

## SECTION OUTCOMES

- Draw vector scale diagrams to visualize and analyze the nature of motion in a plane.
- Analyze motion by using scale diagrams to add and subtract vectors.
- Solve problems involving motion in a horizontal plane.

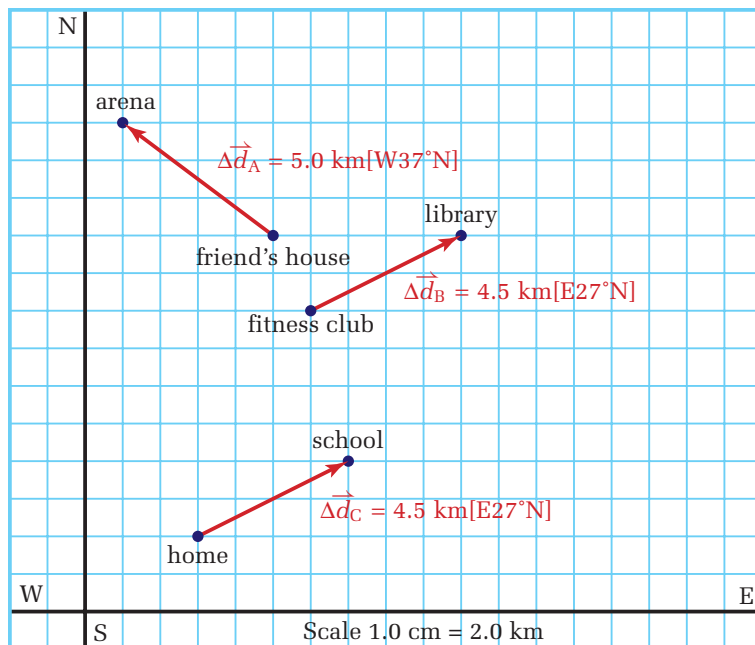
## KEY TERMS

- vector diagram
- coordinate system
- resultant vector

Imagine describing the motion of an expert water-skier to someone who had not watched the skier demonstrate his technique. You would probably do a lot of pointing in different directions. In a sense, you would be using vectors to describe the skier's motion. You have probably used vectors and even **vector diagrams** many times without realizing it. Did you ever draw a “treasure map” as a child? Have you ever gone hiking and drawn routes on a topographical map? If so, you already have a sense of the clarity with which vector diagrams help you to describe, analyze, and plan motion.

## Representing Vectors

You will represent all vector quantities with arrows that point in the direction of the quantity. The length of the arrow is proportional to the magnitude of the quantity you are representing. Vector quantities have direction, so you need a frame of reference or **coordinate system** to represent a direction. Vectors also have magnitude, so you need a scale to indicate and calculate magnitude.



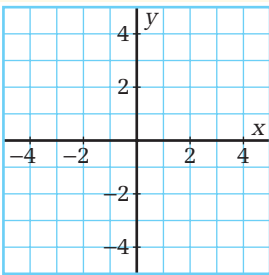
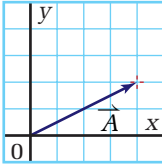
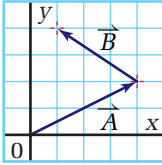
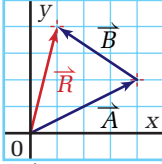
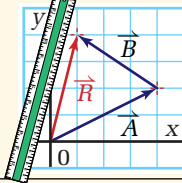
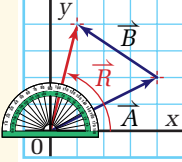
**Figure 3.6** The three displacement vectors are drawn to a scale of 1.0 cm to 2.0 km. To draw the directions from the symbols used here, start by pointing in the first compass direction. Then, rotate the stated number of degrees toward the last compass direction.

Since you are probably comfortable with maps, they provide a good frame of reference to familiarize yourself with the rules for drawing vectors. For example, the map in Figure 3.6 shows displacement vectors between your home and school and between the fitness club and the library. The vectors are drawn to scale. On the map, 1.0 cm represents 2.0 km. Have you noticed that two of the displacement vectors are identical? The direction of displacement vectors B and C is 27° north of east and the magnitude of each is 4.5 km. The beginning and ending points of a vector do not define the vector, only the length and direction. You can move a vector in its frame of reference as long as you do not change the length or the direction in which it points.

## Adding Vectors Graphically

Addition of vectors starts with some basic rules of arithmetic and then includes a few more rules. You have known for a long time that you cannot add “apples and oranges” or centimetres and metres. Similarly, you can add only vectors that represent the same quantity and are drawn to the same scale. Follow the steps listed in Table 3.2 to learn the graphical method for adding vectors. Any two vectors having the same units can be added according to the procedure in Table 3.2. The vector representing the sum is often called the **resultant vector**.

**Table 3.2** Graphical Vector Addition ( $\vec{R} = \vec{A} + \vec{B}$ )

Procedural step	Graph
Establish a coordinate system and choose a scale.	
Place the first vector ( $\vec{A}$ ) in a coordinate system.	
Place the tail of the second vector ( $\vec{B}$ ) at the tip of the first vector.	
Draw a vector from the tail of the first vector ( $\vec{A}$ ) to the tip of the second ( $\vec{B}$ ). Label it “ $\vec{R}$ .”	
With a ruler, measure the length of $\vec{R}$ .	
With a protractor, measure the angle between $\vec{R}$ and the horizontal axis.	

### ELECTRONIC LEARNING PARTNER



Go to your Electronic Learning Partner to visualize the process of adding vectors.

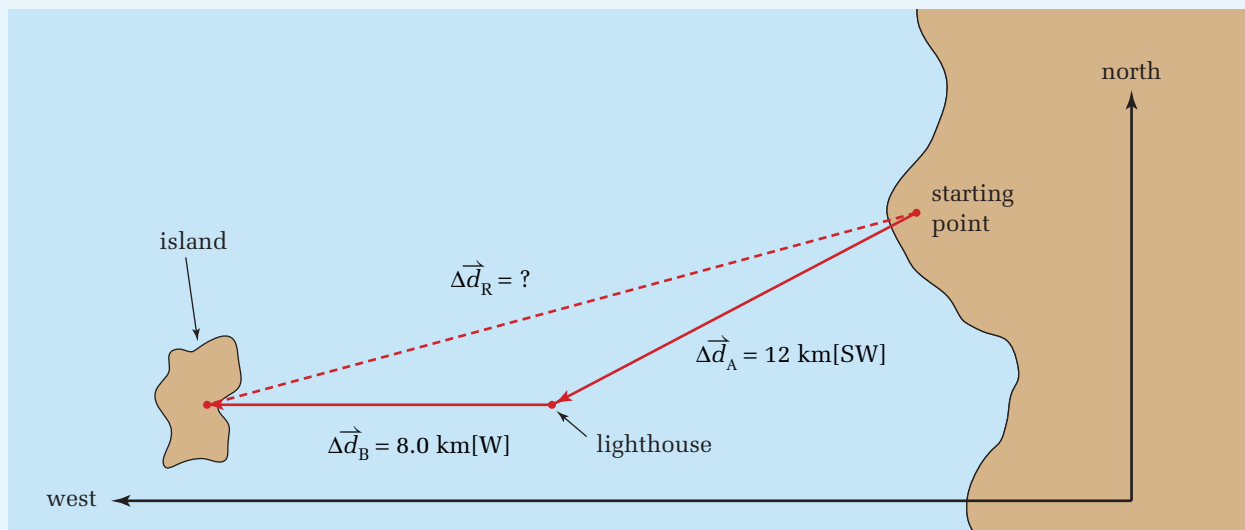
### Adding Vectors

A kayaker sets out for a paddle on a broad stretch of water. She heads toward the west, but is blown off course by a strong wind. After an hour of hard paddling, she arrives at a lighthouse that she knows is 12 km southwest of her starting point. She lands and waits for the wind to die down. She then paddles toward the setting sun and lands on a small island that is 8 km west of the lighthouse. In the calm of the evening, the kayaker plans to paddle straight back to her starting point. Use a vector diagram to determine her displacement from her starting point to the island. In which direction should she now head and how far will she have to paddle to go directly to the point from which she originally started paddling?



### Frame the Problem

- Make a scale diagram of the problem from the following sketch (not to scale). Your scale should be 1.0 cm : 4.0 km.



- The kayaker's journey consisted of two separate steps represented by the *two displacement vectors* in the diagram.
- The *vector sum* of the two vectors yields one *resultant vector* that shows the kayaker's final displacement.
- To return to the point she left earlier in the day, the kayaker will have to paddle a *displacement* that is *equal in magnitude* to her *resultant vector* and *opposite in direction*.

#### PROBLEM TIP

Make the scale on a vector diagram as large as possible — the larger the scale, the more accurate your result will be. Use graph paper when drawing vectors, because it provides many reference lines.

## Identify the Goal

- (a) The displacement,  $\Delta\vec{d}_R$ , of the first two legs of the kayaker's trip
- (b) The displacement,  $\Delta\vec{d}$ , needed to return to the point from which she originally started

## Variables and Constants

### Known

$$\Delta\vec{d}_A = 12 \text{ km[SW]}$$

$$\Delta\vec{d}_B = 8.0 \text{ km[W]}$$

### Unknown

$$\Delta\vec{d}_R$$

$$\Delta\vec{d}$$

## Strategy

Measure the length of the resultant displacement vector in the scale diagram.

Multiply the length of the vector by the scale factor.

With a protractor, measure the angle between the horizontal axis and the resultant vector.

- (a) The resultant displacement is 19 km[W27°S].
- (b) To return to the point from which she originally started her trip, the kayaker would have to paddle 19 km in the direction opposite to [W27°S], or 19 km[E27°N].

## Calculations

$$\Delta\vec{d}_R = 4.75 \text{ cm}$$

$$\Delta\vec{d}_R = 4.75 \text{ cm} \left( \frac{4.0 \text{ km}}{1.0 \text{ cm}} \right)$$

$$\Delta\vec{d}_R = 19 \text{ km}$$

$$\theta = 27^\circ$$

## Validate

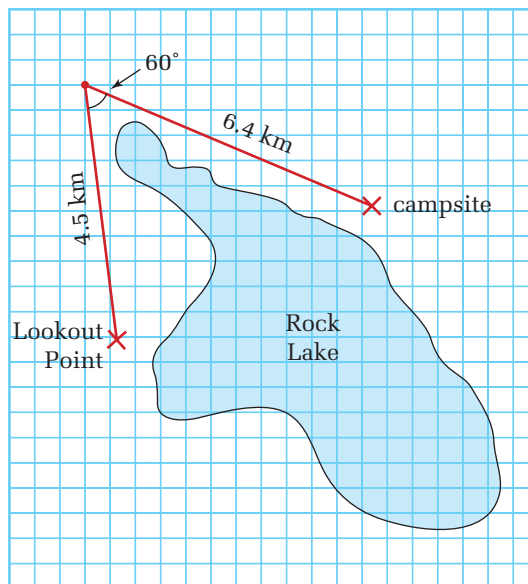
The total distance that the kayaker paddled was 20 km (8 km + 12 km). However, her path was not straight. In paddling back to shore, her trip formed a triangle. Since any side of a triangle must be shorter than the sum of the other two, you expect that her direct return trip will be shorter than 20 km. In fact, it was 1.0 km shorter.

## PRACTICE PROBLEMS

8. An airplane flies with a heading of [N58°W] from Sydney, NS to Newcastle, NB, a distance of 618 km. The airplane then flies 361 km on a heading of [E35°S] to New Glasgow, NS.
- (a) Determine the airplane's displacement for the trip.
- (b) In what direction will the plane have to fly in order to return directly to Sydney?
9. A canoeist starts from her campsite, paddles 3.0 km due north, and then 4.0 km due west.
- (a) Determine her displacement for the trip.
- (b) In what direction would she have to head her canoe in order to paddle straight home?

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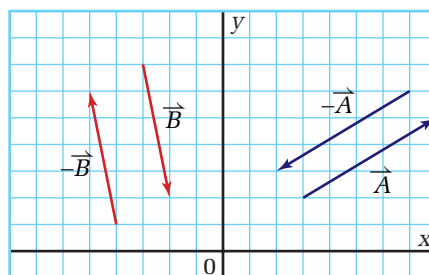
10. From a lookout point, a hiker sees a small lake ahead of her. In order to get around it, she walks 4.5 km in a straight line toward the end of the lake. She turns right making a  $60^\circ$  angle with her original path, and walks to a campsite that is 6.4 km in the new direction. Determine her displacement from the lookout point when she has reached the campsite. (See the map on the right.)
11. A boat heads out from port for a day's fishing. It travels 21.0 km due north to the first fishing spot. It then travels 30.0 km[W $30.0^\circ$ S] to a second spot. Finally, it turns and heads [W $10.0^\circ$ N] for 36.0 km.
- (a) Determine the boat's displacement for the entire journey.
- (b) In what direction should the boat point so as to head straight to its home port?



### Subtracting Vectors Graphically

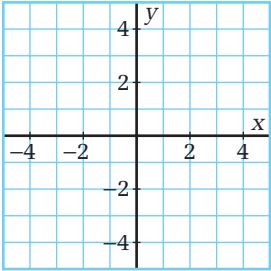
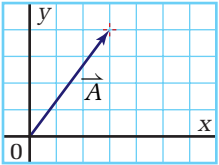
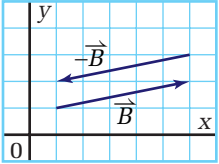
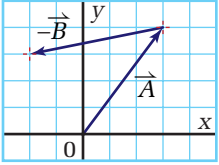
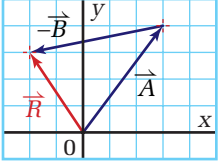
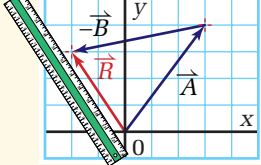
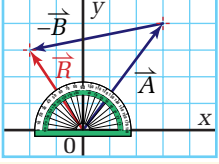
Problems involving motion often include situations in which you must subtract vector quantities. For example, displacement is defined as the difference of position vectors ( $\Delta \vec{d} = \vec{d}_2 - \vec{d}_1$ ). To find acceleration, you must first find a change in velocity, which is a difference in vector quantities ( $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$ ). In Chapter 2, you applied these equations to one dimension, so subtracting these quantities involved only arithmetic subtraction. However, when you work in two (or three) dimensions, you must account for the vector nature of these quantities.

Fortunately, vector subtraction is very similar to vector addition. Recall from basic math the expression, “ $A - B$  is equivalent to  $A + (-B)$ .” The only additional piece of information that you need is the definition of the negative of a vector. In Figure 3.7, you can see that the negative of a vector is the same in magnitude and opposite in direction. Now, follow the directions in Table 3.3 for the first method for subtracting vectors graphically.



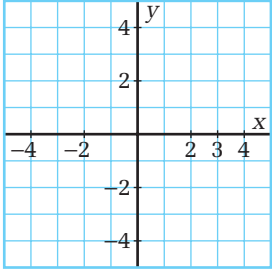
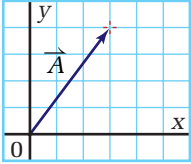
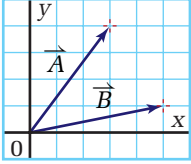
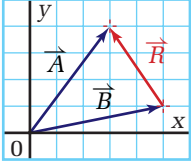
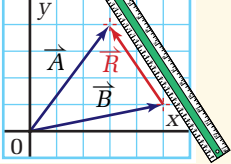
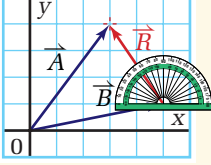
**Figure 3.7** To draw the negative of a vector, draw a line parallel to the positive vector and with an identical length. Then put an arrow head on the opposite end relative to the original vector.

**Table 3.3** Graphical Vector Subtraction ( $\vec{R} = \vec{A} - \vec{B}$ ): Method 1

Procedural step	Graph
Establish a coordinate system and choose a scale.	
Place the first vector ( $\vec{A}$ ) in the coordinate system.	
Draw the negative of vector $\vec{B}$ .	
Place the tail of $-\vec{B}$ at the tip of $\vec{A}$ .	
Draw a vector from the tail of $\vec{A}$ to the tip of $-\vec{B}$ . Label it " $\vec{R}$ ."	
With a ruler, measure the length of $\vec{R}$ .	
With a protractor, measure the angle between $\vec{R}$ and the horizontal axis.	

When subtracting vectors, you have an option of two different methods. Table 3.4 describes the second procedure.

**Table 3.4** Graphical Vector Subtraction ( $\vec{R} = \vec{A} - \vec{B}$ ): Method Two

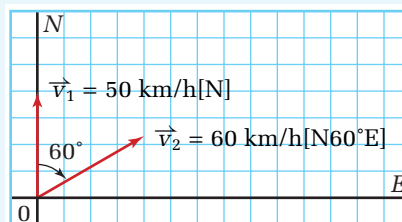
Procedural step	Graph
Establish a coordinate system and choose a scale.	
Place the first vector ( $\vec{A}$ ) in the coordinate system.	
Place the tail of the second vector ( $\vec{B}$ ) at the <i>tail</i> of the first vector ( $\vec{A}$ ).	
Draw a vector from the <i>tip</i> of the second vector ( $\vec{B}$ ) to the <i>tip</i> of the first vector ( $\vec{A}$ ). Label this vector " $\vec{R}$ ."	
With a ruler, measure the length of $\vec{R}$ .	
With a protractor, measure the angle between $\vec{R}$ and the horizontal axis.	

### Subtracting Vectors

A water-skier begins his ride by being pulled straight behind the boat. Initially, he has the same velocity as the boat ( $50 \frac{\text{km}}{\text{h}}$  [N]). Once up, the water-skier takes control and cuts out to the side. In cutting out to the side, the water-skier changes his velocity in both magnitude and direction. His new velocity is  $60 \frac{\text{km}}{\text{h}}$  [N60°E]. Find the water-skier's *change in velocity*.

#### Frame the Problem

- Make a scale diagram of the water-skier's initial and final velocities.
- The water-skier follows directly behind the boat, while rising to the surface.
- He then *changes the magnitude and direction* of his motion.



#### Identify the Goal

The change in the velocity,  $\Delta\vec{v}$ , of the water-skier

#### Variables and Constants

**Known**

$$\vec{v}_1 = 50 \frac{\text{km}}{\text{h}} [\text{N}]$$

$$\vec{v}_2 = 60 \frac{\text{km}}{\text{h}} [\text{N}60^\circ\text{E}]$$

**Unknown**

$$\Delta\vec{v}$$

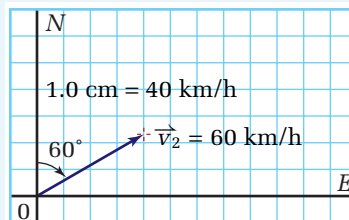
#### Strategy

Write the mathematical definition for the change in velocity.

Draw a coordinate system and choose a scale. Put  $\vec{v}_2$  in the coordinate system.

#### Calculations

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

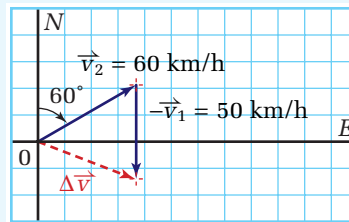


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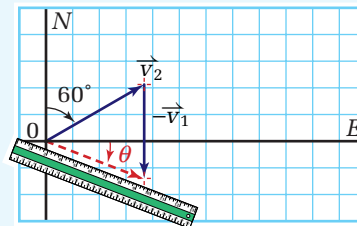


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Place the tail of vector  $-\vec{v}_1$  at the tip of  $\vec{v}_2$  and draw  $\Delta\vec{v}$ .



Measure the magnitude and direction of  $\Delta\vec{v}$ .



$$|\Delta\vec{v}| = 1.4 \text{ cm}, \theta = 21^\circ$$

Multiply the magnitude of the vector by the scale factor  $1.0 \text{ cm} = 40 \text{ km/h}$ .

$$|\Delta\vec{v}| = 1.4 \text{ cm} \left( \frac{40 \frac{\text{km}}{\text{h}}}{1.0 \text{ cm}} \right)$$

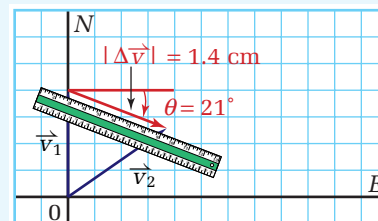
The water-skier's change in velocity was  $56 \text{ km/h[E}21^\circ\text{S]}$ .

$$|\Delta\vec{v}| = 56 \frac{\text{km}}{\text{h}}$$

## Validate

The magnitude of the change in the water-skier's velocity is smaller than the sum of the two velocities, which it should be.

Use the alternative method to check the answer. Draw a coordinate system to the same scale. Draw vectors  $\vec{v}_1$  and  $\vec{v}_2$  with their tails together. Draw a vector from the tip of  $\vec{v}_1$  to the tip of  $\vec{v}_2$ . Measure the magnitude and direction of the resultant vector. As you can see in the diagram, these values are identical to those obtained using the first method.



## PRACTICE PROBLEMS

Use both methods of vector subtraction to solve the following problems.

12. Given:  $\vec{P} = 12 \text{ km[N]}$ ,  $\vec{Q} = 15 \text{ km[S]}$ ,  
 $\vec{R} = 10 \text{ km[N}30^\circ\text{E]}$

- (a) Use method 1 of graphical vector subtraction to solve each of the following:  
 (i)  $\vec{P} - \vec{Q}$     (ii)  $\vec{R} - \vec{Q}$     (iii)  $\vec{Q} - \vec{R}$
- (b) Use method 2 of graphical vector subtraction to solve each of the following:  
 (i)  $\vec{P} - \vec{Q}$     (ii)  $\vec{R} - \vec{Q}$     (iii)  $\vec{P} - \vec{R}$

13. A car is travelling east at  $45 \text{ km/h}$ . It then heads north at  $50 \text{ km/h}$  ( $5.0 \times 10^1 \text{ km/h}$ ). Determine its change in velocity.
14. An airplane is flying at  $2.00 \times 10^2 \text{ km/h}$  [ $30.0^\circ\text{W}$ ]. It makes a smooth wide turn and heads east at  $2.00 \times 10^2 \text{ km/h}$ . Find its change in velocity.
15. A hockey puck hits the boards at a velocity of  $12 \text{ m/s}$  at an angle of  $30^\circ$  to the boards. It is deflected with a velocity of  $10 \text{ m/s}$  at an angle of  $25^\circ$  to the boards. Determine the puck's change in velocity.

16. A runner's velocity is recorded at four different points along the route,  $\vec{v}_1 = 3.5 \text{ m/s[S]}$ ,  $\vec{v}_2 = 5.0 \text{ m/s[N}12^\circ\text{W]}$ ,  $\vec{v}_3 = 4.2 \text{ m/s[W]}$ , and  $\vec{v}_4 = 2.0 \text{ m/s[S}76^\circ\text{E]}$ .
- (a) Graphically determine the change in velocity between  $\vec{v}_1$  and  $\vec{v}_2$ .
- (b) Calculate the change in velocity between  $\vec{v}_1$  and  $\vec{v}_3$ .
- (c) Graphically determine the change in velocity between  $\vec{v}_1$  and  $\vec{v}_4$ .

### • Conceptual Problems

- Review your findings in the “Pushed Around” lab at the beginning of this chapter. Assume that the marble rolls with a constant speed of 25 cm/s. Use method 2 for subtracting vectors to show  $\Delta\vec{v}$  for the rolling marble when it changes direction. Do you see a relationship between the direction you find for  $\Delta\vec{v}$  and the direction in which you blew on the marble to make it change directions?
- A marble is rolling across a lab bench. In what direction would you need to give the marble a sharp poke so that it would turn and travel at an angle of  $90^\circ$  to its original direction? Illustrate your thinking by drawing a sketch to show the original direction of the marble, the new direction of the marble, and the direction of the poke. Verify your thinking by conducting a mini-experiment.
- Two identical cars, travelling at the same speed, approach an intersection at right angles to each other on a day when the streets are very icy. Unfortunately, neither car is able to stop. During the collision, the cars are jammed together and the combined wreckage slides off the street. Draw a sketch of the accident and illustrate the direction in which the wreckage will travel.

## Multiplying and Dividing Vectors by Scalars

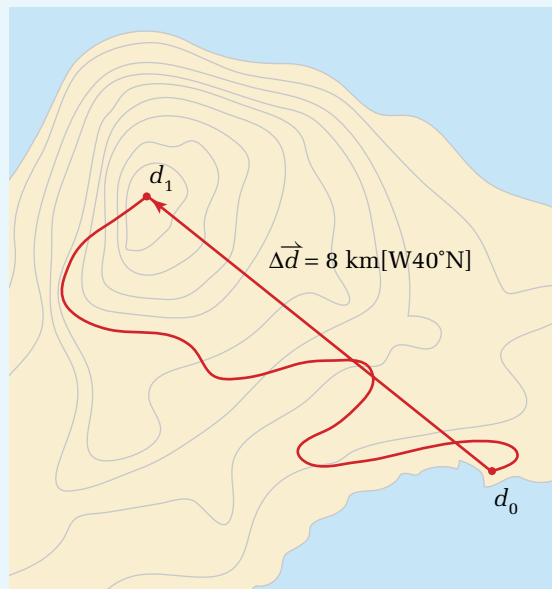
A brief review of the equations of motion will reveal one more important type of mathematical operation on vector quantities. These quantities are often multiplied or divided by the scalar quantity,  $\Delta t$ , the time interval. What happens to a vector when it is multiplied or divided by a scalar? To help answer that question, review the definition of average velocity:  $\vec{v}_{\text{ave}} = \frac{\Delta\vec{d}}{\Delta t}$ . When the displacement vector is divided by the time interval, the magnitude changes and the units change. The only thing that does *not* change is the direction of the vector. In summary, when a vector is multiplied or divided by a scalar, a different quantity is created, but the direction remains the same.

## Dividing Vectors by Scalars

1. A hiker sets out for a trek along a mountain trail. After 2 h, she checks her global positioning system and finds that she is 8 km[W40°N] from her starting point. What was her average velocity for the trek?

### Frame the Problem

- The hiker follows a meandering path; however, *displacement* is a *vector* that depends only on the *initial* and *final* positions.
- *Velocity* is the vector quotient of *displacement* and the *time interval*.
- The *direction* of the hiker's *velocity* is therefore the same as the *direction* of her *displacement*.



### Identify the Goal

The hiker's average velocity,  $\vec{v}_{\text{ave}}$ , for the trip

### Variables and Constants

#### Known

$$\Delta t = 2 \text{ h}$$

$$\Delta \vec{d} = 8 \text{ km}[W40^\circ\text{N}]$$

#### Unknown

$$\vec{v}_{\text{ave}}$$

### Strategy

Use the equation that defines average velocity to calculate the magnitude of the average velocity.

The direction of the velocity is the same as the direction of the displacement.

The average velocity for the trip was 4 km/h[W40°N].

### Calculations

$$\vec{v}_{\text{ave}} = \frac{\Delta \vec{d}}{\Delta t}$$

$$\vec{v}_{\text{ave}} = \frac{8 \text{ km}[W40^\circ\text{N}]}{2 \text{ h}}$$

$$\vec{v}_{\text{ave}} = 4 \frac{\text{km}}{\text{h}}[W40^\circ\text{N}]$$

### Validate

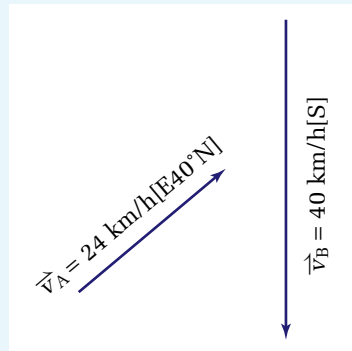
The magnitude, 4 km/h, is quite reasonable for a hike. The direction can only be the same as the direction of the displacement.

2. A hot-air balloon rises into the air and drifts with the wind at a rate of 24 km/h[E40°N] for 2 h. The wind shifts, so the balloon changes direction and drifts south at a rate of 40 km/h for 1.5 h before landing. Determine the balloon's displacement for the flight.



### Frame the Problem

- Make a diagram of the problem.
- The balloon trip takes place in *two stages* (Phase A and Phase B).
- Data for the trip is given in terms of *velocities* and *time intervals*.
- The *total displacement* is the *vector sum* of the two *displacement vectors*.



### PROBLEM TIP

Average velocity is the *displacement* divided by *time interval*. You should *never* attempt to find an overall average velocity by taking the vector sum of two velocities.

### Identify the Goal

Total displacement,  $\Delta\vec{d}_{\text{total}}$ , for the balloon trip

### Variables and Constants

#### Known

$$\Delta t_A = 2.0 \text{ h}$$

$$\Delta t_B = 1.5 \text{ h}$$

$$\vec{v}_{A \text{ ave}} = 24 \frac{\text{km}}{\text{h}}$$

$$\vec{v}_{B \text{ ave}} = 40 \frac{\text{km}}{\text{h}}$$

#### Unknown

$$\Delta\vec{d}_A$$

$$\Delta\vec{d}_B$$

$$\Delta\vec{d}_{\text{total}}$$

### Strategy

Use the velocity for Phase A to calculate the displacement for Phase A.

Select a scale and draw scale diagrams for Phase A.

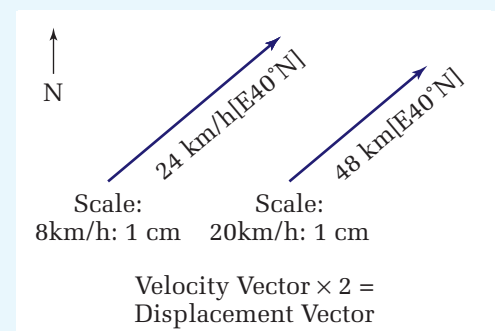
### Calculations

$$\Delta\vec{d}_A = \vec{v}_A \Delta t_A$$

$$\Delta\vec{d}_A = 24 \frac{\text{km}}{\text{h}} [\text{E}40^\circ\text{N}] (2.0 \text{ h})$$

$$\Delta\vec{d}_A = 48 \text{ km} [\text{E}40^\circ\text{N}]$$

### Phase A



continued ►

continued from previous page

### Strategy

Use the velocity for Phase B to calculate the displacement for Phase B.

Draw diagrams for Phase B using the scale you selected for Phase A.

On a graph, draw displacement vector A with its tail at the origin.

Draw displacement vector B with its tail at the tip of vector A.

Draw the resultant displacement vector from the tail of A to the tip of B. Label it " $\Delta\vec{d}_{\text{total}}$ ."

Measure the magnitude and direction of the total displacement vector.

Multiply the magnitude of the vector by the scale factor (1 cm = 20 km)

### Calculations

$$\Delta\vec{d}_B = \vec{v}_B \Delta t_B$$

$$\Delta\vec{d}_B = 40 \frac{\text{km}}{\text{h}} [\text{S}] (1.5 \text{ h})$$

$$\Delta\vec{d}_B = 60 \text{ km} [\text{S}]$$

$$|\Delta\vec{d}_{\text{total}}| = 2.35 \text{ cm} \times \left( \frac{20 \text{ km}}{\text{cm}} \right)$$

$$|\Delta\vec{d}_{\text{total}}| = 47 \text{ km}$$

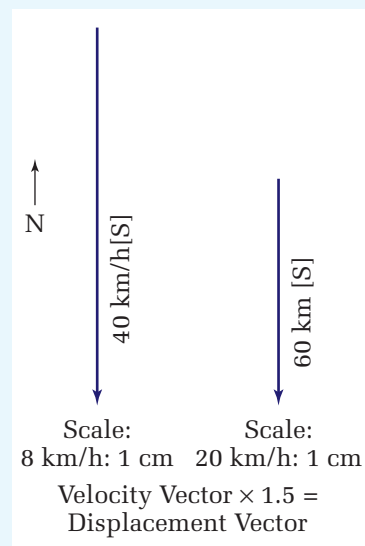
$$\theta = 38^\circ$$

The balloon trip had a displacement of 47 km[E38°S].

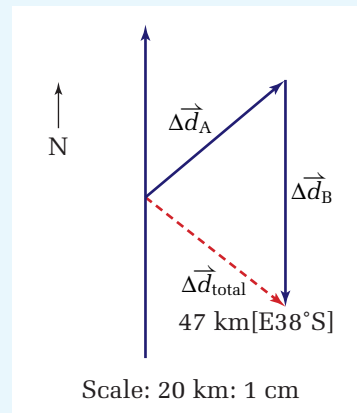
### Validate

The wind caused a significant change in course. Since the balloon made a fairly sharp turn, you would expect that the total displacement would be shorter than the sum of the two legs of the trip. This was, in fact, the case (47 km is much shorter than 48 km + 60 km).

### Phase B



### Total Displacement



## PRACTICE PROBLEMS

17. A hang-glider launches herself from a high cliff and drifts 6.0 km south. A wind begins to blow from the southeast, which pushes her 4.0 km northwest before she lands in a field. Calculate her average velocity for the trip if she is in the air for 45 min.



18. A light plane leaves Shelburne, NS and flies 195 km [N15°W] to Saint John, NB. After picking up a passenger, it flies to Moncton, NB which is 149 km [N33°E]. The entire trip took 3.75 h.



- (a) Calculate the average velocity of the plane for the entire trip from Shelburne, NS to Moncton, NB.

- (b) If the pilot wants to fly straight back to Shelburne, NS from Moncton, NB in 2.85 h, at what velocity will she need to fly? Assume that there is negligible wind.

19. A canoeist paddles across a calm lake with a velocity of 3.0 m/s north for 30.0 min. He then paddles with a velocity of 2.5 m/s west for 15.0 min. Determine his displacement from where he began paddling.
20. A hiker sets out on a trek heading [N35°E] at a pace of 5.0 km/h for 48.0 min. He then heads west at 4.5 km/h for 40.0 min. Finally, he heads [N30°W] for 6.0 km, until he reaches a campground 1.5 hours later.
- (a) Draw a displacement vector diagram to determine his total displacement.
- (b) Determine his average velocity for the trip.
- (Hint: Remember the definition of average velocity.)

## 3.2 Section Review

- (C) Describe the difference between

  - vectors and scalars
  - positive and negative vectors of the same magnitude
  - a resultant vector for displacement during a trip and the vector representing the return trip
  - vector addition and vector subtraction
  - a coordinate system and a frame of reference
- (K/U) A map has a scale in which 50 km is equal to 1 cm. If two towns are 7 cm apart on the map, what is their actual separation in km?
- (I) Investigate how the resultant vector will change when vector  $A$  is added to:

  - vector  $\vec{A}$
  - vector  $-\vec{A}$
  - vector  $2\vec{A}$
  - vector  $-5\vec{A}$
- (K/U) Define average velocity.
- (K/U) A hiker sets out down the trail at a pace of 5 km/h for one hour and then returns to her camp at the same pace to pick up her trail map. What is her average velocity upon arriving back at camp?