# 2.4 Acceleration

Many times in the last section, you read that an object's velocity was increasing, decreasing, or that it was changing direction. Once again, physicists have a precise way of stating the changes in velocity. **Acceleration** is a vector quantity that describes the rate of change of velocity.

### ACCELERATION

Acceleration is the quotient of the change in velocity and the time interval over which the change takes place.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

| Quantity | Symbol | SI unit |
|---|---|---|
| acceleration | $\vec{a}$ | $\frac{\text{m}}{\text{s}^2}$ (metres per second squared) |
| change in velocity | $\Delta\vec{v}$ | $\frac{\text{m}}{\text{s}}$ (metres per second) |
| time interval | $\Delta t$ | s (seconds) |

**Unit Analysis**

$$\frac{\frac{\text{metres}}{\text{second}}}{\text{second}} = \frac{\frac{\text{m}}{\text{s}}}{\text{s}} = \frac{\text{m}}{\text{s}^2}$$

The units of acceleration — metres per second squared — do not have an obvious meaning. If you think about the basic definition of acceleration, however, the meaning becomes clear. The velocity of an object changes by a certain number of metres per second *every second*. For example, analyze the statement, "A truck is travelling at a constant velocity of 20 m/s[E], then accelerates at 1.5 m/s$^2$[E]." This acceleration means that the truck's velocity increases by 1.5 m/s[E] every second. One second after it starts accelerating, it will be travelling at 21.5 m/s[E]. One second later, it will be travelling at 23 m/s[E]. The truck's velocity increases by 1.5 m/s[E] *every second*, as long as it is accelerating.

## Direction of Acceleration Vectors

The direction of the acceleration vector is the direction of the *change* in the velocity and not the direction of the velocity itself. To determine the direction of the acceleration vector, it is helpful to visualize the direction in which you would have to push on an object to *cause* a particular change in velocity.

**SECTION OUTCOMES**

- Use vectors to represent position, displacement, velocity, and acceleration.
- Identify and investigate questions that arise from practical problems involvig motion.
- Analyze word problems, solve algebraically for unknowns, and interpret patterns in data.

**KEY TERMS**

- acceleration
- constant (uniform) acceleration
- non-uniform acceleration
- average acceleration
- instantaneous acceleration

**ELECTRONIC LEARNING PARTNER**

To enhance your understanding of the language of acceleration go to your Electronic Learning Partner for an interactive activity.

Figure 2.19 shows the motion of a van that starts from rest, speeds up, travels at a constant velocity, slows down, and then stops. The frame of reference shows the origin at the left, with the *x*-axis pointing in a positive direction to the right. When the van is speeding up, the average velocity vectors and the average acceleration vector point in the same direction (+). When the van is travelling at a constant speed, the average acceleration is zero. When the van is slowing down, the average velocity vectors (+) and the average acceleration vector (–) are in opposite directions.

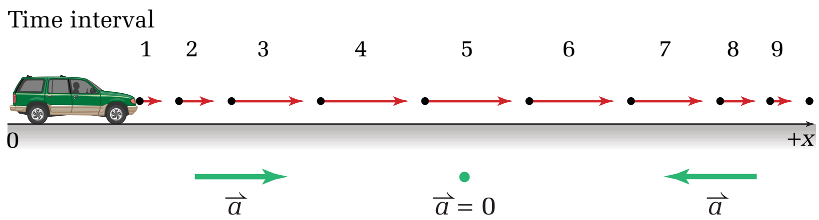



**Figure 2.19** When the van is moving in a positive direction but slowing down, the direction of the acceleration is negative.

## MISCONCEPTION

**They *Don't* Mean the Same Thing!**

Many people think that *negative acceleration* and *deceleration* mean the same thing — that an object is slowing down. "Deceleration" is not a scientific term but a common term that people use for slowing down. "Negative acceleration" is a scientific term meaning that the acceleration vector is pointing in the negative direction. However, an object with a negative acceleration might be speeding up.

Consider the directions that the average velocity and average acceleration vectors point if the van turns around and travels back to its starting point. As shown in Figure 2.20, when the van is speeding up in a negative direction, both the average velocity vectors and the acceleration vector point in the negative direction. While the van travels at constant velocity, the average velocity vectors are negative and the acceleration vector is zero. As the van slows down to stop, the average velocity vectors are pointing in the negative direction and the average acceleration vector is pointing in the positive direction.

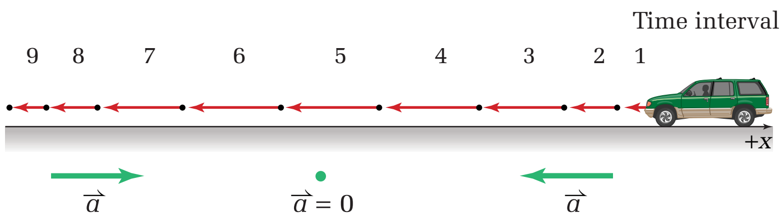



**Figure 2.20** When the van is moving in a negative direction and slowing down, the direction of acceleration is positive.

An object can accelerate without either speeding up or slowing down. If the magnitude of the velocity does not change but the direction does change, the object is accelerating. To visualize the direction of the acceleration vector in such cases, study Figure 2.21. Imagine the direction that you would have to push on the tip of the initial velocity vector to make it overlap with the final velocity vector. The direction of the acceleration vector is from the tip of the initial velocity vector toward the tip of the final velocity vector.

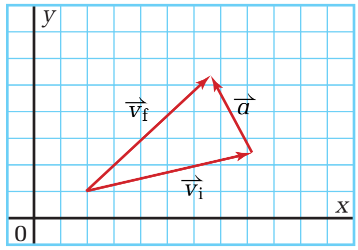


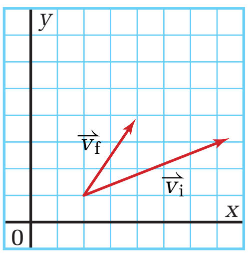

**Figure 2.21** Envision pushing on the tip of $\vec{v}_i$, until it overlaps with $\vec{v}_f$.

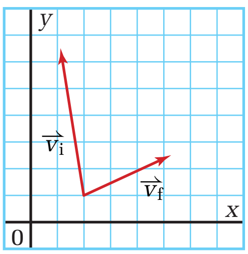
### *Conceptual Problems*

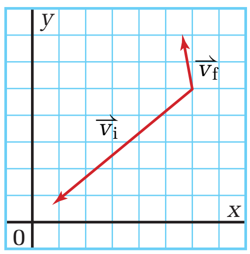
- The following charts refer to the van's journeys in Figures 2.19 and 2.20. Redraw the charts below and, using as examples the two rows that have been completed, fill in the remaining rows.

| Images in figure | Direction of velocity vector | Direction of acceleration vector | Description of motion |
|---|---|---|---|
| **Figure 2.19 Van is moving in the positive direction.** | | | |
| 1-2-3 | positive | positive | speeding up in positive direction |
| 4-5-6 | | | |
| 7-8-9 | | | |
| **Figure 2.20 Van is moving in the negative direction** | | | |
| 1-2-3 | | | |
| 4-5-6 | | | |
| 7-8-9 | negative | positive | slowing down in negative direction |

- Sketch each of the combinations of initial and final velocity vectors, and add to your sketch another vector showing the direction of the acceleration vector.



**PHYSICS FILE**

Physicists often use the term "uniform motion" to apply to motion with a constant velocity, and "uniformly accelerated motion" to apply to motion with a constant acceleration.

## Uniform and Non-Uniform Acceleration

Have you noticed the similarity in the mathematical expressions for velocity and acceleration?

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1} \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

The mathematical operations you performed on position vectors to find velocity are nearly the same as those you will perform on velocity vectors to find acceleration. The similarity applies to both equations and graphs. For example, the slope of a velocity-time graph is the acceleration. If the velocity graph is curved, the slope of the tangent to the velocity-time graph at a specific time is the acceleration of the object at that time. The terms applied to velocity also apply to acceleration. **Constant** or **uniform acceleration** means that the acceleration does not change throughout specified time intervals. As well, **non-uniform acceleration** means that the acceleration is changing with time.

The terms, "average," "constant," and "instantaneous" apply to acceleration in the very same way that they apply to velocity. **Average acceleration** is an acceleration calculated from initial and final velocities and the time interval. Constant acceleration means that the acceleration is not changing over a certain interval of time. The velocity-time graph for the time interval is a straight line. **Instantaneous acceleration** is the acceleration found at one moment in time, and is equal to the slope of the tangent to velocity-time graph at that point in time.

To see the connections among time, position, velocity, and acceleration of a moving object, consider the example of a ball that is thrown straight up in the air with an initial velocity of 20.0 m/s. The position-time data are listed in Table 2.5 and the graphs of position, velocity, and acceleration are shown in Figure 2.22.

The data for the velocity-time graph were determined by the slope of the position-time graph. (Only four tangent lines are shown.) The velocity-time graph of data taken from the slopes of the position-time graph is a straight line with a negative slope that is the same everywhere. Since the slope of the velocity-time graph is the acceleration, the acceleration has the same negative value throughout the motion. (The value is –9.81 m/s$^2$.)

**Table 2.5** Position-Time Data

| $t$ (s) | $\vec{d}$ (m) |
|---|---|
| 0.0 | 0.0 |
| 0.5 | 8.8 |
| 1.0 | 15.1 |
| 1.5 | 19.0 |
| 2.0 | 20.4 |
| 2.5 | 19.4 |
| 3.0 | 15.9 |
| 3.5 | 10.0 |
| 4.0 | 1.6 |

**Note:** Since the motion is in one dimension, direction is indicated by plus (+) or minus (–).

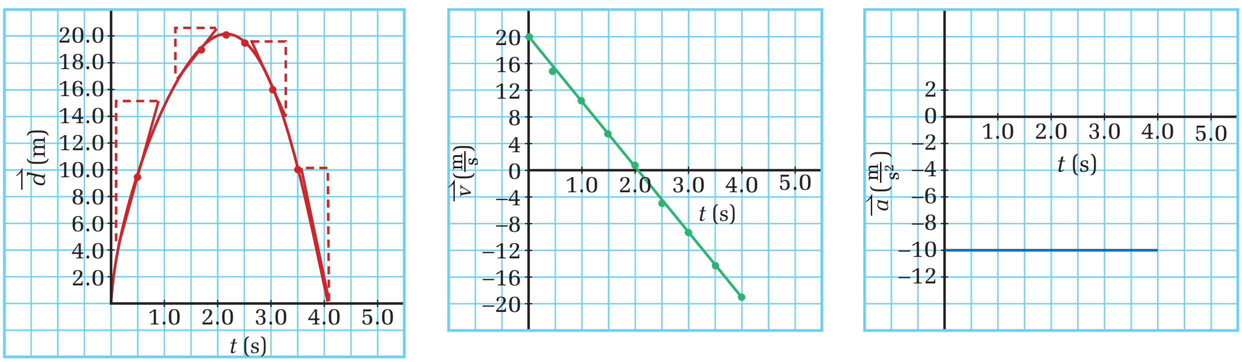



**Figure 2.22** Position, velocity, and acceleration graphs for Table 2.5

QUICK LAB

# Graphing the Motion of a Van

**TARGET SKILLS**
- Analyzing and interpreting
- Communicating results

You qualitatively analyzed the motion of a van earlier. Now, using the example of the ball thrown into the air, you can do a more detailed analysis of the van's motion. The table shown here includes the time and position data, with one worked example for finding acceleration.

## Sample Calculation

Notice that the velocity that will be plotted at $t = 1.0$ s is the average velocity between $t = 0.0$ s and $t = 2.0$ s. The velocity that will be plotted at $t = 3.0$ s is the average velocity between $t = 2.0$ s and 4.0 s. The acceleration that will be plotted at t = 2.0 s is the average acceleration between $t = 1.0$ s and $t = 3.0$ s.

$$\vec{v}_1 = \frac{\Delta\vec{d}_{0\to2}}{\Delta t_{0\to2}} = \frac{\vec{d}_2 - \vec{d}_0}{t_2 - t_0} = \frac{12\text{ m} - 0.0\text{ m}}{2.0\text{ s} - 0.0\text{ s}}$$
$$= \frac{12\text{ m}}{2.0\text{ s}}$$
$$= 6.0\frac{\text{m}}{\text{s}}$$

$$\vec{v}_3 = \frac{\Delta\vec{d}_{2\to4}}{\Delta t_{2\to4}} = \frac{\vec{d}_4 - \vec{d}_2}{t_4 - t_2} = \frac{36\text{ m} - 12\text{ m}}{4.0\text{ s} - 2.0\text{ s}}$$
$$= \frac{24\text{ m}}{2.0\text{ s}}$$
$$= 12\frac{\text{m}}{\text{s}}$$

$$\vec{a}_2 = \frac{\Delta\vec{v}_{1\to3}}{\Delta t_{1\to3}} = \frac{\vec{v}_3 - \vec{v}_1}{t_3 - t_1} = \frac{12\frac{\text{m}}{\text{s}} - 6.0\frac{\text{m}}{\text{s}}}{3.0\text{ s} - 1.0\text{ s}}$$
$$= \frac{6.0\frac{\text{m}}{\text{s}}}{2.0\text{ s}}$$
$$= 3.0\frac{\text{m}}{\text{s}^2}$$

Complete the table for all average velocities and average accelerations. Then plot position-time, velocity-time, and acceleration-time graphs. On the position-time graph, select one point between 0 and 4 s and one point between 6 and 10 s. Draw tangents to the curve and determine their slopes.

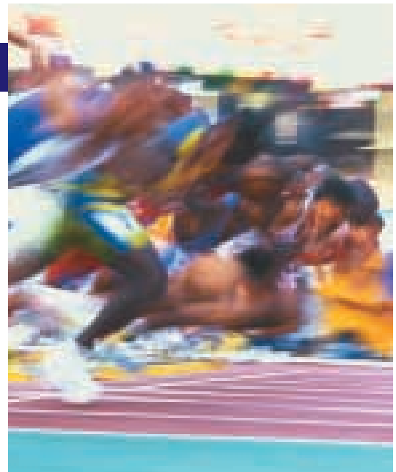
| Time $t$ (s) | Position $\vec{d}$ (m) | Velocity $\frac{\Delta\vec{d}}{\Delta t}$ (m/s) | Acceleration $\frac{\Delta\vec{v}}{\Delta t}$ (m/s$^2$) |
|---|---|---|---|
| 0.0 | 0.0 | | |
| | | 6.0 | |
| 2.0 | 12 | | +3.0 |
| | | 12 | |
| 4.0 | 36 | | |
| | | | |
| 6.0 | 48 | | |
| | | | |
| 8.0 | 96 | | |
| | | | |
| 10.0 | 142 | | |
| | | | |
| 12.0 | 190 | | |
| | | | |
| 14.0 | 226 | | |
| | | | |
| 16.0 | 250 | | |
| | | | |
| 18.0 | 262 | | |

## Analyze and Conclude

1. How well do the average and instantaneous velocities that you calculated agree with each other?
2. Separate the graphs into three sections: (a) 0 s to 8s, (b) 8 s to 12 s, and (c) 12 s to 20 s. For each of these three time periods, compare all three graphs in the following ways.
   - **(a)** How do the shapes of the graphs (curved, straight, horizontal) relate to each other?
   - **(b)** How do the signs of the values (positive, zero, or negative) relate to each other?
3. Under what circumstances can the van be moving but have a zero acceleration?
4. Under what circumstances is the sign of the velocity the same as the sign of the acceleration?
5. What general statement can you make about the motion of the van when the direction of the acceleration vector is opposite to the direction of the velocity vector?

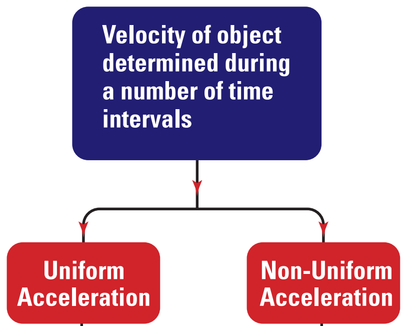
### *Conceptual Problems*

- How do intervals of constant acceleration appear on an acceleration-time graph?
- How do intervals of constant acceleration appear on a velocity-time graph?
- What does a straight-line slope indicate on an acceleration-time graph?
- What would a curved line indicate on an acceleration-time graph?
- Explain circumstances in which an object would be accelerating but have an instantaneous velocity of zero?
- How does uniform acceleration differ from uniform motion?



## Concept Organizer

The acceleration of these sprinters will probably be very large at first and then level off to zero. Their instantaneous acceleration will not be measured and reported. You can only determine an average acceleration if the data taken are only the overall distance and time. What data would you need in order to calculate instantaneous acceleration?



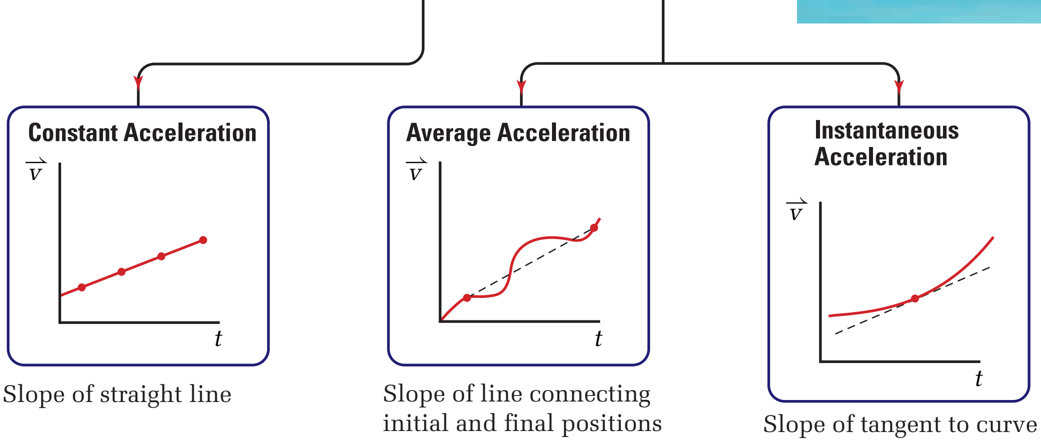




**Figure 2.23** The three ways in which you can describe acceleration are very similar to the ways of describing velocity.

## CANADIANS IN PHYSICS

### Balancing Forces in Structural Engineering

Dr. Jane Thorburn knows about the importance of balancing forces. In her work as a structural engineer, she designed highway bridges for the New Brunswick Department of Transportation. Now a professor at Dalhousie University in Halifax, Nova Scotia, she teaches courses on structural engineering and conducts research on the behaviour of structural steel members.

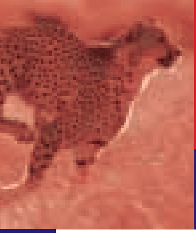
According to Newton's laws, any stable structure — such as a building, bridge, or tower — must produce internal reactions equal in magnitude and opposite in direction to all of the forces acting on it. Structures will be inadequate unless they can balance these external forces, also called "loads."

Structural engineers like Dr. Thorburn determine the right design and materials that will allow a structure to support all possible loads. For example, all structures must be able to support their own mass. They also might have to bear temporary loads related to their use, such as traffic, people, or furniture. At times, a structure might need to balance environmental forces caused by temperature changes, wind, snow accumulation, or earthquakes.

Dr. Thorburn worked on the Hammond River bridge in New Brunswick. When designing this bridge and choosing the materials for its construction, she took several factors into consideration. The bridge had to support the weight of two lanes of traffic. It also had to support its own weight, so the building materials had to be as light as possible. Dr. Thorburn also took environmental factors into account. For example, since the Hammond River is the site of an important salmon run, she wanted to minimize the number of concrete bridge supports, or piers, that were embedded in the riverbed, to reduce any effects that the piers might have on the salmon fishery. In her design, she was able to use only three piers.

Dr. Thorburn also had to determine how the bridge materials could be moved into the construction area, how they would connect together, and how to balance forces during the actual construction of the bridge.

Like most structural engineers, Dr. Thorburn uses special computer software that enables her to create mathematical models of her bridge designs and to determine the impact of external loads on these designs. Her calculation of forces and her understanding of building materials ensure the safety and stability of highway bridges and other structures.



# INVESTIGATION 2-A

## Acceleration Due to Gravity

**TARGET SKILLS**
- Performing and recording
- Identifying variables
- Analyzing and interpreting

One of the most famous stories of physics tells of how Galileo dropped cannonballs of various masses off the leaning Tower of Pisa to disprove the accepted theories of free-fall. Until Galileo's time, natural philosophers (the name for scientists of the time) thought that heavy objects fell "faster" than light objects. However, based on Galileo's thinking and experimentation, scientists now agree that acceleration due to gravity *in a vacuum* is
- uniform
- independent of the mass of the falling object

This investigation challenges you to verify Galileo's model experimentally and to determine the numerical value of the acceleration due to gravity. Since you will be working in air and not a vacuum, your results will provide some information about the effect of air resistance on the acceleration of falling objects.

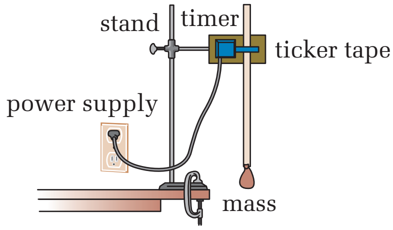
### Problem

Verify that acceleration due to gravity is uniform and independent of mass.
Determine the numerical value of acceleration due to gravity.

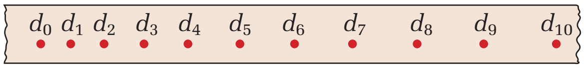
### Equipment

- spark timer
- recording tape
- variety of small objects (rubber stoppers, steel balls, wooden beads, film canisters filled with different amounts of sand) (CAUTION Do not open canisters)
- cellophane tape
- retort stand
- clamp

### Procedure




1. Set your spark timer at 10 Hz so the time between dots will be 0.10 s.
2. Clamp a spark timer to a retort stand. Secure the retort stand close to the edge of a desk or lab bench so that an object pulling the recording tape through the timer can fall to the floor.
3. Attach a small object to a piece of recording tape that is 1 m long.
4. Thread the recording tape through the timer.
5. Hold the object in place. Turn on the timer and release the object.
6. Repeat step 5 for at least three objects of different masses. Collect enough tapes so that each member of your lab group has one tape to analyze.
7. While the object is falling, the timer will record a series of dots on the tape that will look like the diagram on the following page. Locate the first clear dot that marks the beginning of a series of at least 10 time intervals. Label this dot "$\vec{d}_0$" to designate it as the origin of your frame of reference. Label the next 10 dots "$\vec{d}_1$," "$\vec{d}_2$," ..., "$\vec{d}_{10}$" to mark the position of the object at the end of each of 10 time intervals.
8. Make a table with the following headings: Position, Time, Time interval, Displacement, Average velocity, Change in velocity, Acceleration.




9. Use the label "$t_0$" for the instant in time at which the object is at position $\vec{d}_0$. Record the time and position of the object for the sequence of 10 positions following your designation of $\vec{d}_0$.

10. Complete the table by performing the indicated calculations.

11. Construct position-time, average velocity-time, and average acceleration-time graphs.

## Analyze and Conclude

1. Using the slope of your average velocity-time graph, calculate the value of the acceleration.

2. Compare your value of acceleration calculated in the above step to the values in your table that you calculated for individual intervals. Do they agree or is there a significant difference among them? If they differ, which values do you think are more accurate?

3. Compare the average acceleration determined from your velocity-time graph with the values determined by (a) other members of your group and (b) other groups in your class. Do the masses of the objects appear to have any influence on the value calculated for acceleration? If so, what effect does mass appear to have?

4. Considering all of the comparisons you have just made, does your class data support the model that says that acceleration due to gravity is constant and independent of mass? If not, explain how your class data contradict the model. How might you account for any discrepancies?

5. The accepted value for acceleration due to gravity is 9.81 m/s$^2$. Calculate the percent deviation of your own calculated value from the accepted value. If you need to review the method for determining percent deviation, go to Skill Set 1.

6. Calculate an average of the class data for the acceleration due to gravity. Omit any data points that are extremely different from the majority of the values. Calculate the percent deviation of the class average value to the accepted value. How might you account for any discrepancies?

7. Discuss how well (or poorly) your class data support the accepted value of 9.81 m/s$^2$ for the acceleration due to gravity.

8. Describe any factors that might be affecting the free-fall of each object as it pulls the recording tape through the timer.

9. How might air friction affect your data?

10. Discuss any other possible reasons for a deviation from the accepted value for acceleration due to gravity in a vacuum.

11. Identify possible errors that could have arisen during your experiment and suggest refinements to your procedure to minimize these errors.

12. Identify and discuss any evidence that the shape of an object affects its free-fall acceleration.

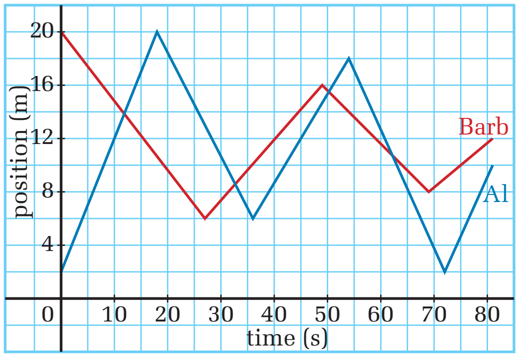
## Apply and Extend

13. Design and conduct an investigation to determine how the shape of an object affects its acceleration due to gravity in air. Determine what shape is the most aerodynamic; that is, determine what shape allows the object to accelerate downward with an acceleration as close to 9.81 m/s$^2$ as possible? (**Note:** To correctly test for the effect of shape, each object must have the same mass.)

**CAUTION** Get your teacher's approval.

# 2.4 Section Review

1. (I) The following graphs represent the motion of two students, Al and Barb, walking back and forth in front of the school, waiting to meet friends.




(a) During what periods of time are Al and Barb walking in the same direction?

(b) At what points do Al and Barb meet?

(c) During what periods of time are Al and Barb facing each other?

(d) Which student is, on the average, walking faster than the other? Explain your reasoning.

2. (K/U) Describe the similarities and differences between:

(a) constant acceleration and non-uniform acceleration.

(b) average acceleration and instantaneous acceleration.

3. (C) Explain the relationship between:

(a) tangent line on a velocity time graph, time interval, and acceleration.

(b) negative acceleration and deceleration.

(c) m/s and m/s$^2$

4. (K/U) How is the direction of an acceleration vector determined?

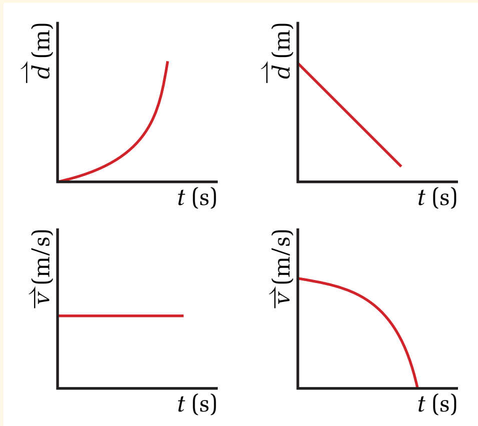
5. (K/U) Describe a motion when:

(a) velocity and acceleration vectors are in the same direction.

(b) velocity and acceleration vectors are in opposite directions.

6. (I) Throw a ball up into the air (from rest) and catch it. Sketch the path of the ball. Label points where the vertical velocity is zero. Label points where the acceleration is zero.

7. (I) Draw conclusions about the acceleration of the motion represented by the following graphs.




8. (I) Design a simulation on interactive physics software to verify that acceleration due to gravity is uniform and independent of mass.

**UNIT PROJECT PREP**

Vehicles and objects are commonly seen speeding up, slowing down, and changing direction in motion pictures.

- What do you feel when a vehicle starts or stops suddenly? How can you use this in your "virtual reality"?
- An object accelerating towards the ground can be a dramatic situation. What effects can you use to simulate this?