Constant, Average, and Instantaneous Velocity

How is the motion of a tortoise similar to the motion of a jet aircraft? How does the motion of a jet cruising at 10 000 m differ from that of a space shuttle lifting off?

Physicists have classified different patterns of motion and developed sets of equations to describe these patterns. For example, **uniform motion** means that the velocity (or rate of change of position) of an object remains constant. A tortoise and a cruising jet might be travelling with extremely different velocities, yet they move for long periods without *changing* their velocities. Therefore, both tortoises and jet airplanes often travel with uniform motion.

Non-uniform motion means that the velocity *is* changing, either in magnitude or in direction. A cruising jet is travelling at a constant velocity, or with uniform motion, while a space shuttle changes velocity dramatically during lift-off. The shuttle travels with non-uniform motion.

In the last section, you studied motion by looking at "snapshots" in time. You had no way of knowing what was happening between the data points. The only way you could report velocity was as an *average* velocity. Clearly, you need more data points to infer that an object is moving with uniform motion, or at a constant velocity. Graphing your data points provides an excellent tool for analyzing patterns of motion and determining whether the motion is uniform or non-uniform.

SECTION

- Identify the frame of reference for a given motion and distinguish between fixed and moving frames.
- Identify and investigate problems involving motion.
- Analyze word problems, solve algebraically for unknowns, and interpret patterns in data.

KEY TERMS

- uniform motion
- non-uniform motion
- instantaneous velocity
- tangent

Constant Velocity

2.3

Is the motion of the skateboarder in Figure 2.11 uniform or non-uniform? To answer questions such as this, you should organize the data in a table (see Table 2.3) and then graph the data as shown in Figure 2.12. This plot is called a "position-time graph." When you plot the points for the skateboarder, you can immediately see that the plot is a straight line, with an upward slope. **Figure 2.11** Is a skateboarder's motion uniform or non-uniform?



Time (s)	Position (m[E])
0.0	0.0
1.3	12
2.6	24
3.9	36
5.2	48
6.5	60



Table 2.3 Position versus Time for a Skateboarder

Figure 2.12 Position-time graph for a skateboarder's motion

To determine the significance of the straight line, consider the meaning of the slope. Start with the mathematical definition of slope.

• Slope is the rise over the run.	slope = $\frac{rise}{run}$
 On a typical <i>x-y</i> plot, the slope is written as 	slope $=\frac{\Delta y}{\Delta x}$
 However, on a position-time plot, the slope is 	slope = $\frac{\Delta \vec{d}}{\Delta t}$
The definition of velocity is	$\overrightarrow{v} = \frac{\Delta \overrightarrow{d}}{\Delta t}$
 Since the slope and the velocity are equal to the same expression, the slope of the line on a position- time graph must be the velocity of the object. 	\overrightarrow{v} = slope
	I



process of graphing motion.

Since you now know that the slope of a position-time graph is the velocity of the moving object, the straight line on the graph of the skateboarder's motion is very significant. The slope is the same everywhere on a straight line, so the skateboarder's velocity must be the same throughout the motion. Therefore, the skateboarder is moving with a constant velocity, or uniform motion. You could take any two points on the graph and calculate the velocity. For example, use the first and last points.

slope =
$$\frac{\Delta \vec{d}}{\Delta t}$$

slope = $\frac{60 \text{ m[E]}}{6.5 \text{ s}}$
slope = $9.2 \frac{\text{m}}{\text{s}}$ [E]
 \vec{v} = slope
 \vec{v} = $9.2 \frac{\text{m}}{\text{s}}$ [E]

Mathematically, this is the same way that you calculated average velocity. What, then, is the difference between *average velocity* and *constant velocity*? Think back to the snapshot analogy and the case of Freda's typical school day. Over several hours, you had only two points to consider — the beginning and the end of the motion.

In the case of the skateboarder, you have several points between the beginning and end of the motion and they are all consistent, giving the same velocity. Nevertheless, you still cannot know exactly what happened between each measured point. Although it is reasonable to think that the motion was uniform throughout, you cannot be sure. Strictly speaking, you can calculate only an average velocity for each small interval. Without continuous data, you cannot be certain that an object's velocity is constant.

To emphasize this point, use your imagination to fill in what could be happening between your observation points for a master skateboarder and for a novice struggling to stay on the board. Examine the graphs in Figures 2.13 and 2.14. What is the average velocity for each time interval on each graph? Is the rate of change of position between time intervals constant on each graph? Based on the data, is there a difference between the average velocities of each skateboarder?



Figure 2.13 The sharp curves in the graph indicate that the skate boarder's velocity was constantly changing. You would expect this jerky motion from a novice skateboarder.



Figure 2.14 Since the line is nearly straight the velocity is almost constant. This motion is what you would expect from a master skateboarder.

оціск Maintaining a LAB Constant Pace

Generate a position-time graph using Method A or Method B.

Method A: Motion Sensor

Use a motion sensor and computer interface to generate a position-time graph of your own motion as you walk toward the sensor, while trying to maintain a constant pace.

Method B: Spark Timer

Pull a piece of recording tape, about 1 m long, through a spark timer, while trying to maintain a constant velocity. Examine the recording tape and locate a series of about 10 dots that seem to illustrate a period of constant velocity. On the recording tape, label the first dot in the series \vec{d}_0 . Mark an arrow on your tape to show the direction of the motion. Make a data table to record the position and time of the 10 dots that immediately follow your labelled starting point, \vec{d}_0 . Draw a position-time graph based on your data table.

Analyze and Conclude

- Explain, using the graph as evidence, whether you were successful in maintaining a constant velocity. If you were not successful for the entire timing, were you successful for at least parts of it?
- **2.** Determine your average velocity for the entire timing period and your constant velocity for appropriate segments of your walk by calculating the relevant slopes of the graph.

TARGET SKILLS

- Identifying variables
- Performing and recording
- Modelling concepts

Apply and Extend

- **3.** Explain for each of the following situations whether you can determine if the person or object is maintaining a constant velocity.
 - (a) A student leaves home at 8:00 a.m. and arrives at school at 8:30 a.m.
 - (b) A dog was observed running down the street. As he ran by the meat store, the butcher noted that it was 10:00 a.m. When he ran by the bakery, it was 10:03 a.m. A woman in the supermarket saw him at 10:05 a.m. Finally, he reached home at 10:10 a.m.
 - (c) A swimmer competes in a 50 m backstroke race. Three judges, each with a stopwatch, timed her swim. Their stopwatches read 28.65 s, 28.67 s, and 28.65 s.
- **4.** A spark timer generated the recording tape, shown here, as a small cart rolled across a lab bench.
 - (a) Set up a data chart to record the positions and times for the 8 dots after \vec{d}_0 . The spark timer was set at a frequency of 60 Hz, thus making 60 dots per second.
 - (b) Draw a position-time graph.
 - (c) State whether the graph shows a constant velocity for the whole time period under observation or for only segments of it. Explain your reasoning.
 - (d) Calculate the value of a segment of constant velocity.
 - (e) Calculate the average velocity for the entire time period.





Figure 2.15 The numerical value of the velocities represented in the graphs are all the same but the concepts are quite different.

Average Velocity and Changing Directions

You have seen how a position-time graph helps you to determine whether motion is uniform or non-uniform. These graphs are even more helpful when doing a detailed analysis of non-uniform motion. Consider the situation illustrated in Figure 2.16. Adrienne drives her friend Jacques home from school. Jacques lives 800 m east of the school and Adrienne lives 675 m west. The diagram shows Adrienne's position at several specific times.

The data in Figure 2.16 are organized in Table 2.4 and graphed in Figure 2.17. Notice that Adrienne stops for 10 s to let Jacques out and then turns around and goes in the opposite direction.







Table 2.4 Data for Adrienne's Trip

Time (s)	Position (m)
0.0	0.0
15	100 [E]
90	800 [E]
105	800 [E]
120	675 [E]
148	300 [E]
171	0.0
194	300 [W]
210	525 [W]
225	625 [W]

Clearly, the position-time graph of Adrienne's journey shows that her velocity is not constant for her entire trip home. You can see from the changing slopes on the graph that not only is she speeding up and slowing down, she is also changing direction.

Note: In the calculations to the right, you will see that the choice of time intervals for determining average velocity can lead to unreasonable results.

Figure 2.17 Position-time graph of Adrienne's car trip

Segment of Trip From school to Jacques' home From the 15 s mark to Adrienne's home	
Initial and	Final Times
t = 0 s to $t = 90$ s	t = 15 s to t = 225 s
(See dashed red line on graph)	(See dashed blue line on graph)
Average	Velocity
slope $= \frac{rise}{run}$	slope $= \frac{rise}{run}$
slope $=\frac{\Delta \overrightarrow{d}}{\Delta t}$	slope $= \frac{\Delta \vec{d}}{\Delta t}$
slope = $\frac{800 \text{ m}[\text{E}] - 0.0 \text{ m}}{90 \text{ s} - 0.0 \text{ s}}$	slope = $\frac{625 \text{ m}[\text{W}] - 100 \text{ m}[\text{E}]}{225 \text{ s} - 15 \text{ s}}$
slope = $\frac{800 \text{ m[E]}}{90 \text{ s}}$	slope = $\frac{(-625 \text{ m[E]}) - 100 \text{ m[E]}}{210 \text{ s}}$
slope = 8.88 $\frac{\text{m}}{\text{s}}$ [E]	slope = $\frac{-725 \text{ m[E]}}{210 \text{ s}}$
$\overrightarrow{v} \approx 9 \frac{\mathrm{m}}{\mathrm{s}} \mathrm{[E]}$	slope = $-3.45 \frac{\text{m}}{\text{s}}[\text{E}]$
	$\overrightarrow{v} \approx 3 \ \frac{\mathrm{m}}{\mathrm{s}} [\mathrm{W}]$
Notice that, if you consider east to be positive,	
then west is equivalent to negative east.	

You have probably concluded that the average velocity from the 15 s point to Adrienne's home does not seem reasonable. If you convert 3 m/s to km/h, the result is approximately 11 km/h. As well, the direction is west, but you know that Adrienne started the trip going east. The seemingly unreasonable average velocity is due to the *definition* of average velocity.

оціск Velocity and LAB Time Intervals

The diagram records the flight of a hawk soaring in the air looking for prey. Using videotape footage, the observer recorded the position of the hawk at 2.0 s intervals. He plotted the points and connected them with a smooth curve.



To estimate the hawk's velocity at precisely t = 8.0 s, determine its average velocity at several intervals that include the 8.0 s mark. Observe any changes in the calculated values for average velocity as the interval becomes smaller. Then, draw conclusions based on the following steps.

- **1**. Reconstruct the graph on a piece of graph paper.
- **2.** Draw straight lines on the curve connecting the following pairs of points.
 - (a) 5.0 s and 11.0 s
 - (b) 6.0 s and 10.0 s
 - (c) 7.0 s and 9.0 s
 - (d) 7.5 s and 8.5 s

TARGET SKILLS

- Analyzing and interpreting
- Communicating results
- **3.** Determine the hawk's average velocity for each of the pairs of points in step 2.
- **4.** Draw one more straight line between points on opposite sides of the 8.0 s point and as close as possible to the 8.0 s mark. Extend the straight line as far as possible on the graph. Determine the slope of the straight line by choosing any two points on the line and calculating $\frac{\vec{d}_2 \vec{d}_1}{t_2 t_1}$, for those points.

Analyze and Conclude

- **1.** Why were the calculated average velocities different for the different pairs of points?
- **2.** What do you think is the meaning of the slope that you calculated in step 4?
- **3**. Describe the relationship among the five velocities that you calculated.
- **4.** Evaluate, in detail, the process you just performed. From your evaluation, propose a method for determining the velocity of an object at one specific time, rather than an average between two time points.
- **5.** Using your method, determine the velocity of the hawk at exactly 3.0 s and 5.0 s.

• Conceptual Problems

- Describe circumstances in which the average velocity of a segment of a trip is very close to the reading you would see on the speedometer of a car.
- Describe circumstances in which the average velocity of a segment of a trip appears to totally contradict reason. Explain why.

Instantaneous Velocity

When you apply the definition of average velocity to points on a graph that are far apart, sometimes the resulting value is extremely unreasonable as you discovered in the example of Adrienne's trip. When you bring the points on the graph closer and closer together, the calculated value of the velocity is nearly always very reasonable. In the Quick Lab, you brought the points so close together that they merged into one point. To perform a calculation of velocity, you had to draw a tangent line and use two points on that line. The value that you obtain in this way is called the **instantaneous velocity**. It might seem strange to define a velocity at one instant in time when velocity was originally defined as a *change* in position over a time *interval*. However, as you saw in the Quick Lab, you can make the time interval smaller and smaller, until the two points actually meet and become one point.

• Conceptual Problems

- A jet-ski is able to maintain a constant speed as it turns a corner. Describe how the instantaneous speed of the jet-ski will differ from its instantaneous velocity during the turning process?
- The concept organizer on page 50 illustrates how to calculate the average velocity from a position versus time graph.
 - Sketch a position time graph with a smooth curve having increasing slope. Select and mark two points on the curve. Draw in a dotted line to represent the average velocity between those two points.
 - **2.** Now mark a point directly between the first two points. How would the average velocity for the very small time interval represented by the single dot look? Sketch it.
- Draw a concept organizer to show how position, displacement, average velocity, instantaneous velocity, and time are related.

The straight lines that you drew between points on the curve are called *chords* of the curve. When the straight line finally touches only one point on the curve, it becomes a **tangent** line. The magnitude of the velocity of an object at the point where the tangent line touches the graph is the slope of the tangent. You now have the tools to do a detailed analysis of position-time data.

INSTANTANEOUS VELOCITY

The *instantaneous velocity* of an object, at a specific point in time, is the *slope of the tangent* to the curve of the position-time graph of the object's motion at that specific time.

Note: Although average velocity is symbolized as \overrightarrow{v}_{ave} , a subscript is not typically used to indicate instantaneous velocity. When a subscript is present, it usually refers to the time or circumstances represented by that specific instantaneous velocity.



D Math Link

One, and only one, tangent line exists at any one point on a curve. Notice in the diagram that if the slope of the tangent line is changed, either increased or decreased, the line then cuts two points on the curve. It is no longer a tangent line but is now a *secant* line. A secant line intersects a curve at two points and continues beyond those points. How is the tangent line related to the trigonometric function named tangent?



MODEL PROBLEM

Determining Instantaneous Velocity

The plot shown here is a position-time graph of someone riding a bicycle. Assume that position zero is the cyclist's home. Find the instantaneous velocity for at least nine points on the curve. Use the calculated values of velocity to draw a velocity-time graph. In your own words, describe the bicycle ride.

600 E 400 1 200 0 200 400 600 800 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Frame the Problem

- Between 0 s and 100 s, the graph is a straight line. Therefore, the velocity for that period of time is constant. Label the segment of the graph "A."
- Between 100 s and 350 s, the graph is curved, indicating that the velocity is changing. Label the segment of the graph "B."
- Between 350 s and 450 s, the graph is horizontal. There is no change in position, so the velocity is zero. Label the segment of the graph "C."
- At 450 s, the cyclist turns around and starts toward home. Up to 600 s, the graph is a curve, so the velocity is changing. Label the segment "D."

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- Between 600 s and 700 s and again between 700 s and 900 s, the graph forms straight lines. The velocity is constant during each period. Label those sections "E" and "F."
- At 900 s, the cyclist is back home.
- The motion is in one dimension so denote direction by positive and negative values.

Identify the Goal

Find the value of the instantaneous velocity at nine points in time. Draw a velocity-time graph.

Variables and Constants

Known	Unknown
\overrightarrow{d}_n	\overrightarrow{V}_n
t_n	

For all points (n) from t = 0 s to t = 900 s

Strategy

Redraw the graph with enough space below it to draw the velocity-time graph on the same time scale.



Time period or point

 t_0 to

 t_{150}

 t_{225}

 t_{300}

 t_{350}

 t_{475}

Identify the linear segments of the position-time graph. Select at least five points on the non-linear segments. Draw lines that are tangent to the graph at these points.

Calculate the velocity for each linear segment of the graph and the points at which you have drawn tangent lines. Record the velocities in a table.

Plot the points on the velocity-time graph. Connect the points with a smooth curve where the points do not form a straight line.

P	
	$\frac{\overrightarrow{d}_2 - \overrightarrow{d}_1}{t_2 - t_1} = \overrightarrow{v}$
t_{100}	$\frac{200 \text{ m} - 0 \text{ m}}{100 \text{ s} - 0 \text{ s}} = 2.0 \frac{\text{m}}{\text{s}}$
	$\frac{350 \text{ m} - 200 \text{ m}}{200 \text{ s} - 100 \text{ s}} = 1.5 \frac{\text{m}}{\text{s}}$
	(not shown) = 0.80 $\frac{\text{m}}{\text{s}}$
	$\frac{450 \text{ m} - 360 \text{ m}}{450 \text{ s} - 250 \text{ s}} = 0.45 \frac{\text{m}}{\text{s}}$
to <i>t</i> ₄₅₀	slope is zero, $\overrightarrow{v} = 0.0 \frac{\text{m}}{\text{s}}$
	$\frac{210 \text{ m} - 400 \text{ m}}{530 \text{ s} - 450 \text{ s}} = -2.4 \frac{\text{m}}{\text{s}}$

t_{550}	$\frac{180 \text{ m} - 300 \text{ m}}{700 \text{ s} - 490 \text{ s}} = -0.57 \frac{\text{m}}{\text{s}}$
t_{600} to t_{700}	$\frac{100 \text{ m} - 250 \text{ m}}{700 \text{ s} - 600 \text{ s}} = -1.5 \frac{\text{m}}{\text{s}}$
t_{700} to t_{900}	$\frac{0.0 \text{ m} - 100 \text{ m}}{900 \text{ s} - 700 \text{ s}} = -0.50 \frac{\text{m}}{\text{s}}$

- **A**: The cyclist is riding at a constant velocity, away from home.
- **B**: The cyclist slows down and, at the end of the segment, stops.
- **C**: The cyclist is not moving.
- **D**: The cyclist starts toward home at a high velocity, then slows. The position vector is positive, because the cyclist is at a positive position in relation to home. However, the velocity is negative because the cyclist is moving in a negative direction, toward home.
- **E**: The cyclist is still heading toward home, but at a constant velocity.
- **F**: The cyclist slows even more but is still at a positive position and a negative velocity.

Validate

The slopes of the curve in A and B are positive (line and tangents go up to the right); therefore, the velocities should all be positive. They are.

The slope in C is zero so the velocity should be zero. It is.

The slopes of the curve in D, E, and F are all negative (lines and tangents go down to the right); therefore, the velocities should be negative. They are.

PRACTICE PROBLEMS

4. Redraw the position-time graph shown here. Determine the velocity in each of the linear segments and for at least three points along the curved section. Use the calculated velocities to draw a velocity-time graph of the motion. Explain the circumstances that make the position vector negative and the velocity vector positive.



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5. Using the data table, draw a position-time graph. For points that do not lie on a straight line, connect the points with a smooth curve. Calculate the velocity for a sufficient number of points so that you can draw a good velocity-time graph.

Time (s)	Position (m)	Time (s)	Position (m)
0.0	0.0	16.0	0.0
2.0	-10.0	17.0	10.0
4.0	-20.0	18.0	20.0
6.0	-30.0	20.0	25.0
8.0	-36.0	22.0	30.0
10.0	-38.0	24.0	26.6
12.0	-32.0	26.0	23.3
13.0	-27.0	28.0	20.0
14.0	-10.0	30.0	0.0

Concept Organizer

What type of velocity is the police officer measuring with the radar gun? A radar gun takes data points that are so close together, that it measures instantaneous velocity. If the car is moving with a constant velocity, the instantaneous velocity is the same as the constant velocity. If the car's velocity is changing, the radar gun will not measure average velocity. How would you measure a car's average velocity? To understand and report data correctly, you need to know how a measuring instrument works as well as knowing the precise meaning of specific terms.



Figure 2.18 This concept organizer will help you understand and remember three ways in which you can describe velocity.

QUICK Rocket Motion

Imagine that you fire a toy rocket straight up into the air. Its engine burns for 8.0 s before it runs out of fuel. The rocket continues to climb for 4.0 s, then stops and begins to fall back down. After falling freely for 4.0 s, a parachute opens and slows the descent. The rocket then reaches a terminal velocity of 6.0 m/s[down]. Using videotape footage, you determined the rocket's altitude, h, at 2 s intervals. Your data are shown in the table.



Make a motion analysis table like the one shown here, but add three more columns and label them: Time interval, Displacement, and Average velocity. Perform the indicated calculations for all intervals between the points listed, then complete the table.

Plot an average velocity-time graph. Remember, when you plot *average* velocity, you plot the point that lies at the midpoint of the time interval. For example, when you plot the average velocity for the interval from 2 s to 4 s, you plot the point at 3 s. Draw a smooth curve through the points.

Determine the instantaneous velocity at t = 7 s, t = 13 s, t = 21 s, and t = 25 s.

Analyze and Conclude

 In your own words, describe the motion of the rocket during each of the five stages.

TARGET SKILLS

- Analyzing and interpreting
- Communicating results

Phase	Time (seconds)	Position (metres[up])
1	0	0
engine on	2	10
	4	40
	6	90
	8	160
2	10	220
engine off (rising)	12	240
3	14	220
engine off (falling)	16	160
4	18	92
parachute opens	20	48
	22	28
5	24	20
terminal velocity	26	12
5	28	4
	30	0

- **2.** What is the condition of the position-time graph when the velocity-time graph passes through zero? Explain the meaning of this point.
- **3.** Under what specific conditions is the velocity-time graph a straight, horizontal line?
- 4. Compare the instantaneous velocities that you calculated for times 7 s, 13 s, 21 s, and 25 s, with the average velocities for the intervals that included those times. In which cases are the instantaneous and average velocities nearly the same? Quite different? Explain why.
- 5. Explain why it is reasonable to draw the line connecting the points on the position-time graph as a smooth curve rather than connecting the dots with a straight line.

2.3 Section Review

- **1.** C Explain why the following situations do not represent uniform motion.
 - (a) driving through downtown at rush hour
 - (b) start and stop sport drills that are executed at top speed
 - (c) pendulum swinging with a constant frequency
 - (d) a ball rolling down a ramp
 - (e) standing on a merry go round that rotates at a constant number of revolutions per minute
- 2. D Analyze the following position time graph and sketch the velocity time graph of the same data.



- **3**. **C** Explain the relationship between:
 - (a) slope and position time graphs.
 - (b) slope and velocity time graphs.
 - (c) average velocity, constant velocity, and instantaneous velocity.
 - (d) tangent line on a position time graph, and time interval.
 - (e) negative time and a position time graph.
 - (f) velocity, acceleration, and terminal velocity.
 - (g) m/s and km/h.
- 4. **Me** Both physicists and mathematicians use observations from the physical world to create theories. Suggest criteria that separates physicists from mathematicians.
- 5. We A boy and a girl are going to race twice around a track. The girl's strategy is to run each lap with the same speed. A boy is going to run the first lap slower so he can run the final lap faster. Suppose that the girl's speed is x m/s for both laps and the boy's speed for the first lap is (x 2) m/s, and for the second lap is (x + 2) m/s. Decide who will win and justify your response. If you need help, try using real numbers for the speeds.
- 6. **K**D By what factor does velocity change if the time interval is increased by a factor of three and the displacement is decreased by a factor of two?