3.1 Mathematical Models of Motion

SECTION OUTCOMES

- Identify and investigate questions that arise from practiced problems involving motion.
- Analyze word problems and solve algebraically for unknowns.

KEY TERM

- equations of motion

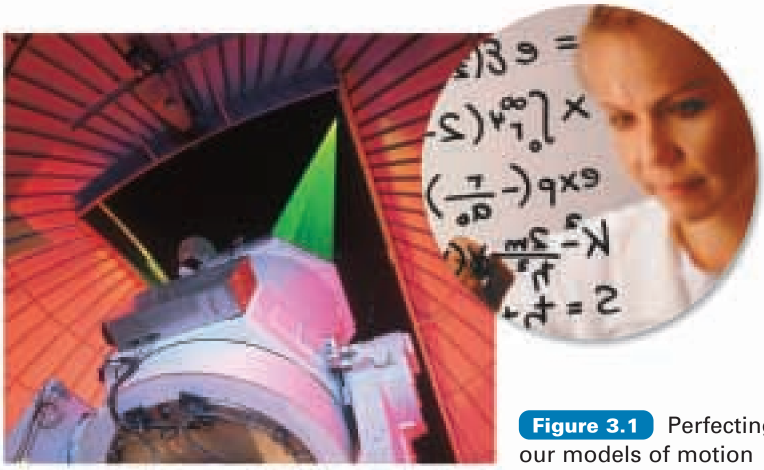


Figure 3.1 Perfecting our models of motion

Throughout the previous chapter, you were developing models of motion. Your models took the form of stick figures, mathematical definitions of displacement, velocity, and acceleration, and graphs. In this section, you will call on all of those models to build a set of mathematical equations called the **equations of motion** (or of kinematics) for uniform acceleration. As the name implies, these equations apply *only* to situations in which the *acceleration is constant*.

A very important feature of the equations of motion is that they apply independently to each dimension, so you will use them to analyze motion in one direction at a time. For example, you will analyze only north-south motion or only vertical (up and down) motion. Consequently, the variables in the equations represent only the parts, or components, of the vector quantities, position, displacement, velocity, and acceleration.

Figure 3.2 shows a position and displacement vector separated into components. Since components of vectors apply to only one dimension, they are not vectors themselves. Therefore, vector notations will not be used in the equations of motion.

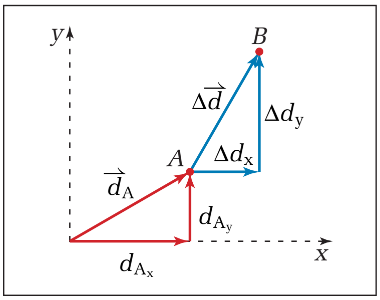


Figure 3.2 The displacement of an object that moved from point A to point B is represented by the vector $\Delta\vec{d}$. The object moved a distance, Δd_x, in the x direction and a distance, Δd_y in the y direction. Therefore, Δd_x is called the x-component of the displacement vector and Δd_y is the y-component. Any vector can be divided into components. The x- and y-components of position vector $\vec{d}_A$ are also shown, and are labelled as "d_{Ax}" and "d_{Ay}".

Deriving the Kinematic Equations

The fundamental definitions of displacement, velocity, and acceleration form the basis of the set of equations you will develop and apply. Start with the definition of acceleration in one dimension.

$$a = \frac{\Delta v}{\Delta t}$$

In some cases, you will know the initial and final velocities for a certain time interval and want to determine the acceleration. So, you will use the expanded form of the mathematical definition for the change in velocity: $\Delta v = v_f - v_i$. Substituting this expression into the original equation for acceleration, you obtain a useful equation.

$$a = \frac{v_f - v_i}{\Delta t}$$

In many cases, you will know the initial velocity and acceleration and want to find the final velocity for a time interval. Algebraically rearranging the above equation will give you another useful form.

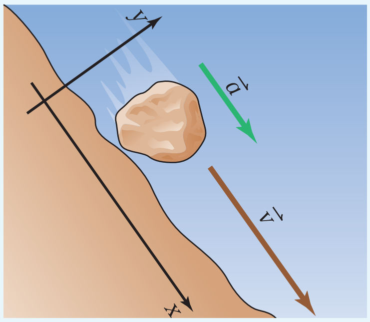
- Mulitply both sides of the equation by Δt and simplify.
$$a\Delta t = \left(\frac{v_f - v_i}{\cancel{\Delta t}}\right)\cancel{\Delta t}$$
$$v_f - v_i = a\Delta t$$
- Add v_i to both sides of the equation and simplify.
$$v_f - v_i + v_i = a\Delta t + v_i$$
$$v_f = v_i + a\Delta t$$

MODEL PROBLEMS

Changing Velocities

1. A slight earth tremor causes a large boulder to break free and start rolling down the mountainside with a constant acceleration of 5.2 m/s^2. What was the boulder's velocity after 8.5 s?

Frame the Problem



- Sketch and label a diagram of the motion.
- Choose the *direction of the motion* of the boulder as the *positive x* direction so the displacement will be positive.
- The boulder was *stationary* before the tremor, so its *initial velocity* was zero.
- The boulder's *acceleration* was constant, so the *equations of motion* apply to the problem.

Identify the Goal

The velocity (in one dimension), v, of the boulder 8.5 s after it started rolling

Variables and Constants

Known	Implied	Unknown
$a = 5.2\ \text{m/s}^2$	$v_i = 0\ \text{m/s}$	v_f
$\Delta t = 8.5\ \text{s}$		

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Strategy

Select the equation that relates the final velocity to the initial velocity, acceleration and time interval.

$v_f = v_i + a\Delta t$

All of the needed quantities are known so substitute them into the equation.

$$v_f = 0.0\,\frac{\text{m}}{\text{s}} + \left(5.2\,\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(8.3\,\cancel{\text{s}})$$

Simplify.

$$v_f = 43.16\,\frac{\text{m}}{\text{s}}$$

The final velocity of the boulder was 43 m/s.

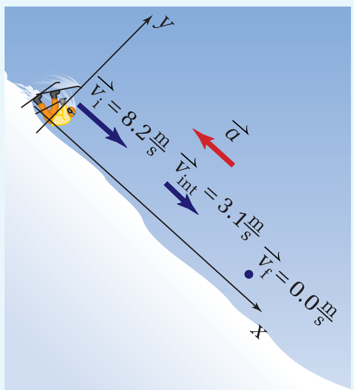
Validate

The units cancel to give metres per second, which is correct for velocity. The product of 5 × 8 is 40, so you would expect the answer to be slightly larger than 40 m/s.

2. A skier is going 8.2 m/s when she falls and starts sliding down the ski run. After 3.0 s, her velocity is 3.1 m/s. How long after she fell did she finally come to a stop? (Assume that her acceleration was constant.)

Frame the Problem

- Sketch and label the situation.
- Choose a coordinate system that places the skier at the origin when she falls and places her motion in the positive *x* direction.
- When the skier falls and begins to slide, her *initial velocity* is the same as her skiing velocity.
- Friction begins to *slow* her down and, eventually, she will come to a *stop*.
- Her *final velocity* will be *zero*.
- Since her *acceleration is constant*, the *equation of motion* relating initial and final velocities to acceleration and time applies to this problem.



Identify the Goal

The total time, Δt, it takes for the skier to stop sliding

Variables and Constants

Known	Implied	Unknown
$v_{\mathrm{i}} = 8.2\ \frac{\mathrm{m}}{\mathrm{s}}$	$v_{\mathrm{f}} = 0.0\ \frac{\mathrm{m}}{\mathrm{s}}$	a
$v_{\mathrm{int}} = 3.1\ \frac{\mathrm{m}}{\mathrm{s}}$		Δt

Strategy

You know the initial and final velocities of the first phase of the motion, but you need to find the acceleration in order to solve for the time interval. In this case, it is best to use the definition for acceleration.

Use the information about the intermediate velocity to find the acceleration. For the first phase of the motion, the "final" velocity is 3.1 m/s. Substitute values into the equation.

Calculations

$$a = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{\Delta t}$$

$$a = \frac{3.1\ \frac{\mathrm{m}}{\mathrm{s}} - 8.2\ \frac{\mathrm{m}}{\mathrm{s}}}{3.0\ \mathrm{s}}$$

$$a = \frac{-5.1\ \frac{\mathrm{m}}{\mathrm{s}}}{3.0\ \mathrm{s}}$$

$$a = -1.7\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Knowing the acceleration, you can use it to find the length of the entire time interval from the initial fall to the time the skier stopped. Use the same form of the equation, but use the calculated acceleration. Also, in this part of the problem, the final velocity is zero.

Substitute first

$$a = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{\Delta t}$$

$$-1.7\ \frac{\mathrm{m}}{\mathrm{s}^2} = \frac{0.0\ \frac{\mathrm{m}}{\mathrm{s}} - 8.2\ \frac{\mathrm{m}}{\mathrm{s}}}{\Delta t}$$

$$\left(-1.7\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\Delta t = \frac{0.0\ \frac{\mathrm{m}}{\mathrm{s}} - 8.2\ \frac{\mathrm{m}}{\mathrm{s}}}{\cancel{\Delta t}}\cancel{\Delta t}$$

$$\frac{\cancel{\left(-1.7\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)}\Delta t}{\cancel{\left(-1.7\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)}} = \frac{-8.2\ \frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}}}}{\left(-1.7\ \frac{\cancel{\mathrm{m}}}{\mathrm{s}^{\cancel{2}}}\right)}$$

$$\Delta t = 4.823\ \mathrm{s}$$

Solve for Δt first

$$a = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{\Delta t}$$

$$a\Delta t = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{\cancel{\Delta t}}\cancel{\Delta t}$$

$$\frac{\cancel{a}\Delta t}{\cancel{a}} = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{a}$$

$$\Delta t = \frac{0.0\ \frac{\mathrm{m}}{\mathrm{s}} - 8.2\ \frac{\mathrm{m}}{\mathrm{s}}}{-1.7\ \frac{\mathrm{m}}{\mathrm{s}^2}}$$

$$\Delta t = \frac{-8.2\ \frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}}}}{-1.7\ \frac{\cancel{\mathrm{m}}}{\mathrm{s}^{\cancel{2}}}}$$

$$\Delta t = 4.823\ \mathrm{s}$$

It took 4.8 s for the skier to come to a stop.

Validate

The units cancelled to give seconds, which is correct for a time interval. Eight seconds is a reasonable time period for a skier to slide to a stop.

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PRACTICE PROBLEMS

1. An Indy car's velocity increases from +6.0 m/s to +38 m/s over a 4.0 s time interval. What is its average acceleration?
2. A stalled car starts to roll backward down a hill. At the instant that it has a velocity of 4.0 m/s down the hill, the driver is able to start the car and start accelerating back up. After accelerating for 3.0 s, the car is travelling uphill at 3.5 m/s. Determine the car's acceleration once the driver got it started. (Assume that the acceleration was constant.)
3. A bus is travelling along a street at a constant velocity when the driver steps on the brakes and brings the bus to a stop in 3.0 s. If the brakes cause the bus to accelerate at $-8.0\ \text{m/s}^2$, at what velocity was the bus travelling when the brakes were applied?

Building Equations

The next logical step in building a set of equations would be to rearrange the equation that defines velocity. However, a problem arises when you try to use the equation. Can you see why?

$$v = \frac{\Delta d}{\Delta t}$$

$$v\Delta t = \left(\frac{\Delta d}{\cancel{\Delta t}}\right)\cancel{\Delta t}$$

$$\Delta d = v\Delta t$$

This equation is valid *only if the velocity is constant*, that is, if the motion is uniform. The equations of motions are developed for *constant acceleration*. So, unless that constant acceleration is zero, the velocity will be changing.

To find a relationship between displacement and velocity for a changing velocity, turn, once again, to graphs. First, consider an object moving at a constant velocity. The velocity-time graph is simply a horizontal line, as shown in Figure 3.3. As well, displacement is the product of the constant velocity and the time interval. Notice on the graph that velocity and time interval form the sides of a rectangle. Since the area of a rectangle is the product of its sides, velocity times time must be the same as the area of the rectangle.

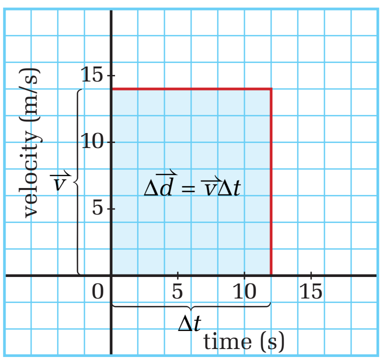


Figure 3.3 The area under the curve of a velocity-time graph is the displacement.

In fact, the displacement of an object is always the same as the area under the velocity-time graph. When the graph is a curve, you can approximate displacement by estimating the area under the curve. In Figure 3.4 (A), the area of each of the small squares is 5.0 m/s times 1.0 s or 5.0 m. Counting the number of squares and multiplying by 5.0 m gives a good estimate of displacement. You can make your estimate more accurate by dividing up the area into small rectangles as shown in Figure 3.4 (B). Notice that the corner of each rectangle above the curve on the left is nearly the same area as the space between the rectangle and the curve on the right. So the area of the rectangle is very nearly the same as the area under the curve. When you add the areas of all of the rectangles, you have a very close approximation of the displacement.

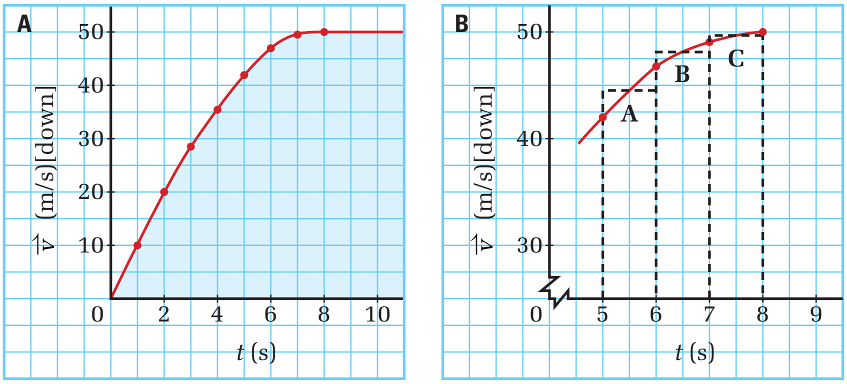


Figure 3.4 **(A)** The area under the curve is the displacement. **(B)** You can increase the accuracy of the area determination by making the columns narrower. (Why?)

How does the knowledge that the area under the curve is the same as the displacement help you to develop precise equations for displacement under constant acceleration? Consider the shape of the velocity-time graph for an object travelling with uniform acceleration. The graph is a straight line, as shown in Figure 3.5. If you draw a rectangle so that the line forming the top is precisely at the midpoint between the initial velocity and the final velocity, you will find that the line intersects the velocity curve exactly at the midpoint of the time interval. The top of the rectangle and the velocity line create congruent triangles. If you cut out the triangle on the right (below the graph), it would fit perfectly into the triangle on the left (above the graph). The area of the rectangle is *exactly* the same as the displacement. The height of the rectangle is the average of the initial and final velocities for the time interval or $v_{ave} = \frac{v_i + v_f}{2}$. You can now use this expression for velocity in the equation developed for displacement, above.

$$\Delta d = v\Delta t$$

$$\Delta d = \frac{v_i + v_f}{2}\Delta t$$

You have just developed another useful equation of motion for uniform acceleration.

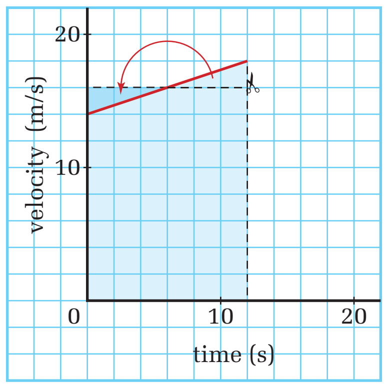


Figure 3.5 Confirm the results of the analysis by using the formula for the area of a trapezoid. $A = \frac{(\text{side } 1 + \text{side } 2)}{2}(\text{width})$

Often, you know the initial velocity of an object and its acceleration but not the final velocity. You can develop an expression for displacement that does not include final velocity.

- Start with the equation above. $\Delta d = \frac{v_i + v_f}{2}\Delta t$
- Recall the expression you developed for final velocity. $v_f = v_i + a\Delta t$
- Substitute this value into the first equation. $\Delta d = \frac{v_i + (v_i + a\Delta t)}{2}\Delta t$
- Combine like terms. $\Delta d = \left(\frac{2v_i + a\Delta t}{2}\right)\Delta t$
- Multiply through by Δt. $\Delta d = \frac{\cancel{2}v_i\Delta t}{\cancel{2}} + \frac{a\Delta t^2}{2}$
- Simplify. $\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$

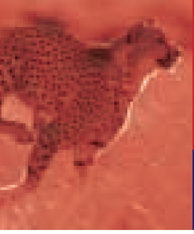
Table 3.1 summarizes the equations of motion and indicates the variables that are related by each equation. Notice that, in every case, the equation relates four of the five variables. Therefore, if you know three of the variables, you can find the other two. First, use an equation that relates the three known variables to a fourth. Then, find an equation that relates any three of the four you now know to the fifth.

Table 3.1 Equations of Motion under Uniform Acceleration

Equation	Variables				
	Δd	v_i	v_f	a	Δt
$a = \frac{v_f - v_i}{\Delta t}$		x	x	x	x
$v_f = v_i + a\Delta t$		x	x	x	x
$\Delta d = \frac{v_i + v_f}{2}\Delta t$	x	x	x		x
$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$	x	x		x	x

Conceptual Problem

- Derive an equation that relates v_i, v_f, Δd, and a. (Hint: Notice that Δt is not involved.) Solve for Δt in the first equation. Substitute that value into Δt in the third equation. Solve for v_f^2. Can you prove that $v_f^2 = v_i^2 + 2a\Delta d$?



INVESTIGATION 3-A

Stop on a Dime

TARGET SKILLS
- Initiate and planning
- Hypothesizing
- Analyzing and interpreting

In this investigation, you will be challenged to build a vehicle that, when launched from a ramp, will travel a horizontal distance of 3.0 m and come to rest on a dime!

Problem

Design, build, and test a vehicle. Enter it into a competition.

Equipment

- flat 1.0 m ramp
- dime
- materials of your choice for building a vehicle

Procedure

Designing and Building

1. With a partner or a small group, discuss potential designs for your vehicle, according to the following criteria.
   (a) Prefabricated kit and prefabricated wheels are not allowed.
   (b) Propulsion may come only from the energy gained by rolling the vehicle down a 1.0 m ramp.
   (c) You may adjust the angle of the ramp.
   (d) The vehicle must be self-contained. No external guidance systems, such as tracks, guide wires, or strings, are permitted.

   Be creative. Do not limit your thinking to a traditional four-wheeled vehicle.
2. Collect materials and build your vehicle according to your design. Get your teacher's approval before testing.
3. Test your vehicle and make adjustments until you are satisfied with its performance. Collect data on at least three trial runs.

Entering the Competition

4. As a class, establish criteria for being allowed to enter the competition. For example, the vehicle must stop within 10 cm of the dime in at least one test run.
5. Submit a written application for entry into the competition. The application must include the following.
   (a) description of design features
   (b) outline of any major problems encountered in testing the vehicle, accompanied by a discussion of the solutions you discovered
   (c) data from trial runs, including position-time, velocity-time, and acceleration-time graphs. (Data must include at least three time intervals while accelerating down the ramp and five time intervals after the vehicle begins its horizontal motion.)

The Competition

6. As a class, decide on the scoring system for the competition. Decide how many points will be given for such results as coming within 6.0 cm of the dime. Decide on other possible criteria for points. For example, points could be given for sturdiness, creative use of materials, originality, or aesthetic appeal.
7. Hold the competition.

Analyze and Conclude

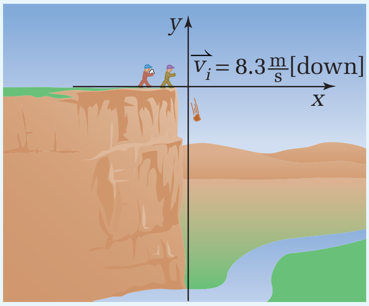
1. Analyze the performance of your own vehicle in comparison with your own criteria.
2. Analyze your vehicle in comparison with the vehicle that won the competition.
3. Considering what you learned from the competition, how would you design your vehicle differently if you were to begin again?
4. Summarize what you have learned about motion from this challenge.

MODEL PROBLEMS

Applying the Equations of Motion

1. You throw a rock off a cliff, giving it a velocity of 8.3 m/s, straight down. At the instant you released the rock, your hiking buddy started a stopwatch. You heard the splash when the rock hit the river below, exactly 6.9 s after you threw the rock. How high is the cliff above the river?

Frame the Problem



- Make a sketch of the problem and assign a coordinate system.
- The rock had an *initial velocity downward*. Since you chose downward as negative, the initial velocity is *negative*.
- The rock is *accelerating* due to gravity.
- The acceleration due to gravity is *constant*. Therefore, the equations of motion for uniform acceleration apply to the problem.

Identify the Goal

The displacement, Δd, from the top of the cliff to the river below

Variables and Constants

Known	Implied	Unknown
$v_i = -8.3$ m/s	$a = -9.81$ m/s^2	Δd
$\Delta t = 6.9$ s		

Strategy

Use the equation of motion that relates the unknown variable, Δd, to the three known variables, v_i, a, and Δt.

$$\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

Substitute the known variables.

$$\Delta d = \left(-8.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(6.9\ \cancel{\text{s}}) + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(6.9\ \text{s})^2$$

Simplify.

$$\Delta d = -57.27\ \text{m} + \left(-4.905\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(47.61\ \cancel{\text{s}^2})$$

$$\Delta d = -57.27\ \text{m} - 233.53\ \text{m}$$

$$\Delta d = -290.8\ \text{m}$$

Calculations

The cliff was 2.9×10^2 m above the river. The negative sign indicates that the distance is in the negative direction, or down, from the origin, the point from which you threw the rock.

Validate

All of the units cancel to give metres, which is the correct unit. The sign is negative, which you would expect for a rock going down. The rock fell a long distance (more than a quarter of a kilometre), so 6.9 s is a reasonable length of time for the rock's fall to have taken.

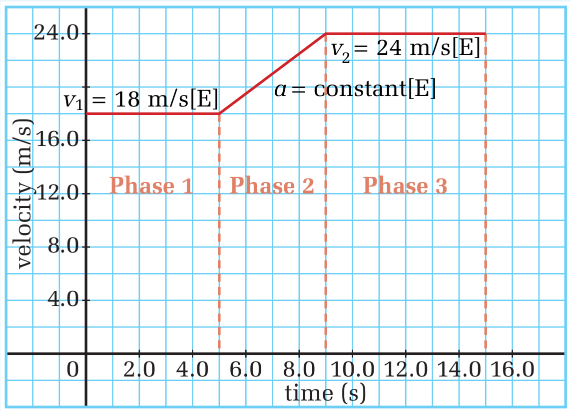
2. **A car travels east along a straight road at a constant velocity of 18 m/s. After 5.0 s, it accelerates uniformly for 4.0 s. When it reaches a velocity of 24 m/s, the car proceeds with uniform motion for 6.0 s. Determine the car's total displacement during the trip.**

PROBLEM TIP

When the type of motion of an object changes, the problem must be split into phases. Each phase is treated as a separate problem. The "final" conditions of one phase become the "initial" conditions of the next phase.

Frame the Problem

- Make a diagram of the motion of the car that includes the known variables during each phase of the car's motion.



- During phase 1, the car is moving with *uniform motion*. Since the *acceleration* is *zero*, the equations of motion with uniform acceleration are not needed. The equation *defining velocity* applies to this phase.
- During phase 2 of the motion, the car is *accelerating*. Therefore, use the equation of motion with *uniform acceleration* that relates time, initial velocity, and final velocity to displacement.
- During phase 3, the car is again moving with *uniform motion*.
- The *total displacement* of the car is the *sum* of the displacements for all three phases.

Identify the Goal

The total displacement, Δd, of the car for the duration of the motion

Variables and Constants

Known		**Unknown**
$v_1 = 18$ m/s	$\Delta t_2 = 4.0$ s	Δd_{total}
$v_2 = 24$ m/s	$\Delta t_3 = 6.0$ s	Δd_1
$\Delta t_1 = 5.0$ s		Δd_2
		Δd_3

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Strategy / Calculations

Use the equation that defines velocity.

$$\Delta d = v\Delta t$$

Substitute values.

$$\Delta d_1 = v_1\Delta t_1$$

Simplify.

$$\Delta d_1 = \left(18\ \frac{\text{m}}{\cancel{\text{s}}}[\text{E}]\right)(5\ \cancel{\text{s}})$$

$$\Delta d_1 = 90\ \text{m}[\text{E}]$$

The displacement for phase 1 was 90 m east.

Use the equation of motion that relates time, initial velocity, and final velocity to displacement.

$$\Delta d = \frac{v_\text{i} + v_\text{f}}{2}\Delta t$$

$$\Delta d_2 = \frac{v_1 + v_2}{2}\Delta t_2$$

Substitute values.

$$\Delta d_2 = \frac{18\ \frac{\text{m}}{\text{s}}[\text{E}] + 24\ \frac{\text{m}}{\text{s}}[\text{E}]}{2}(4\ \text{s})$$

Simplify.

$$\Delta d_2 = 21\ \frac{\text{m}}{\cancel{\text{s}}}[\text{E}]\ 4\ \cancel{\text{s}}$$

$$\Delta d_2 = 84\ \text{m}[\text{E}]$$

The displacement during phase 2 was 84 m east.

Use the equation that defines velocity.

$$\Delta d = v\Delta t$$

$$\Delta d_3 = v_2\Delta t_3$$

Substitute values.

$$\Delta d_3 = \left(24\ \frac{\text{m}}{\cancel{\text{s}}}[\text{E}]\right)6\ (\cancel{\text{s}})$$

Simplify.

$$\Delta d_3 = 144\ \text{m}[\text{E}]$$

The displacement during phase 3 was 144 m east.

Find the sum of the displacements for all three phases.

$$\Delta d_\text{total} = \Delta d_1 + \Delta d_2 + \Delta d_3$$

$$\Delta d_\text{total} = 90\ \text{m}[\text{E}] + 84\ \text{m}[\text{E}] + 144\ \text{m}[\text{E}]$$

$$\Delta d_\text{total} = 318\ \text{m}[\text{E}]$$

The total displacement for the trip was 3.2×10^2 m east.

Validate

In every case, the units cancelled to give metres, which is the correct unit for displacement. The duration of the trip was short (15 s), so the displacement cannot be expected to be very long. The answer of 318 m[E] is very reasonable.

3. A truck is travelling at a constant velocity of 22 m/s north. The driver sees a traffic light turn from red to green soon enough, so he does not have to alter his speed. Meanwhile, a woman in a sports car is stopped at the red light. At the moment the light turns green and the truck passes her, she begins to accelerate at 4.8 m/s^2. How far have both vehicles travelled when the sports car catches up with the truck? How long did it take for the sports car to catch up with the truck?

Frame the Problem

- The truck and the sports car leave the traffic signal at the *same time*. Define this time as $t = 0.0$ s.
- The truck passes the sports car at the traffic light. Let this point be $d = 0.0$ m.
- The truck travels with *uniform motion*, which means *constant velocity*. The truck's motion can therefore be described using the *equation that defines velocity*.
- The car's *initial velocity is zero*. Then, the car travels with *uniform acceleration*. The *equation of motion* that relates displacement, initial velocity, acceleration, and time interval describes the car's motion.
- When the sports car catches up with the truck, both vehicles have travelled for the *same length of time* and the *same distance*.

Identify the Goal

The displacement, Δd, that the sports car and truck travel from the traffic light to the point where the sports car catches up with the truck

The time interval, Δt, that it takes for the sports car to catch up with the truck

Variables and Constants

Known	Implied	Unknown	
$v_{\text{truck}} = 22$ m/s	$v_{\text{i(car)}} = 0.0$ m/s	Δd_{car}	Δt_{car}
$a_{\text{car}} = 4.8$ m/s^2		Δd_{truck}	Δt_{truck}

Strategy

Write the equation that defines velocity for the motion of the truck.

Calculations

$$\Delta d = v\Delta t$$

$$\Delta d_{\text{truck}} = 22\ \frac{\text{m}}{\text{s}}\Delta t_{\text{truck}}$$

Write the equation of motion for the sports car.

$$\Delta d = v_{\text{i}}\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta d_{\text{car}} = 0.0\ \frac{\text{m}}{\text{s}}\Delta t_{\text{car}} + \frac{1}{2}4.8\ \frac{\text{m}}{\text{s}^2}\Delta t_{\text{car}}^2$$

$$\Delta d_{\text{car}} = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta t_{\text{car}}^2$$

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Strategy | **Calculations**

The displacement for the sports car and the truck are the same. Call them both Δd.

$$\Delta d_{\text{car}} = \Delta d_{\text{truck}} = \Delta d$$

The time interval is the same for the sports car and the truck. Call them both Δt.

$$\Delta t_{\text{car}} = \Delta t_{\text{truck}} = \Delta t$$

You now have two equations and two unknowns. Solve for Δt in the equation for the sports car. Then substitute that expression into Δt for the truck. This will give you one equation with only one unknown, Δd.

$$\Delta d = 22\ \frac{\text{m}}{\text{s}}\Delta t$$

$$\frac{\Delta d}{22\ \frac{\text{m}}{\text{s}}} = \frac{\cancel{22\ \frac{\text{m}}{\text{s}}}}{\cancel{22\ \frac{\text{m}}{\text{s}}}}\Delta t$$

$$\Delta t = \frac{\Delta d}{22\ \frac{\text{m}}{\text{s}}}$$

$$\Delta d = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta t^2$$

$$\Delta d = 2.4\ \frac{\text{m}}{\text{s}^2}\left(\frac{\Delta d}{22\ \frac{\text{m}}{\text{s}}}\right)^2$$

Solve for displacement.

$$\Delta d\left(22^2\ \frac{\text{m}^2}{\text{s}^2}\right) = 2.4\ \frac{\text{m}}{\text{s}^2}\frac{\Delta d^2}{\cancel{22^2\ \frac{\text{m}^2}{\text{s}^2}}}\cancel{22^2\ \frac{\text{m}^2}{\text{s}^2}}$$

$$484\ \frac{\text{m}^2}{\text{s}^2}\Delta d = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta d^2$$

Subtract $484\ \frac{\text{m}^2}{\text{s}^2}\Delta d$ from both sides of the equation.

$$484\ \frac{\text{m}^2}{\text{s}^2}\Delta d - 484\ \frac{\text{m}^2}{\text{s}^2}\Delta d = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta d^2 - 484\ \frac{\text{m}^2}{\text{s}^2}\Delta d$$

$$0.0 = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta d^2 - 484\ \frac{\text{m}^2}{\text{s}^2}\Delta d$$

Factor out the Δd.

$$0.0 = \left(2.4\ \frac{\text{m}}{\text{s}^2}\Delta d - 484\ \frac{\text{m}^2}{\text{s}^2}\right)\Delta d$$

Set each of the factors equal to 0.

$$2.4\ \frac{\text{m}}{\text{s}^2}\Delta d - 484\ \frac{\text{m}^2}{\text{s}^2} = 0 \quad \text{or} \quad \Delta d = 0.0\ \frac{\text{m}}{\text{s}}$$

Solve for Δd.

$$2.4\ \frac{\text{m}}{\text{s}^2}\Delta d = 484\ \frac{\text{m}^2}{\text{s}^2}$$

$$\frac{\cancel{2.4\ \frac{\text{m}}{\text{s}^2}}\Delta d}{\cancel{2.4\ \frac{\text{m}}{\text{s}^2}}} = \frac{484\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{2.4\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$

$$\Delta d = 201.67\ \text{m}$$

You found two solutions for displacement of the sports car and truck when setting the time intervals and displacements of the two vehicles equal to each other. The value of zero for displacement simply means that they had the same displacement (zero) at time zero. The displacement of the sports car and truck was 2.0×10^2 m when the sports car caught up with the truck.

Strategy

To find the time it took for the sports car to catch up with the truck, substitute the displacement into the equation relating displacement of the truck and time interval.

Calculations

$$\Delta t = \frac{\Delta d}{22\frac{\text{m}}{\text{s}}}$$

$$\Delta t = \frac{201.66\ \cancel{\text{m}}}{22\frac{\cancel{\text{m}}}{\text{s}}}$$

$$\Delta t = 9.167\ \text{s}$$

It took 9.2 s for the sports car to catch up with the truck.

Validate

The units cancelled to give metres for displacement and seconds for time interval, which is correct. A second equation exists for calculating time interval from displacement. Substitute the displacement into the equation relating time and displacement for the car, and solve for time interval. It should give the same value, 9.167 s.

The values are in agreement.

$$\Delta d_{\text{car}} = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta t_{\text{car}}^2$$

$$201.67\ \text{m} = 2.4\ \frac{\text{m}}{\text{s}^2}\Delta t_{\text{car}}^2$$

$$\frac{201.67\ \cancel{\text{m}}}{2.4\ \frac{\cancel{\text{m}}}{\text{s}^2}} = \frac{\cancel{2.4\ \frac{\text{m}}{\text{s}^2}}}{\cancel{2.4\ \frac{\text{m}}{\text{s}^2}}}\Delta t_{\text{car}}^2$$

$$\Delta t_{\text{car}}^2 = 84.027\ \text{s}^2$$

$$\Delta t = 9.167\ \text{s}$$

PRACTICE PROBLEMS

4. A field hockey player starts from rest and accelerates uniformly to a speed of 4.0 m/s in 2.5 s

(a) Determine the distance she travelled.

(b) What is her acceleration?

5. In a long distance race, Michael is running at 3.8 m/s and is 75 m behind Robert, who is running at a constant velocity of 4.2 m/s. If Michael accelerates at 0.15 m/s^2, how long will it take him to catch Robert?

6. A race car accelerates at 5.0 m/s^2. If its initial velocity is 2.0×10^2 km/h, how far has it travelled after 8.0 s?

7. A motorist is travelling at 20.0 m/s when she observes that a traffic light 1.50×10^2 m ahead of her turns red. The traffic light is timed to stay red for 10.0 seconds. If the motorist wishes to pass the light without stopping just as it turns green again, what will be the speed of her car just as it passes the light?

3.1 Section Review

1. K/U Define kinematics.

2. MC Refer to model problem #1 on page 84. Given the explanation of timing, is 6.9 s likely to be exact? Explain the types of errors that could lead to an over measurement of the time interval. How might the hiking buddies minimize their error?

3. C Develop an appropriate problem for each of the following formulas.

(a) $a = \dfrac{(v_\text{f} - v_\text{i})}{\Delta t}$

(b) $\Delta d = \dfrac{(v_\text{i} + v_\text{f})}{2}\Delta t$

(c) $\Delta d = v_\text{i}\Delta t + \frac{1}{2}a\Delta t^2$