\|\|}$	$F_{g\perp}$
$\mu = 0.10$	$a_{\|\|} = 0\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_N$	$\vec{F}_f$	$\vec{F}_a$
$\theta = 6.5°$				
v = constant				

Strategy

Calculations

Apply Newton's second law to the forces perpendicular to the slope. Refer to the diagram to find all of the forces that are perpendicular to the slope. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_N + F_{g\perp} = ma_\perp$$
$$F_N - mg\cos\theta = ma_\perp$$
$$F_N = mg\cos\theta + ma_\perp$$

Insert values and solve. Note that the acceleration perpendicular to the slope ($a_\perp$) is zero.

$$F_N = (82\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 6.5° + 0$$
$$F_N = 799.25\ \text{N}$$

Apply Newton's second law to the forces parallel to the slope. Refer to the diagram to find all of the forces that are parallel to the slope. Solve for the force that the rope exerts on the sled.

$$\vec{F} = m\vec{a}$$
$$F_f + F_a + F_{g||} = ma_{||}$$
$$\mu F_N + F_a + mg\sin\theta = ma_{||}$$
$$F_a = ma_{||} - \mu F_N - mg\sin\theta$$

Insert values and solve.

$$F_a = (82\ \text{kg})\left(0\ \frac{\text{m}}{\text{s}^2}\right) - (0.10)(799.25\ \text{N}) - (82\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 6.5°$$
$$F_a = -79.925\ \text{N} - 91.063\ \text{N}$$
$$F_a = -170.988\ \text{N}$$
$$|\vec{F}_a| \cong 1.7 \times 10^2\ \text{N}$$

The tension force in the rope is about 1.7×10^2 N.

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Validate

The tension is much less than the force of gravity on the sled, since most of the weight of the sled is being supported by the ground. The tension is also greater than the parallel component of the sled's weight, because the rope must balance both the force of friction and the component of the force of gravity parallel to the slope.

PRACTICE PROBLEMS

27. You flick a 5.5 g coin up a smooth board propped at an angle of 25° to the floor. If the initial velocity of the coin is 2.3 m/s up the board and the coefficient of kinetic friction between the coin and the board is 0.40, how far does the coin travel before stopping?

28. You are pushing a 53 kg crate at a constant velocity up a ramp onto a truck. The ramp makes an angle of 22° with the horizontal. If your applied force is 373 N, what is the coefficient of friction between the crate and the ramp?

5.3 Section Review

1. K/U State Newton's third law in words and in mathematical symbols.

2. K/U State the equal-and-opposite force pairs in each of the following situations:

(a) kicking a soccer ball

(b) a pencil resting on a desk

(c) stretching an elastic band

3. K/U Apply Newton's third law to each situation in order to determine the reaction force, magnitude, and direction.

(a) A soccer ball is kicked with 85 N[W].

(b) A bulldozer pushes a concrete slab directly south with 45 000 N of force.

(c) A 450 N physics student stands on the floor.

4. C Three identical blocks, fastened together by a string, are pulled across a frictionless surface by a constant force, *F*.

(a) Compare the tension in string A to the magnitude of the applied force, *F*.

(b) Draw a free-body diagram of the forces acting on block 2.

5. K/U Explain why your apparent weight is sometimes not the same as your true weight.

6. C Suppose you are standing on a scale in a moving elevator and notice that the scale reading is *less* than your true weight.

(a) Draw a free-body diagram to represent the forces acting on you.

(b) Describe the elevator motion that would produce the effect.

7. C Describe a situation in which you could be standing on a scale and the reading on the scale would be zero. (**Note:** The scale is functioning properly and is accurate.) What is the name of this condition?

8. K/U Sketch a free-body diagram and an additional diagram showing the parallel and perpendicular components of gravitational force acting on an object on a ramp inclined at an angle of θ to the horizontal. State the equation used to calculate each force component.

9. K/U Which component of gravitational force affects each of the following?

(a) acceleration down a frictionless incline

(b) the force of friction acting on an object on a ramp

(c) the tension in a rope holding the object motionless

(d) the tension in a rope pulling the object up the ramp