5.2 Motion and Newton's Second Law

SECTION OUTCOMES

- Apply Newton's laws of motion to explain the relationships among force, mass, and acceleration.
- Carry out procedures controlling the major variables.
- Interpret patterns and trends in data and infer or calculate linear relationships.

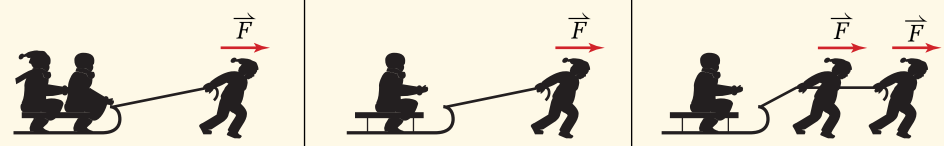
The deafening roar of the engine of a competitor's tractor conveys the magnitude of the force that is applied to the sled in a tractor-pull contest. As the sled begins to move, weights shift to increase frictional forces. Despite the power of their engines, most tractors are slowed to a standstill before reaching the end of the 91 m track. In contrast to the brute strength of the tractors, dragsters "sprint" to the finish line.

The tires of the tractor in the photograph below are probably exerting a much greater force on the ground than are the tires of the dragster. Nevertheless, the dragster is accelerating to much greater speeds than the tractor. Newton's first law only tells you that a net force will cause a change in the motion of an object, but it does not tell you exactly what that change will be. Newton's second law, on the other hand, will allow you not only to explain why there is such a difference between the motion of the tractor and the dragster, but it will allow you to make exact predictions about their motion.

Figure 5.6 In a tractor pull, vehicles develop up to 9000 horsepower to accelerate a sled, until they can no longer overcome the constantly increasing frictional forces. Dragsters, on the other hand, accelerate right up to the finish line.

Newton's Second Law

Imagine three young friends playing in the snow with a toboggan on flat ground. Assume that (a) each friend has the same mass, m, (b) each friend can pull with the same force, $\vec{F}$, (c) the toboggan's mass is so small that it can be ignored, and (d) the toboggan glides so easily on top of the snow that friction can be ignored. The friends take turns pulling and being pulled on the toboggan. A comparison of the resulting accelerations is shown in the chart below. Look for a pattern that relates force, mass, and acceleration.



Net force	$\lvert\vec{F}\rvert$	$\lvert\vec{F}\rvert$	$2\lvert\vec{F}\rvert$
Mass on toboggan	$2m$	m	m
Acceleration	$\frac{1}{2}\lvert\vec{a}\rvert$	$\lvert\vec{a}\rvert$	$2\lvert\vec{a}\rvert$

Newton observed motions as simple as those of the toboggan described above and as complex as the motion of planets around the sun. From these observations, he developed his second law of motion. As you may have deduced from the data in the chart above, when a net force, $\vec{F}$, acts on a mass, m, the resulting acceleration of the mass, $\vec{a}$, is proportional to the magnitude of the force and inversely proportional to the amount of mass. The direction of the acceleration is the same as that of the net force. The form of the law described here and the more familiar form are detailed in the box on the next page.

COURSE CHALLENGE: SPACE-BASED POWER

The Cost of Altitude

Research current costs associated with getting objects into space. Later, you may use this data to help estimate the cost of delivering electric energy through a space-based power system.

Conceptual Problem

- Picture this. You and your family are moving. There are boxes everywhere. You just carried a very heavy box out to the truck and have come back for another one. You reach for a box that you believe to be full and very heavy. However, it is empty. What do you think will happen when you start to lift it? Explain, in terms of forces and acceleration, what happens when anyone starts to lift an object that they believe to be much heavier than it actually is.

ELECTRONIC LEARNING PARTNER

Your Electronic Learning Partner has an interactive activity that explores Newton's second law of motion.

PROBEWARE

www.mcgrawhill.ca/links/atlphysics

If your school has probeware equipment, visit the Internet site above and follow the links for laboratory activities on Newton's second law and on stopping distances.

NEWTON'S SECOND LAW

Force is the product of mass and acceleration, or, acceleration is the quotient of the force and the mass.

$$\vec{F} = m\vec{a} \quad \text{or} \quad \vec{a} = \frac{\vec{F}}{m}$$

Quantity	Symbol	SI unit
acceleration	$\vec{a}$	$\frac{m}{s^2}$ (metre per second squared)
force	$\vec{F}$	N (newton)
mass	m	kg (kilogram)

Unit Analysis

(mass) (acceleration) = $\text{kg}\,\frac{\text{m}}{\text{s}^2} = \text{N}$

Note: The $\vec{F}$ in Newton's second law always represents the vector sum of all the forces, or the net force, acting on the mass.

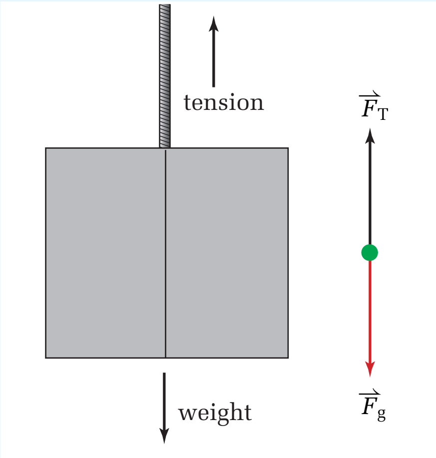
MODEL PROBLEM

Applying Newton's Second Law

A man is riding in an elevator. The combined mass of the man and the elevator is 7.00×10^2 kg. Calculate the magnitude and direction of the elevator's acceleration if the tension ($\vec{F}_T$) in the supporting cable is 7.50×10^3 N ($\vec{F}_T$ is the applied force).

Frame the Problem

- Sketch a free body diagram of the man and elevator.
- The cable exerts an *upward force* on the man and elevator.
- *Gravity* exerts a *downward force* on the man and elevator.
- The net force on the man and elevator will determine the *acceleration* according to *Newton's second law*.



Identify the Goal

The acceleration, $\vec{a}$, of the elevator

Variables and Constants

Known	Implied	Unknown
$\vec{F}_T = 7.50 \times 10^3$ N	$\vec{g} = 9.81 \frac{m}{s^2}$[down]	$\vec{F}_g$
$m = 7.00 \times 10^2$ kg		$\vec{F}_{net}$
		$\vec{a}$

Strategy

Since the motion is all along one line, up and down, denote direction with signs only. Let up be positive and down be negative.

Calculations

Find the force of gravity acting on the man and elevator using the equation for weight. Since "down" was chosen as negative, the acceleration due to gravity becomes negative.

$$\vec{F}_g = m\vec{g}$$
$$\vec{F}_g = (7.00 \times 10^2 \text{ kg})\left(-9.81 \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_g = -6.867 \times 10^3 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$
$$\vec{F}_g = -6.87 \times 10^3 \text{ N}$$

Find the net force acting on the man and elevator by finding the vector sum of the tension and force of gravity acting on the elevator and man.

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_g$$
$$\vec{F}_{net} = +7.50 \times 10^3 \text{ N} - 6.867 \times 10^3 \text{ N}$$
$$\vec{F}_{net} = +6.33 \times 10^2 \text{ N}$$

Apply Newton's second law in terms of acceleration and solve.

$$\vec{a} = \frac{\vec{F}}{m}$$

Write N as $\frac{\text{kg} \cdot \text{m}}{\text{s}^2}$ so you can cancel units.

$$\vec{a} = \frac{+6.33 \times 10^2 \text{ N}}{7.00 \times 10^2 \text{ kg}}$$
$$\vec{a} = +9.043 \times 10^{-1} \frac{\frac{\cancel{\text{kg}} \cdot \text{m}}{\text{s}^2}}{\cancel{\text{kg}}}$$
$$\vec{a} = +9.043 \times 10^{-1} \frac{\text{m}}{\text{s}^2}$$

The elevator was accelerating upward at 0.904 m/s^2.

Validate

The tension was greater than the weight, causing a net upward force to exist. A net force will cause an object to accelerate upward. The units cancelled to give metres per square second, which is correct for acceleration.

PRACTICE PROBLEMS

1. A 4.0 kg object experiences a net force of 2.2 N[E]. Calculate the acceleration of the object.
2. A 6.0 kg object experiences an applied force of 4.4 N[E] and an opposing frictional force of 1.2 N[W]. Calculate the acceleration of the object.
3. A stretched elastic exerts a force of 2.5 N[E] on a wheeled cart, causing it to accelerate at 1.5 m/s^2[E]. Calculate the mass of the cart, ignoring frictional effects.

INVESTIGATION 5-A

Force and Acceleration

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

Children playing with a toboggan in the snow will gain an intuitive sense that force, mass, and acceleration are related. Early scientists developed the same intuition, but only through experimentation could they formulate mathematical models that accurately predicted observed results. In this investigation, you will determine the validity of Newton's second law.

Problem

Obtain experimental evidence to support Newton's second law.

Hypothesis

State Newton's second law in the form of an hypothesis.

Equipment

- elastic bands
- dynamics cart
- metre stick
- ticker-tape timer or motion sensor

Procedure

1. Tape the elastic band onto the end of the metre stick as shown. Determine the length of an unstretched elastic band. Mark the length on the ruler with a piece of tape.

2. Attach the free end of the elastic to the dynamics cart.
3. Set up your equipment to collect distance versus time data on the cart as you pull it along with the elastic.
4. Obtain data with the elastic stretched to 1.0 cm, 2.0 cm, 3.0 cm, and 4.0 cm. It is crucial that you ensure that the stretch in the elastic remains exactly the same for the entire trip.

CAUTION As you run trials with more stretch in the elastic, the cart's final speed will increase dramatically. Have a partner waiting to catch the moving cart. Ensure that your path is free of obstacles or doors that could swing open.

5. Generate velocity versus time graphs for each length trial.

Analyze and Conclude

1. What was the purpose of ensuring that the amount of stretch in the elastic band remained constant?
2. Use the velocity versus time graphs to obtain an average acceleration for each trial. Ensure that you select a time interval that is the same length for each trail when determining the average acceleration. (A larger time interval will yield better results.) Use the results to generate a force versus acceleration graph. You can express the force as centimetres of stretch of the elastic band.
3. Find the slope of the best-fit line from the force versus acceleration graph. What does this slope represent?
4. From your results, develop a mathematical model that relates force, mass, and acceleration.
5. Use your mathematical model to predict how the slope of the line in the force versus acceleration graph would appear if (a) two carts and (b) three carts were pulled using the same elastic. If time allows, test your prediction.

Combining Dynamics and Kinematics

When analyzing motion, you often need to solve a problem in two steps. You might have information about the forces acting on an object, which you would use to find the acceleration. In the next step, you would use the acceleration that you determined in order to calculate some other property of the motion. In other cases, you might analyze the motion to find the acceleration and then use the acceleration to calculate the force applied to a mass. The following model problems will illustrate this process.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of acceleration and velocity.

MODEL PROBLEMS

Finding Velocity from Dynamics Data

1. In television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass 9.1×10^{-31} kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Frame the Problem

- The electrons are moving *horizontally*, from the back to the front of the tube, under an *electric force*.
- Since the electrons move so quickly, the time interval of the entire flight is very short. Therefore, the *effect of the force of gravity is too small to be detected* and you can consider the electric force to be the only force affecting the electrons.
- Information about *dynamics data* allows you to find the electrons' *acceleration*.
- Each electron is *initially at rest*, meaning that the *initial velocity is zero*.
- Given the acceleration, the equations of motion lead to other variables of motion.
- Let the direction of the force, and therefore the direction of the acceleration, be positive.

Identify the Goal

The final velocity, $\vec{v}_2$, of an electron when exiting the electron gun

Variables and Constants

Known	Implied	Unknown
$m_e = 9.1 \times 10^{-31}$ kg	$\vec{v}_1 = 0\ \frac{\text{m}}{\text{s}}$	$\vec{a}$
$\vec{F} = 5.8 \times 10^{-15}$ N [toward front of tube]		$\vec{v}_2$
$\Delta\vec{d} = 3.5 \times 10^{-3}$ m [horizontally toward front of tube]		

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Strategy

Write Newton's second law in terms of acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$

Substitute and solve.

$$\vec{a} = \frac{+5.8 \times 10^{-15}\ \text{N}}{9.1 \times 10^{-31}\ \text{kg}}$$

$\frac{\text{N}}{\text{kg}}$ is equivalent to $\frac{\text{m}}{\text{s}^2}$.

$$\vec{a} = 6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\text{[toward the front of tube]}$$

Apply the kinematic equation that relates initial velocity, acceleration, and displacement to final velocity.

$$v_2^2 = v_1^2 + 2a\Delta d$$

$$v_2^2 = 0 + 2\left(6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\right)(3.5 \times 10^{-3}\ \text{m})$$

$$v_2 = 6.67\ 967 \times 10^6\ \frac{\text{m}}{\text{s}}$$

$$v_2 \cong 6.7 \times 10^6\ \frac{\text{m}}{\text{s}}$$

Calculations

The final velocity of the electrons is about 6.7×10^6 m/s in the direction of the applied force, which is toward the front of the tube.

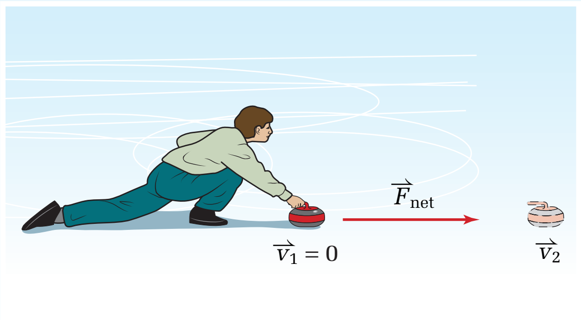
Validate

Electrons, with their very small inertial mass, could be expected to reach high speeds.

2. **A curler exerts an average force of 9.50 N[S] on a 20.0 kg stone. (Assume that the ice is frictionless.) The stone started from rest and was in contact with the girl's hand for 1.86 s.**

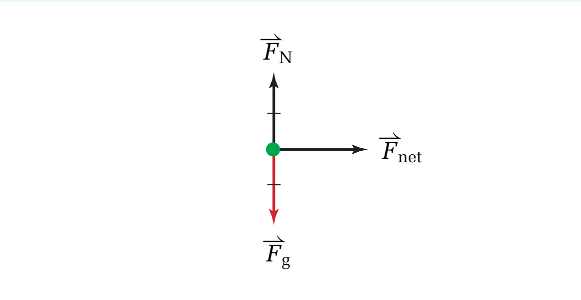
(a) Determine the average acceleration of the stone.

(b) Determine the velocity of the stone when the curler releases it.



Frame the Problem

- Draw a free body diagram of the problem.
- The *downward force of gravity* is balanced by the *upward normal force*. Therefore there is *no net force* in the vertical direction. These forces do not affect the acceleration of the stone.
- The only *horizontal force* on the stone is the *force exerted* by the curler. Therefore it is the *net force* on the stone.
- The *net force* determines the *acceleration* of the stone according to *Newton's second law* of motion.



- After the stone leaves the curler's hand, there is *no* longer a horizontal *force* on the stone and thus, it is *no* longer *accelerating*.
- The *equations of motion* for uniform acceleration apply to the motion of the stone.

Identify the Goal

(a) The acceleration, $\vec{a}$, of the curling stone

(b) The final velocity, $\vec{v}_2$, of the stone as it leaves her hand

Variables and Constants

Known	Implied	Unknown
$\vec{F}_{\text{applied}} = 9.5\ \text{N[S]}$	$\vec{v}_1 = 0.0\ \frac{\text{m}}{\text{s}}\text{[S]}$	$\vec{a}$
$m = 20.0\ \text{kg}$	$\vec{v}_2$	
$\Delta t = 1.86\ \text{s}$		

Strategy

You know the net force so use Newton's second law in terms of acceleration.

Calculations

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{a} = \frac{9.50\ \text{N[S]}}{20.0\ \text{kg}}$$

$$\vec{a} = 0.475\ \frac{\frac{\cancel{\text{kg}} \cdot \text{m}}{\text{s}^2}}{\cancel{\text{kg}}}\text{[S]}$$

$$\vec{a} = 0.475\ \frac{\text{m}}{\text{s}^2}\text{[S]}$$

(a) The average acceleration of the stone was 0.475[S].

Use the equation that relates initial and final velocities, acceleration, and the time interval.

Substitute in the known variables and solve for $\vec{v}_2$.

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{v}_2 = 0.0\ \frac{\text{m}}{\text{s}^2}\text{[S]} + \left(0.475\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\text{[S]}\right)(1.86\ \cancel{\text{s}})$$

$$\vec{v}_2 = 0.8835\ \frac{\text{m}}{\text{s}}\text{[S]}$$

(b) The velocity of the stone when it left the curlers hand was $0.884\frac{\text{m}}{\text{s}}$[S].

Validate

The curling stone experienced a net force acting toward the south. A net force causes acceleration in the direction of the force. The stone accelerated in the direction of the force, gaining speed as it went. The units cancelled to give $\frac{\text{m}}{\text{s}^2}$[S] for acceleration and $\frac{\text{m}}{\text{s}}$[S] for velocity. These are the correct units.

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PRACTICE PROBLEMS

4. A 15 kg object experiences an applied force of 5.5 N[N] and an opposing frictional force of 2.5 N[S]. If the object starts from rest, how far will it have travelled after 4.0 s?

5. A 45 kg student rides his 4.0 kg bicycle, exerting an applied force of 325 N[E].

(a) Calculate the acceleration of the cyclist if frictional resistance sums to 50.0 N[W].

(b) How far will the student have travelled if he started with a velocity of 3.0 m/s[E] and accelerated for 8.0 s?

6. The driver of a 1.2×10^3 kg car travelling 45 km/h[W] on a slippery road applies the brakes, skidding to a stop in 35 m. Determine the coefficient of friction between the road and the car tires.

7. A linear accelerator accelerated a germanium ion ($m = 7.2 \times 10^{-25}$ kg) from rest to a velocity of 7.3×10^6 m/s[E] over a time interval of 5.5×10^{-6} s. What was the magnitude of the force that was required to accelerate the ion?

8. A hockey stick exerts an average force of 39 N[S] on a 0.20 kg hockey puck over a displacement of 0.22 m. If the hockey puck started from rest, what is the final velocity of the puck? Assume that the friction between the puck and the ice is negligible.

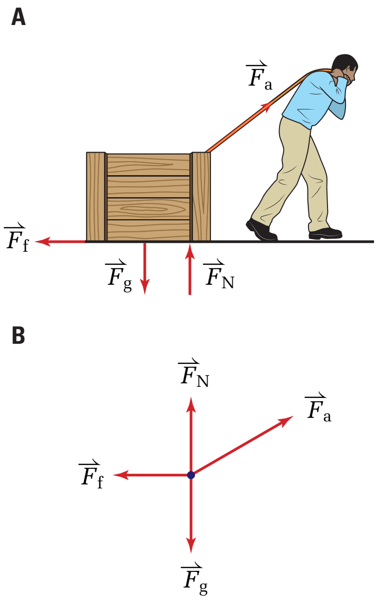


Figure 5.7 **(A)** The forces of gravity ($\vec{F}_g$), friction ($\vec{F}_f$), the normal force of the floor ($\vec{F}_N$), and the applied force of the rope ($\vec{F}_a$) all act on the crate at the same time. **(B)** The free-body diagram includes *only* those forces acting *on* the crate and none of the forces that the crate exerts on other objects.

Determining the Net Force

In almost every instance of motion, more than one force is acting on the object of interest. To apply Newton's second law, you need to find the resultant force. When some of the forces are acting at an angle with the direction of motion, a free-body diagram is a critical tool that will help to ensure that you have correctly identified and combined the forces. Figure 5.7 (A) illustrates a crate being pulled across a floor by a rope attached to the edge of the crate. Figure 5.7 (B) is a free-body diagram representing the forces acting on the crate.

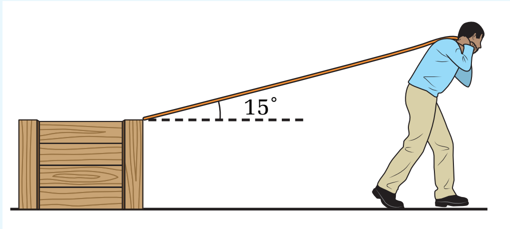
As you begin to solve problems involving forces at various angles, you will be working in one dimension at a time. You will select a coordinate system and then determine the component, or part, of each force acting in each dimension. Positive and negative signs completely describe the motion in one dimension. Thus, when you apply Newton's laws to the components of the forces in one dimension, you will not use vector notations.

Another convention used in this textbook involves writing the sum of all of the forces in one dimension. In the first step, when the forces are identified as, for example, gravitational, frictional, or applied, only plus signs will be used. Then, when information about that specific force is inserted into the calculation, a positive or negative sign will be included to indicate the direction of that specific force. Watch for these conventions in model problems.

MODEL PROBLEM

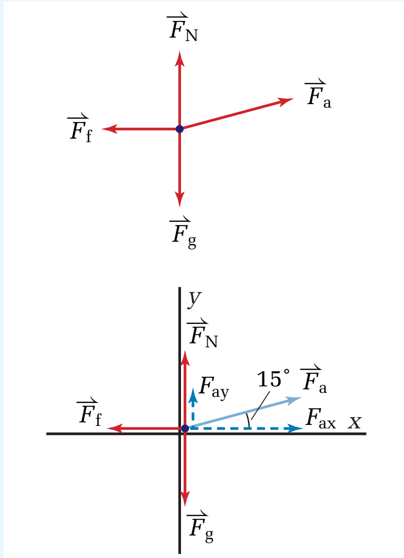
Working with Three Forces

To move a 45 kg wooden crate across a wooden floor (μ = 0.20), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of 115 N, it makes an angle of 15° with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of 1.4 m/s?



Frame the Problem

- Draw a free-body diagram.
- *Motion* is in the horizontal direction, so the net *horizontal force* is causing the crate to *accelerate*.
- Let the *direction* of the motion be the *positive* horizontal direction.
- There is no motion in the vertical direction, so the *vertical acceleration is zero*. If the acceleration is zero, the *net vertical force* must be *zero*. This information leads to the value of the *normal force*. Let "up" be the *positive* vertical direction.
- Since the beginning of the time interval in question is the instant at which the crate begins to move, the *coefficient of kinetic friction* applies to the motion.
- Once the *acceleration* is found, the *kinematic equations* allow you to determine the values of other quantities involved in the motion.



Identify the Goal

The time, Δt, required to reach a velocity of 1.4 m/s

Variables and Constants

Known		**Implied**	**Unknown**	
$\vec{F}_a = +115$ N	$m = 45$ kg	$\vec{v}_i = 0\ \frac{\text{m}}{\text{s}}$	$\vec{F}_N$	$\vec{a}$
$\theta = 15°$	$\vec{v}_f = 1.4\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	Δt
$\mu = 0.20$			$\vec{F}_f$	

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Strategy

To find the normal force, apply Newton's second law to the vertical forces. Analyze the free-body diagram to find all of the vertical forces that act on the crate.

Calculations

$$\vec{F} = m\vec{a}$$
$$F_{a(\text{vertical})} + F_g + F_N = ma$$
$$F_g = -mg$$
$$F_{a(\text{vertical})} - mg + F_N = ma$$
$$F_N = ma + mg - F_{a(\text{vertical})}$$
$$F_N = 0 + (45\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) - (115\text{ N})\sin 15^\circ$$
$$F_N = 441.45\text{ N} - 29.76\text{ N}$$
$$F_N = 411.69\text{ N}$$

To find the acceleration, apply Newton's second law to the horizontal forces. Analyze the free-body diagram to find all of the horizontal forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(\text{horizontal})} + F_f = ma$$
$$F_f = -\mu F_N$$
$$a = \frac{F_{a(\text{horizontal})} - \mu F_N}{m}$$
$$a = \frac{(115\text{ N})\cos 15^\circ - (0.20)(411.69\text{ N})}{45\text{ kg}}$$
$$a = \frac{111.08\text{ N} - 82.34\text{ N}}{45\text{ kg}}$$
$$a = 0.6387\ \frac{\text{m}}{\text{s}^2}$$

To find the time interval, use the kinematic equation that relates acceleration, initial velocity, final velocity, and time.

$$a = \frac{v_f - v_i}{\Delta t}$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$\Delta t = \frac{1.4\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{0.6387\ \frac{\text{m}}{\text{s}^2}}$$
$$\Delta t = 2.19\text{ s}$$
$$\Delta t \cong 2.2\text{ s}$$

You will be pulling the crate at 1.4 m/s at 2.2 s after the crate begins to move.

Validate

Check the units for acceleration: $\frac{\text{N}}{\text{kg}} = \frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}}{\cancel{\text{kg}}} = \frac{\text{m}}{\text{s}^2}$. The units are correct. A velocity of 1.4 m/s is not very fast, so you would expect that the time interval required to reach that velocity would be short. The answer of 2.2 s is very reasonable.

PRACTICE PROBLEMS

9. In a tractor-pull competition, a tractor applies a force of 1.3 kN to the sled, which has mass 1.1×10^4 kg. As the tractor moves, weights shift to increase the friction. When the coefficient of kinetic friction between the sled and the ground has increased to 0.80, what is the acceleration of the sled? Explain the significance of the sign of the acceleration.

10. A curling stone with mass 20.0 kg leaves the curler's hand at a speed of 0.885 m/s. It slides 31.5 m down the rink before coming to rest.

(a) Find the average force of friction acting on the stone.

(b) Find the coefficient of kinetic friction between the ice and the stone.

11. Pushing a grocery cart with a force with a magnitude of 95 N, applied at an angle of 35° down from the horizontal, makes the cart travel at a constant speed of 1.2 m/s. What is the frictional force acting on the cart?

12. A man walking with the aid of a cane approaches a skateboard (mass 3.5 kg) lying on the sidewalk. Pushing with an angle of 60° down from the horizontal with his cane, he applies a force of 115 N, which is enough to roll the skateboard out of his way.

    (a) Calculate the horizontal force acting on the skateboard.

    (b) Calculate the initial acceleration of the skateboard.

13. A mountain bike with mass 13.5 kg, with a rider having mass 63.5 kg, is travelling at 32 km/h when the rider applies the brakes, locking the wheels. How far does the bike travel before coming to a stop if the coefficient of friction between the rubber tires and the asphalt road is 0.60?

QUICK LAB

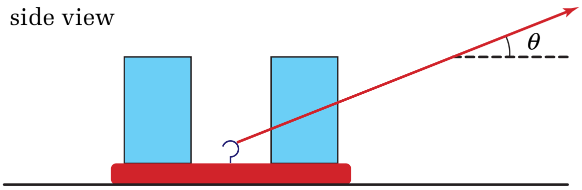
Best Angle for Pulling a Block

TARGET SKILLS

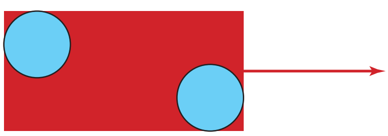
- **Predicting**
- **Performing and recording**
- **Analyzing and interpreting**

Set two 500 g masses on a block of wood. Attach a rope and drag the block along a table. If the rope makes a steeper angle with the surface, friction will be reduced (why?) and the block will slide more easily. Predict the angle at which the block will move with least effort. Attach a force sensor to the rope and measure the force needed to start dragging the block at a variety of different angles. Graph your results to test your prediction.

side view



top view



Analyze and Conclude

1. Identify from your graph the "best" angle at which to move the block.
2. How close did your prediction come to the experimental value?
3. Identify any uncontrolled variables in the experiment that could be responsible for some error in your results.
4. In theory, the "best" angle is related to the coefficient of static friction between the surface and the block: $\tan \theta_{best} = \mu_s$. Use your results to calculate the coefficient of static friction between the block and the table.
5. What effect does the horizontal component of the force have on the block? What effect does the vertical component have on the block?
6. Are the results of this experiment relevant to competitors in a tractor pull, such as the one described in the text and photograph caption at the beginning of this section? Explain your answer in detail.

Figure 5.8 Which way does the net force act?

Thus far, you have performed force calculation for cases in which the direction of the net force was obvious. However, in many cases, the direction of the net force is not clear. Before you solve the problem, you must first find the magnitude and direction of the net force.

Young children provide wonderful examples of the vector nature of forces. If you have ever had two children pulling each of your arms in different but not opposite directions, then you have witnessed the vector nature of forces. One child pulls you in one direction, the other pulls you in another direction, and you end up moving in a third direction that is actually determined by the vector sum of the original two forces.

MODEL PROBLEM

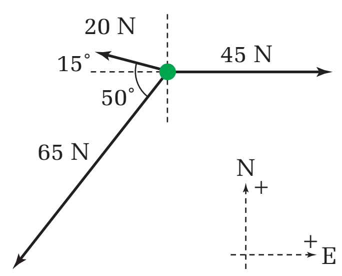
Forces in Two Dimensions

Three children are each pulling on their older sibling, who has a mass of 65 kg. The forces exerted by each child are listed here. Use a scale diagram to determine the resultant acceleration of the older sibling.

$\vec{F}_1 = 45\text{ N[E]}$
$\vec{F}_2 = 65\text{ N[S40°W]}$
$\vec{F}_3 = 20\text{ N[N75°W]}$

Frame the Problem

- The force of gravity on the older sibling is balanced by the normal force of the ground. Therefore, you can neglect vertical forces because there is no motion in the vertical plane.
- Draw a free body diagram representing horizontal forces on the older sibling.
- The *net force* in the horizontal plane will determine the magnitude and direction of the *acceleration* of the older sibling,
- *Newton's second law* applies to this problem.



Identify the Goal

The acceleration, $\vec{a}$, of the older sibling

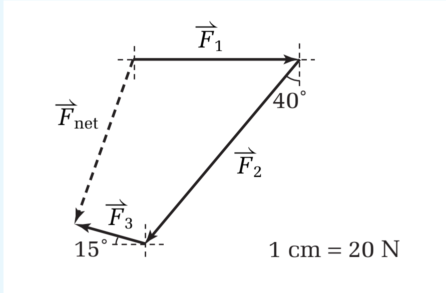
Variables and Constants

Known	Unknown
$\vec{F}_1 = 45\text{ N[E]}$	$\vec{a}$
$\vec{F}_2 = 65\text{ N[S40°W]}$	θ
$\vec{F}_3 = 20\text{ N[N75°W]}$	

Strategy

Draw a scale diagram, adding the vectors "tip to tail." If you need review, turn to Table 3.1 on page 82.

Calculations



Measure the length of the resultant force vector.

$|\vec{F}_{net}| = 2.4\text{ cm}$

Use the scale factor to determine the magnitude of the force.

$|\vec{F}_{net}| = (2.4\ \cancel{\text{cm}})\left(\frac{20\text{ N}}{\cancel{\text{cm}}}\right)$

$|\vec{F}_{net}| = 48\text{ N}$

Use a protractor to measure the angle.

$\theta = [\text{S}20°\text{W}]$

Use Newton's second law in terms of acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{a} = \frac{48\text{ N}[\text{S}20°\text{W}]}{65\text{ kg}}$$

$$\vec{a} = 0.7385\ \frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\ [\text{S}20°\text{W}]}{\cancel{\text{kg}}}$$

$$\vec{a} = 0.7385\ \frac{\text{m}}{\text{s}^2}[\text{S}20°\text{W}]$$

The older sibling will have an acceleration of 0.74 $\frac{\text{m}}{\text{s}^2}$[S20°W].

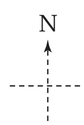
Validate

The acceleration value is reasonable. Units cancelled to give $\frac{\text{m}}{\text{s}^2}$ which is correct for acceleration.

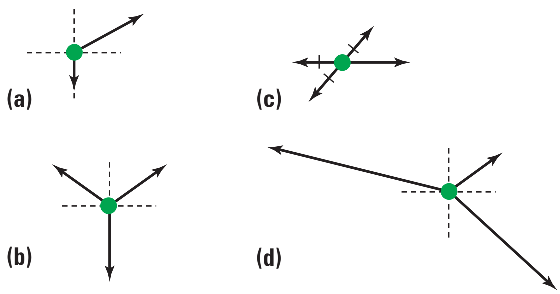
PRACTICE PROBLEMS

14. A swimmer is propelled directly north by a force of 35.0 N. Moving water exerts a second force of 20 N[E]. Use a scale diagram to determine the net force acting on the swimmer.

15. Find the resultant force acting on each object pictured. Obtain values by measuring the vectors.



1 cm = 50 N



16. A train car is pulled along the tracks by a force of 1500 N from a pickup truck driving beside the tracks. The rope connecting the

continued ▶

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truck and the train car makes an angle of 15° to the direction of travel.

(a) Find the component of the pulling force in the direction of travel.

(b) Find the component of the pulling force perpendicular to the direction of travel.

17. A student pushes a 25 kg lawn mower with a force of 150 N. The handle makes an angle of 35° to the horizontal.

(a) Find the vertical and horizontal components of the applied force.

(b) Calculate the normal force supporting the lawn mower while it is being pushed.

(c) Calculate the net force propelling the mower if a frictional force of 85 N exists.

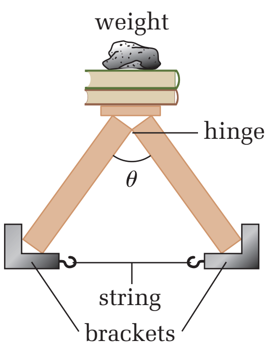
(d) Calculate the horizontal acceleration of the lawn mower. (Remember: Only part of the $F_{applied}$ is parallel to the direction of horizontal acceleration.)

QUICK LAB

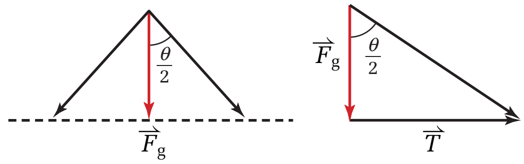
The Vector Nature of Force

TARGET SKILLS

- **Analyzing and interpreting**
- **Communicating results**



By using the apparatus shown, you can investigate the vector nature of force. Cut five lengths of thread to connect the base of the apparatus. Shorter lengths will create a taller isosceles triangle; longer lengths will create a shorter, wider isosceles triangle. Measure the angle at the top of the triangle and then pile small masses onto the top of the apparatus until the thread snaps. Record the angle and the amount of mass. Repeat this procedure for each length of thread.



$$\tan\frac{\theta}{2} = \frac{T}{F_g}$$

$$T = F_g \tan\frac{\theta}{2}$$

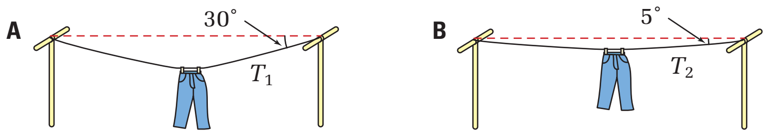
For a given amount of mass, the weight remains constant. The weight is the vertical component of the force compressing the wooden post. For the mass to be supported, the weight (vertical component) and the tension in the thread (horizontal component) must add "tip to tail" to form a closed triangle with the resultant force in the post. Notice that the weight, for a given amount of mass, does not change; therefore, as the angle is increased, the tension force must get progressively larger to keep the system in equilibrium. The tension force, T, can be found using simple trigonometry.

Analyze and Conclude

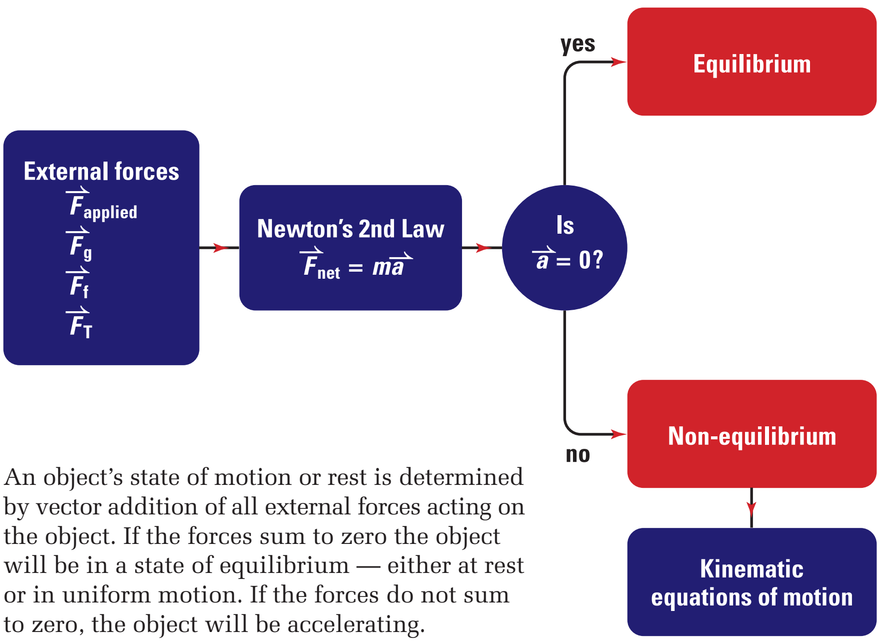
1. Find the weight of the total amount of mass required to break each thread.
2. The following diagram illustrates how the weight of the supported mass can be used to determine the tension in the thread. Find the breaking tension for each length of thread.
3. Organize your data to compare breaking weight to interior angle.
4. Draw conclusions about the two-dimensional nature of force.

Conceptual Problem

- Which clothesline will be under the greatest tension, assuming that both pairs of pants are identical? Why? [Hint: Review your results from the Quick Lab.]



Concept Organizer



An object's state of motion or rest is determined by vector addition of all external forces acting on the object. If the forces sum to zero the object will be in a state of equilibrium — either at rest or in uniform motion. If the forces do not sum to zero, the object will be accelerating.

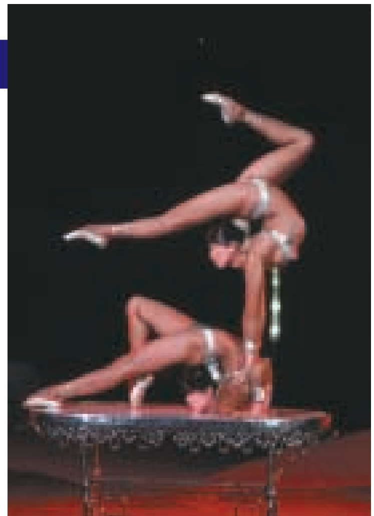


Figure 5.9 Understanding Newton's second law of force and motion.

5.2 Section Review

1. K/U State Newton's second law and give two examples.

2. K/U By how much will an object's acceleration change if
   (a) the force is doubled?
   (b) the mass of the object is halved?
   (c) the mass is doubled and the force is halved?

3. K/U Determine the net force in each of the following situations:
   (a) A race car travels at 185 km/h[W].
   (b) Two tug-of-war teams are at a standoff, each pulling with 1200 N of force.
   (c) The Voyager 1 space probe moves at 25 000 km/h in deep space beyond our solar system.

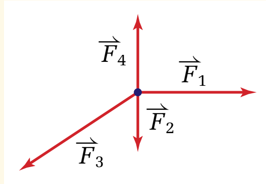
4. K/U How is direction represented when analyzing linear motion?

5. K/U Explain how to calculate
   (a) the horizontal component (F_x) of a force F
   (b) the vertical component (F_y) of a force F
   (c) the coefficient of friction (μ) between two surfaces
   (d) the gravitational force (F_g) acting on an object

6. K/U A 0.30 kg lab cart is observed to accelerate twice as fast as a 0.60 kg cart. Does that mean that the net force on the more massive cart is twice as large as the force on the smaller cart? Explain.

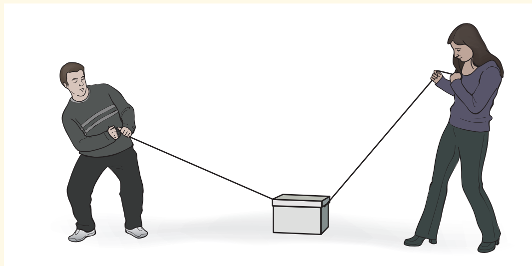
7. K/U A force F produces an acceleration a when applied to a certain body. If the mass of the body is doubled and the force is increased fivefold, what will be the effect on the acceleration of the body?

8. K/U An object is being acted on by forces pictured in the diagram.



   (a) Could the object be accelerating horizontally? Explain.
   (b) Could the object be moving horizontally? Explain.

9. K/U A tall person and a short person pull on a load at different angles but with equal force, as shown.



   (a) Which person applies the greater *horizontal* force to the load? What effect does this have on the motion of the load?
   (b) Which person applies the greater *vertical* force to the load? What effect does this have on frictional forces? On the motion of the load?