2.2 Displacement and Velocity

In the last section, you saw how diagrams allow you to describe motion qualitatively. It is not at all difficult to determine whether an object or person is at rest, speeding up, slowing down, or moving at a constant speed. Physicists, however, describe motion quantitatively by taking measurements.

From the diagrams you have analyzed, you can see that the two fundamental measurements involved in motion are distance and time. You can measure the distance from a reference point to the object in each frame. Since a known amount of time elapsed between each frame, you can determine the total time that passed, in relation to a reference time, when the object reached a certain location. From these fundamental data, you can calculate an object's position, speed, and rate of change of speed at any particular time during the motion.

SECTION OUTCOMES

- Use vectors to represent position, displacement, and velocity.
- Describe and provide examples of how the position and displacement of an object are vector quantities.
- Analyze word problems and solve algebraically for unknowns.

KEY TERMS

- scalar
- vector
- position
- displacement
- velocity
- acceleration
- time interval
- speed

Vectors and Scalars

Most measurements that you use in everyday life are called scalar quantities, or **scalars**. These quantities have only a magnitude, or size. Mass, time, and energy are scalars. You can also describe motion in terms of scalar quantities. The distance an object travels and also the speed at which it travels are scalar quantities.

In physics, however, you will usually describe motion in terms of vector quantities, or **vectors**. In addition to magnitude, vectors have *direction*. Whereas distance and speed are scalars, the **position**, **displacement**, **velocity**, and **acceleration** of an object are vector quantities. Table 2.1 lists some examples of vector and scalar quantities. A vector quantity is represented by an arrow drawn in a frame of reference. The length of the arrow represents the magnitude of the quantity and the arrow points in the direction of the quantity within that reference frame.

Table 2.1 Examples of Scalar and Vector Quantities

Scalar quantities		Vector quantities	
Quantity	**Example**	**Quantity**	**Example**
distance	15 km	displacement	15 km[N45°E]
speed	30 m/s	velocity	30 m/s [S]
		acceleration	9.81 m/s^2 [down]
time interval	10 s		
mass	6 kg		

Note: There is no scalar equivalent of acceleration.

Position Vectors

A position vector locates an object within a frame of reference. You will notice in Figure 2.6A that an *x*-*y* coordinate system has been added to the diagram of the sprinting stick figures. The coordinate system allows you to designate the zero point for the variables under study and the direction in which the vectors are pointing. It establishes the *origin* from which the position of an object can be measured. The position arrow starts at the origin and ends at the location of the object at a particular instant in time. In this case, the sprinter is the object.

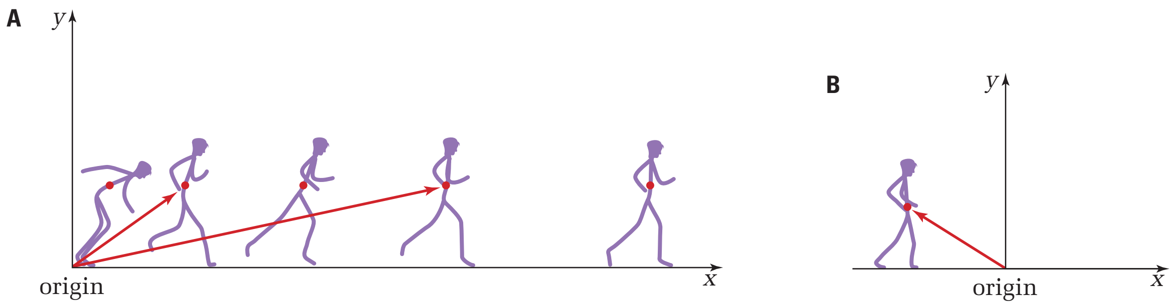


Figure 2.6 **(A)** A coordinate system and position vectors have been added to the stick diagram. **(B)** As the sprinter walks toward the origin, the sprinter's position is negative in this coordinate system.

POSITION VECTOR

A position vector, $\vec{d}$, points from the origin of a coordinate system to the location of an object at a particular instant in time.

As you can see in Figure 2.6A, vectors locate the sprinter's position for two of the five different points in time. Time zero is selected as the instant at which the sprint started. However, as shown in Figure 2.6B, you can show the sprinter several seconds before the race. Her position is to the left of the origin as she is walking up to the starting position. Thus, it is possible to have negative values for positions and times in a particular frame of reference.

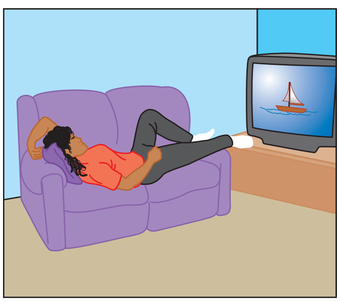


Figure 2.7 Could this be an example of displacement?

Displacement

Although you might think you know when an object is moving or has moved, you can be fooled! Pay close attention to the scientific definition of displacement and you will have a ready denial for the next time you are accused of lying around all day.

The **displacement** of an object, $\Delta\vec{d}$, is a vector that points from an initial position, $\vec{d}_1$, from which an object moves *to* a second position, $\vec{d}_2$, in a particular frame of reference. The vector's magnitude is equal to the straight-line distance between the two positions.

DISPLACEMENT

Displacement is the vector difference of the final position and the initial position of an object.

$$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$$

Quantity	**Symbol**	**SI unit**
displacement	$\Delta\vec{d}$	m (metre)
final position	$\vec{d}_2$	m (metre)
initial position	$\vec{d}_1$	m (metre)

Notice in the boxed definition that displacement depends only on the initial and final positions of the object or person. It is like taking snapshots of a person at various points during the day and not knowing or caring about anything that happened in between.

To see how the definition of displacement affects your perception of motion, follow a typical student, Freda, through a normal day. Figure 2.8 is a map of Townsville, where Freda lives. The map is framed by a coordinate system with its origin at Freda's home, position $\vec{d}_0$. In Table 2.2, you are given her position at six times during the day. What can you learn about her displacement from these data?

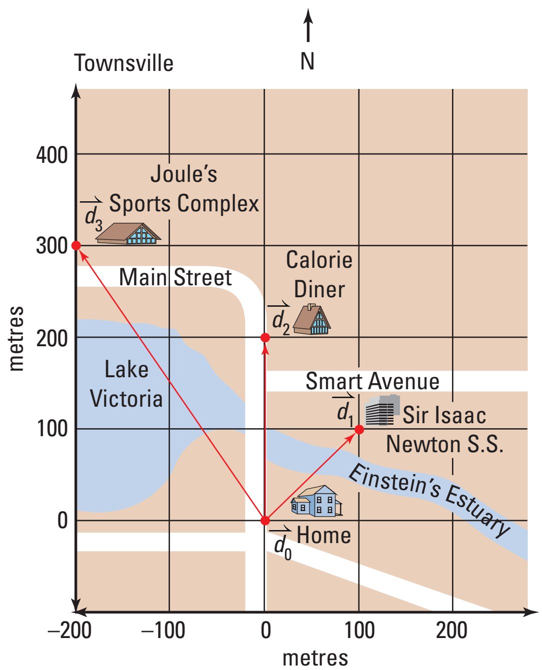


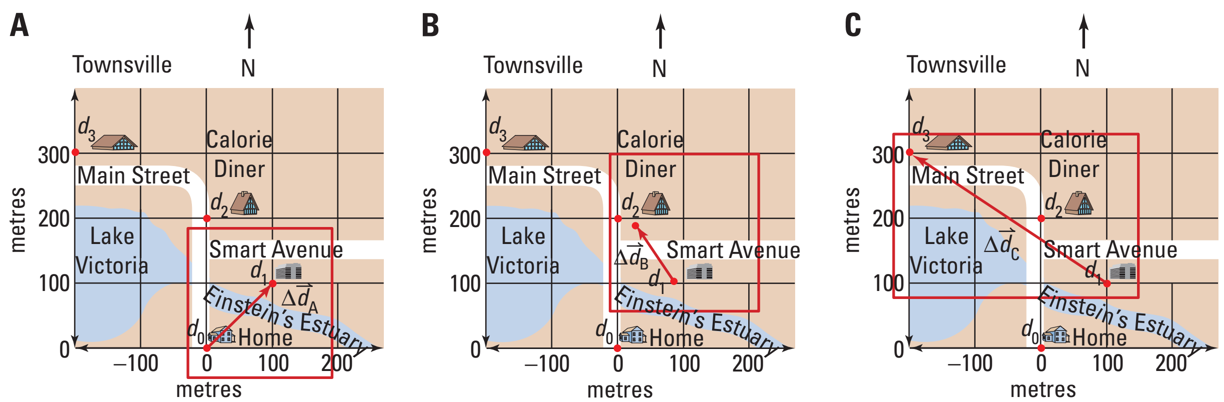
Figure 2.8 Freda's town

Table 2.2 Freda's Typical Daily Schedule

Time	Location	Position	Activity
6:30 a.m.	home	$\vec{d}_0$	sleeping
9:00 a.m.	school	$\vec{d}_1$	studying physics
12:00 noon	diner	$\vec{d}_2$	eating lunch
2:00 p.m.	school	$\vec{d}_1$	studying physics
5:00 p.m.	sports complex	$\vec{d}_3$	playing squash
10:00 p.m.	home	$\vec{d}_0$	sleeping

You can determine Freda's displacement for any pair of position vectors. To develop a qualitative understanding of displacement, consider the following examples.

Figure 2.9 Freda's displacement from **(A)** home to school, **(B)** school to diner, and **(C)** school to sports complex



Home to School

6:30 a.m. to 9:00 a.m.

$\vec{d}_0$ to $\vec{d}_1$

$\Delta\vec{d}_A = \vec{d}_1 - \vec{d}_0$

Since $\vec{d}_0 = 0$, $\Delta\vec{d}_A = \vec{d}_1$

Scale measurement would show that $\Delta\vec{d}_A = 140$ m and points northeast.

School to Lunch

9:00 a.m. to 12:00 noon

$\vec{d}_1$ to $\vec{d}_2$

$\Delta\vec{d}_B = \vec{d}_2 - \vec{d}_1$

Scale measurement would show that $\Delta\vec{d}_B = 140$ m, but points northwest.

School to Sports Complex

9:00 a.m. to 5:00 p.m.

$\vec{d}_1$ to $\vec{d}_3$

$\Delta\vec{d}_C = \vec{d}_3 - \vec{d}_1$

Scale measurement would show that $\Delta\vec{d}_C = 360$ m, and points a little west of northwest.

By now, you have probably discovered the important difference between measuring the *distance* a person travels and determining the person's *displacement* between two points in time. You know that Freda covered a much greater distance during the day than these displacements indicate. Suppose that someone observed Freda only at 6:30 a.m. and at 10:00 p.m. Her position at both of those times was the same — she was in bed. Despite the fact that she had a very energetic day, her displacement for this time interval is zero. Imagine what her reaction would be if she was accused of lying around all day.

Conceptual Problems

- Use the scale map of Townsville in Figure 2.8 to estimate the minimum distance that Freda would walk while following her daily schedule. Compare this value to her displacement for the day.
- Determine Freda's displacement when she walks from the sports complex to her home.
- On a piece of graph paper, draw a scale map of your home and school area. Mark on it the major locations that you would visit on a typical school day. Frame the map with a coordinate system that places your home at the origin, the positive *x*-axis pointing east and the positive *y*-axis pointing north. Label your home position $\vec{d}_0$ and designate the other locations $\vec{d}_1$, $\vec{d}_2$, and so on. Determine your displacement and estimate the distance you travel
  (a) from home to school
  (b) from school to home
  (c) from school to a location that you visit after school
  (d) from a location that you visit after school to home
  (e) from the time you get out of bed to the time you get back into bed
- In the following situation, choose the correct answer and explain your choice. A basketball player runs down the court and shoots at the basket. After she arrives at the end of the court, her displacement is
  (a) either greater than or equal to the distance she travelled
  (b) always greater than the distance she travelled
  (c) always equal to the distance she travelled
  (d) either smaller than or equal to the distance she travelled
  (e) always smaller than the distance she travelled
  (f) either smaller or larger than, but not equal to, the distance she travelled

Time and Time Intervals

The second fundamental measurement you will use to describe motion is *time*. In the example of Freda's schedule, you used clock time. However, in physics, clock time is very inconvenient, even if you use the 24 h clock. In physics, the time at which an event begins is usually designated as time zero. You might symbolize this as $t_0 = 0$ s. Other instants in time are measured in reference to t_0 and designated as t_{n}. The subscript "n" indicates the time at which a certain incident occurred during the event.

The elapsed time between two instants of time is called a **time interval**, Δt. Notice the difference between t_n and Δt: t_n is an instant of time and Δt is the time that elapses between two incidents.

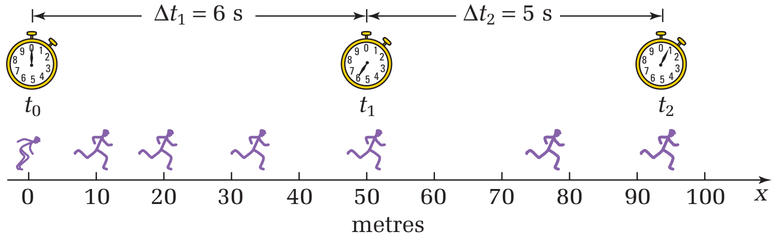


Figure 2.10 A time interval is symbolized as Δt. The symbol t with a subscript indicates an instant in time related to a specific event.

Conceptual Problems

- Write an equation to show the mathematical relationship between the time interval Δt that elapsed while you were travelling to school this morning and the instants in time at which you left home and at which you arrived at school.
- Draw a sketch, similar to Figure 2.10, of a sprinter running a 100 m race and label it with the following information. (Remember, if you are not a good artist, you can use dots to show the sprinter at the specified positions.)

Time (s)	Position (m)
0	0
3.6	10
5.7	25
10.0	50
12.8	80
14.0	100

(a) Determine the time interval that elapsed between the runner passing the following positions.
  - the beginning of the race and the 10 m point
  - the 10 m point and the 80 m point
  - the 80 m point and the 100 m point

(b) Compare the time interval taken for the first 50 m and the second 50 m of the sprint. Explain why they are different.

Velocity

You have probably known the meaning of "speed" since you were very young. Speed is a scalar quantity that is simply defined as the distance travelled divided by the time spent travelling. For example, a car that travels 250 m in 10 s has an average speed of 25 m/s.

In physics, you will use the vector quantity *velocity* much more frequently than *speed*. Velocity not only describes how fast an object moves from one position to another, but also indicates the direction in which the object is moving. Physicists define velocity as the *rate of change of position*.

As you have discovered, when you determine the displacement (change of position) of an object (or person), you do not consider anything that has occurred between the initial and final positions. Consequently, you do not know whether the velocity has been changing during that time. Therefore, when you calculate velocity by dividing displacement by time, you are, in reality, finding the *average* velocity and ignoring any changes that might have occurred during the time interval.

VELOCITY

Velocity is the quotient of displacement and the time interval.

$$\vec{v}_{\text{ave}} = \frac{\Delta\vec{d}}{\Delta t} \quad \text{or} \quad \vec{v}_{\text{ave}} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

Quantity	Symbol	SI unit
average velocity	$\vec{v}_{\text{ave}}$	$\frac{\text{m}}{\text{s}}$ (metres per second)
displacement	$\Delta\vec{d}$	m (metres)
time interval	Δt	s (seconds)

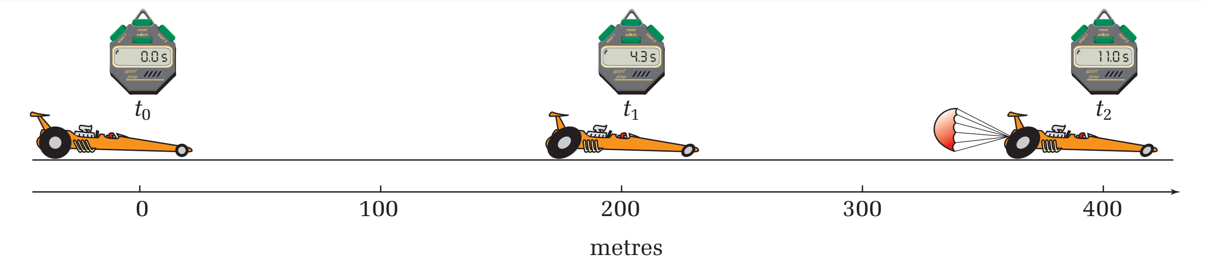
Conceptual Problems

- Consider the speedometer of a car. Does it provide information about speed or velocity?
- A student runs around a 400 m oval track in 80 s. Would the average velocity and average speed be the same? Explain this result using both the definition of average velocity and a distinction between scalars and vectors.
- Consider the definition of average velocity. Describe the effect of reducing the time interval over which average velocity is calculated from a very large value such as several hours compared to a very short interval such as a fraction of a second.

MODEL PROBLEMS

Calculating Average Velocity

1. A dragster in a race is timed at the 200.0 m and 400.0 m points. The times are shown on the stopwatches in the diagram. Calculate the average velocity for (a) the first 200.0 m, (b) the second 200.0 m, and (c) the entire race.



Frame the Problem

- The dragster undergoes a *change in position*.
- The stopwatch shows a reading at *three instants in time*.
- Since you have data for only three instants, you can determine only the *average* velocity.
- The equation for average velocity applies to this problem.

Identify the Goal

(a) The average velocity, $\vec{v}_{\text{ave}}$, for the displacement from 0.0 m to 200.0 m

(b) The average velocity, $\vec{v}_{\text{ave}}$, for the displacement from 200.0 m to 400.0 m

(c) The average velocity, $\vec{v}_{\text{ave}}$, for the displacement from 0.0 m to 400.0 m

Variables and Constants

Known		Unknown		
$\vec{d}_0 = 0.0$ m[E]	$t_0 = 0.0$ s	$\Delta\vec{d}_{0\rightarrow 1}$	$\Delta t_{0\rightarrow 1}$	$\vec{v}_{\text{ave}(0\rightarrow 1)}$
$\vec{d}_1 = 200.0$ m[E]	$t_1 = 4.3$ s	$\Delta\vec{d}_{1\rightarrow 2}$	$\Delta t_{1\rightarrow 2}$	$\vec{v}_{\text{ave}(1\rightarrow 2)}$
$\vec{d}_2 = 400.0$ m[E]	$t_2 = 11$ s	$\Delta\vec{d}_{0\rightarrow 2}$	$\Delta t_{0\rightarrow 2}$	$\vec{v}_{\text{ave}(0\rightarrow 2)}$

Strategy	Calculations
Find the displacement for the first 200.0 m, using the definition of displacement.	$\Delta d_{0\rightarrow 1} = \vec{d}_1 - \vec{d}_0$ $\Delta d_{0\rightarrow 1} = 200.0\text{ m[E]} - 0.0\text{ m[E]}$ $\Delta d_{0\rightarrow 1} = 200.0\text{ m[E]}$
Find the time interval for the first 200.0 m, using the definition of time interval.	$\Delta t = t_1 - t_0$ $\Delta t = 4.3\text{ s} - 0.0\text{ s}$ $\Delta t = 4.3\text{ s}$

Find the average velocity for the first 200.0 m, using the definition of average velocity.

$$\vec{v}_{ave(0\to1)} = \frac{\Delta\vec{d}_{0\to1}}{\Delta t_{0\to1}}$$

$$\vec{v}_{ave(0\to1)} = \frac{200.0\text{ m[E]}}{4.3\text{ s}}$$

$$\vec{v}_{ave(0\to1)} = 46.51\ \frac{\text{m}}{\text{s}}\text{[E]}$$

(a) The average velocity for the first 200.0 m was 47 m/s[E].

Find the displacement for the second 200.0 m, using the definition of displacement.

$$\Delta d_{1\to2} = \vec{d}_2 - \vec{d}_1$$

$$\Delta d_{1\to2} = 400.0\text{ m[E]} - 200.0\text{ m[E]}$$

$$\Delta d_{1\to2} = 200.0\text{ m[E]}$$

Find the time interval for the second 200.0 m, using the definition of time interval.

$$\Delta t = t_2 - t_1$$

$$\Delta t = 11\text{ s} - 4.3\text{ s}$$

$$\Delta t = 6.7\text{ s}$$

Find the average velocity for the second 200.0 m, using the definition of average velocity.

$$\vec{v}_{ave(1\to2)} = \frac{\Delta\vec{d}_{1\to2}}{\Delta t_{1\to2}}$$

$$\vec{v}_{ave(1\to2)} = \frac{200.0\text{ m[E]}}{6.7\text{ s}}$$

$$\vec{v}_{ave(1\to2)} = 29.85\ \frac{\text{m}}{\text{s}}\text{[E]}$$

(b) The average velocity for the second 200.0 m was 3.0×10^1 m/s[E].

Find the displacement for the entire race.

$$\Delta d_{0\to2} = \vec{d}_2 - \vec{d}_0$$

$$\Delta d_{0\to2} = 400.0\text{ m[E]} - 0.0\text{ m[E]}$$

$$\Delta d_{0\to2} = 400.0\text{ m[E]}$$

Find the time interval for the entire race.

$$\Delta t = t_2 - t_0$$

$$\Delta t = 11\text{ s} - 0.0\text{ s}$$

$$\Delta t = 11\text{ s}$$

Find the average velocity for the entire race.

$$\vec{v}_{ave(0\to2)} = \frac{\Delta\vec{d}_{0\to2}}{\Delta t_{0\to2}}$$

$$\vec{v}_{ave(0\to2)} = \frac{400.0\text{ m[E]}}{11\text{ s}}$$

$$\vec{v}_{ave(0\to2)} = 36.36\ \frac{\text{m}}{\text{s}}\text{[E]}$$

(c) The average velocity for the entire race was 36 m/s[E].

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Validate

Velocities with magnitudes between 30 m/s and 47 m/s are very large (108 km/h to 169 km/h), which you would expect for dragsters. As well, the units gave m/s, which is correct for velocity.

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2. A basketball player gains the ball in the face-off at centre court. He then dribbles down to the opponents' basket and scores 6.0 s later. After scoring, he runs back to guard his own team's basket, taking 9.0 s to run down the court. Using centre court as his reference position, calculate his average velocity (a) while he is dribbling up to the opponents' net, and (b) while he is running down from the opponents' net to his own team's net. (A basketball court is 3.0×10^1 m long.)

Frame the Problem

- The basketball player's *starting position* is at *centre court*.
- Two additional *positions* are identified, one up-court and one down-court from his starting position.
- Two *time intervals* are given in the description of his play.
- *Average velocity* is a calculation of his *displacement* for particular *time intervals*.
- Centre court is the *origin* of the coordinate system.
- Since *directions* are required in order to determine *velocities*, define the direction of the opponents' net as *positive* and the direction of the player's own net as *negative*.

Identify the Goal

(a) The average velocity, $\vec{v}_{\text{ave}}$, for the first event

(b) The average velocity, $\vec{v}_{\text{ave}}$, for the second event

Variables and Constants

Known	Implied	Unknown
$\vec{d}_0 = 0.0$ m	$\vec{d}_1 = +15$ m	$\vec{v}_{\text{ave}(0\to 1)}$
$\Delta t_1 = 6.0$ s	$\vec{d}_2 = -15$ m	$\vec{v}_{\text{ave}(1\to 2)}$
$\Delta t_2 = 9.0$ s		$\Delta\vec{d}_{0\to 1}$
		$\Delta\vec{d}_{1\to 2}$
		t_0
		t_1
		t_2

Note: Since the court is 30 m long and the position zero is defined as centre court, each basket must be half of 30 m, or 15 m, from position zero.

Strategy

Calculations

Find the displacement for the first event, using the definition of displacement.

$$\Delta\vec{d}_{0\to1} = \vec{d}_1 - \vec{d}_0$$
$$\Delta\vec{d}_{0\to1} = +15\text{ m} - 0.0\text{ m}$$
$$\Delta\vec{d}_{0\to1} = +15\text{ m}$$

The time interval is given, so calculate the average velocity of the first event by using the definition of average velocity.

$$\vec{v}_{0\to1} = \frac{\Delta\vec{d}_{0\to1}}{\Delta t_{0\to1}}$$
$$\vec{v}_{0\to1} = \frac{+15\text{ m}}{6.0\text{ s}}$$
$$\vec{v}_{0\to1} = +2.5\ \frac{\text{m}}{\text{s}}$$

(a) The average velocity for the first event was +2.5 m/s. The positive sign indicates that the direction of the player's velocity was toward the opponents' net.

Find the displacement for the second event by using the definition of displacement.

$$\Delta\vec{d}_{1\to2} = \vec{d}_2 - \vec{d}_1$$
$$\Delta\vec{d}_{1\to2} = -15\text{ m} - (+15\text{ m})$$
$$\Delta\vec{d}_{1\to2} = -30\text{ m}$$

The time interval is given, so calculate the average velocity of the second event by using the definition of average velocity.

$$\vec{v}_{1\to2} = \frac{\Delta\vec{d}_{1\to2}}{\Delta t_{1\to2}}$$
$$\vec{v}_{1\to2} = \frac{-30\text{ m}}{9.0\text{ s}}$$
$$\vec{v}_{1\to2} = -3.33\ \frac{\text{m}}{\text{s}}$$

(b) The average velocity for the second event was –3.3 m/s. The negative sign indicates that the direction of the player's velocity was toward the player's own net.

Validate

The units in the answer were m/s, which is correct for velocity. The player's velocity was faster when he was going to guard his own net than when he was dribbling toward his opponents' net to make a shot. This is logical because, when planning a shot, a player would take more time. When guarding, it is critical to get to the net quickly.

PRACTICE PROBLEMS

1. Calculate the basketball player's average velocity for the entire time period described in Model Problem 2.

2. Freda usually goes to the sports complex every night after school. The displacement for that walk is 360 m[N57°W]. What is her average velocity if the walk takes her 5.0 min?

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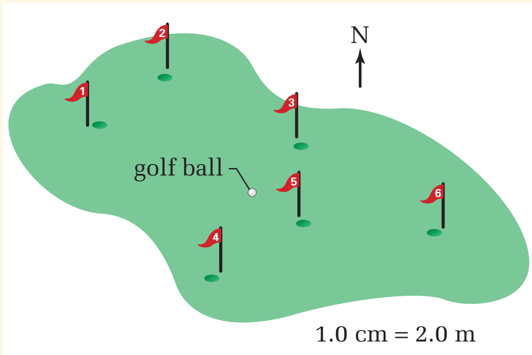
3. Imagine that you are in the bleachers watching a swim meet in which your friend is competing in the freestyle event. At the instant the starting gun fires, the lights go out! When the lights come back on, the timer on the scoreboard reads 86 s. You observe that your friend is now about halfway along the length of the pool, swimming in a direction opposite to that in which he started. The pool is 5.0×10^1 m in length.
   - **(a)** Determine his average velocity during the time the lights were out.
   - **(b)** What are two possible distances that you might infer your friend swam while the lights were out?
   - **(c)** Given that the record for the 100 m freestyle race is approximately 50 s, which is the most likely distance that your friend swam while the lights were out? Explain your reasoning.
   - **(d)** Based on your conclusions in (c), calculate your friend's average speed while the lights were out.

2.2 Section Review

1. K/U List four scalar quantities and five vector quantities.
2. K/U Describe the similarities and difference between:
   - **(a)** time and time interval.
   - **(b)** position, displacement, and distance.
   - **(c)** speed and velocity.
3. K/U What is the displacement of Earth after a time interval of 365 $\frac{1}{4}$ days?
4. MC Create a scale diagram of your route to school. What is the displacement of your house from the school?
5. C Draw a displacement **and** a velocity scale diagram for the following:
   - **(a)** a farmer drives 3 km[N43°W] at 60 km/h.
   - **(b)** a swimmer crosses a still river, heading [S56°W] at 3 m/s, in 75 s.
   - **(c)** an easterly wind blows a plastic bag at 6 km/h over a distance of 100 m.
6. K/U The following diagram represents a putting green at a 9 hole golf course.
   - **(a)** What is the displacement from the furthest hole to the ball?
   - **(b)** What is the displacement from the ball to furthest hole?
   - **(c)** What is the displacement from the closest hole to the ball?
   - **(d)** What is the displacement from the ball to the closest hole?



UNIT PROJECT PREP

Displacement, velocity, and the time interval are important features of any motion picture.

- How do film-makers manipulate these quantities to simulate motion?
- Have you ever been in a vehicle that was stopped but seemed like it was moving?
- What situations and effects can you create by lengthening or decreasing the time interval?