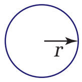
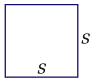
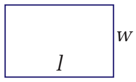
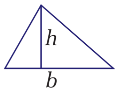
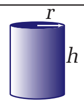
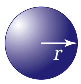
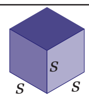
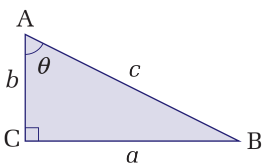
A Math Toolbox

	Circumference/ perimeter	Area	Surface area	Volume
	$C = 2\pi r$	$A = \pi r^2$		
	$P = 4s$	$A = s^2$		
	$P = 2l + 2w$	$A = lw$		
		$A = \frac{1}{2}bh$		
			$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
			$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
			$SA = 6s^2$	$V = s^3$

Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{a}{c}$$

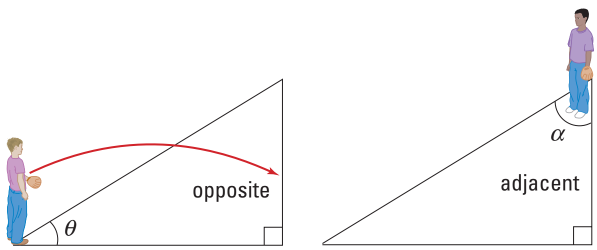
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{a}{b}$$

The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the 90˚ angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the *opposite* side. The side that you could touch and is not the hypotenuse is the *adjacent* side.



A scientific calculator or trigonometry tables can be used to obtain an angle value from the ratio result. Your calculator performs a complex calculation (Maclaurin series summation) when the $\sin^{-1}$, or $\cos^{-1}$, or $\tan^{-1}$ operation is used to determine the angle value. Sin^{-1} is not simply a 1/sin operation.

Definition of the Pythagorean Theorem

The Pythagorean theorem is used to determine side lengths of a right-angle (90°) triangle. Given a right-angle triangle ABC, the Pythagorean theorem states

$$c^2 = a^2 + b^2$$

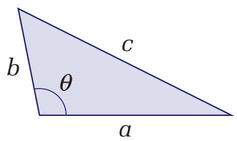
Quantity	Symbol	SI unit
hypotenuse side is opposite the 90°angle	c	m (metres)
side a	a	m (metres)
side b	b	m (metres)

Note: The hypotenuse is always the side across from the right (90°) angle. The Pythagorean theorem is a special case of a more general mathematical law called the "cosine law." The cosine law works for all triangles.

Definition of the Cosine Law

The cosine law is useful when

- determining the length of an unknown side given two side lengths and the contained angle between them
- determining an unknown angle given all side lengths



Angle θ is contained between sides a and b.

The cosine law states $\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\theta}$.

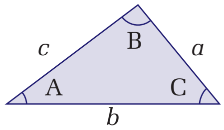
Quantity	Symbol	SI unit
unknown length side c opposite angle θ	c	m (metres)
length side a	a	m (metres)
length side b	b	m (metres)
angle θ opposite unknown side c	θ	(radians)

Note: Applying the cosine law to a right angle triangle, setting $\theta = 90°$, yields the special case of the Pythagorean theorem.

Definition of the Sine Law

The sine law is useful when

- two angles and any one side length are known
- two side lengths and any one angle are known



Given any triangle ABC the sine law states

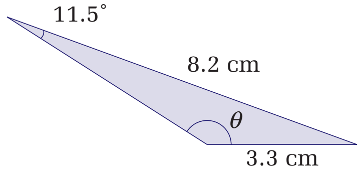
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Quantity	Symbol	SI unit
length side a opposite angle A	a	m (metres)
length side b opposite angle B	b	m (metres)
length side c opposite angle C	c	m (metres)
angle A opposite side a	A	(radians)
angle B opposite side b	B	(radians)
angle C opposite side c	C	(radians)

Note: The sine law generates ambiguous results in some situations because it does not discriminate between obtuse and acute triangles. An example of the ambiguous case is shown below.

Example

Use the sine law to solve for θ.



Sine law: ambiguous case

$$\frac{\sin\theta}{8.2} = \frac{\sin 11.5°}{3.3}$$

$$\sin\theta = 0.5$$

$$\theta = 30°$$

Clearly, angle θ is much greater than 30°. In this case, the supplementary angle is required (180° – 30° = 150°). It is important to recognize when dealing with obtuse angles (> 90°) that the supplementary angle might be required. Application of the cosine law in these situations will help reduce the potential for error.

Algebra

In some situations, it might be preferable to use algebraic manipulation of equations to solve for a specific variable before substituting numbers. Algebraic manipulation of variables follows the same rules that are used to solve equations after substituting values. In both cases, to maintain equality, whatever is done to one side must be done to the other.

Skill Set 5 continued from previous page

Solving for "x" before Numerical Substitution	Solving for "x" after Numerical Substitution
(a) $A = kx$ — x is multiplied by k, so divide by k to isolate x. $\frac{A}{k} = \frac{\cancel{k}x}{\cancel{k}}$ — Divide both sides of the equation by k. $\frac{A}{k} = x$ — Simplify. $x = \frac{A}{k}$ — Rewrite with x on the left side.	**(a)** $8 = 2x$ — x is multiplied by 2, so divide by 2 to isolate x. $\frac{8}{2} = \frac{\cancel{2}x}{\cancel{2}}$ — Divide both sides of the equation by 2. $4 = x$ — Simplify. $x = 4$ — Rewrite with x on the left side.
(b) $B = \frac{x}{g}$ — x is divided by g, so multiply by g to isolate x. $Bg = \frac{x\cancel{g}}{\cancel{g}}$ — Multiply both sides of the equation by g. $Bg = x$ — Simplify. $x = Bg$ — Rewrite with x on the left side.	**(b)** $8 = \frac{x}{4}$ — x is divided by 4, so multiply by 4 to isolate x. $(10)(4) = \frac{4x}{4}$ — Multiply both sides of the equation by 4. $40 = x$ — Simplify. $x = 40$ — Rewrite with x on the left-hand side.
(c) $W = x + f$ — x is added to f, so subtract f to isolate x. $W - f = x + f - f$ — Subtract f on both sides of the equation. $W - f = x$ — Simplify. $x = W - f$ — Rearrange for x.	**(c)** $25 = x + 13$ — x is added to 13, so subtract 13 to isolate x. $25 - 13 = x + 13 - 13$ — Subtract 13 from both sides of the equation. $12 = x$ — Simplify. $x = 12$ — Rewrite with x on the left-hand side.
(d) $W = \sqrt{x}$ — x is under a square root, so square both sides of the equation. $W^2 = (\sqrt{x})^2$ $W^2 = x$ — Simplify. $x = W^2$ — Rearrange for x.	**(d)** $6 = \sqrt{x}$ — x is under a square root, so square both sides of the equation. $6^2 = (\sqrt{x})^2$ $36 = x$ — Simplify. $x = 36$ — Rewrite with x on the left-hand side.

Definition of the Quadratic Formula

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, the roots of it can be found using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Statistical Analysis

In science, data are collected until a trend is observed. Three statistical tools that assist in determining if a trend is developing are *mean*, *median*, and *mode*.

Mean: The sum of the numbers divided by the number of values. It is also called the "average."

Median: When a set of numbers is organized in order of size, the median is the middle number. When the data set contains an even number of values, the median is the average of the two middle numbers.

Mode: The number that occurs most often in a set of numbers. Some data sets will have more than one mode.

See examples of these on the following page.

Example 1:
Odd number of data points

Data Set 1: 12, 11, 15, 14, 11, 16, 13

$$\text{mean} = \frac{12 + 11 + 15 + 14 + 11 + 16 + 13}{7}$$
$$\text{mean} = 13$$
$$\text{reorganized data} = 11, 11, 12, 13, 14, 15, 16$$
$$\text{median} = 13$$
$$\text{mode} = 11$$

Example 2:
Even number of data points

Data Set 2: 87, 95, 85, 63, 74, 76, 87, 64, 87, 64, 92, 64

$$\text{mean} = \frac{(87 + 95 + 85 + 63 + 74 + 76 + 87 + 64 + 87 + 64 + 92 + 64)}{12}$$
$$\text{mean} = 78$$
$$\text{reorganized data} = 63, 64, 64, 64, 74, 76, 85, 87, 87, 87, 92, 95$$
$$\text{median} = \frac{(76 + 85)}{2}$$
$$\text{median} = 80$$

An even number of data points requires that the middle two numbers be averaged.

$$\text{mode} = 64, 87$$

In this example, the data set is bimodal (contains two modes).

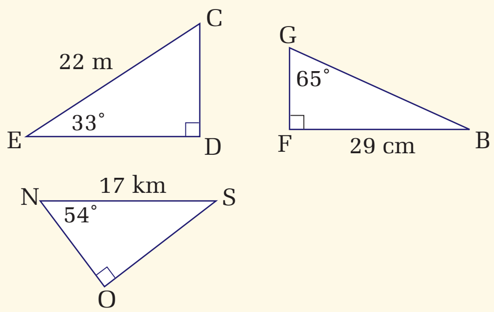
SET 5 Skill Review

1. Calculate the area of a circle with radius 6.5 m.

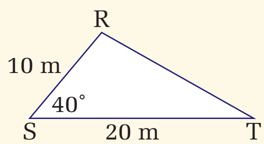
2. By how much does the surface area of a sphere increase when the radius is doubled?

3. By how much does the volume of a sphere increase when the radius is doubled?

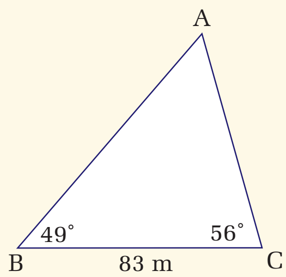
4. Find all unknown angles and side lengths.



5. Use the cosine law to solve for the unknown side.



6. Use the sine law to solve for the unknown sides.



7. Solve for x in each of the following.

(a) $42 = 7x$

(b) $30 = x/5$

(c) $12 = x \sin 30°$

(d) $8 = 2x - 12^4$

8. Solve for x in each of the following.

(a) $F = kx$

(b) $G = hk + x$

(c) $a = bx \cos\theta$

(d) $b = d \cos x$

(e) $a = bc + x^2$

(f) $T = 2\pi\sqrt{\frac{1}{x}}$

9. Use the quadratic equation to find the roots of the function.
$4x^2 + 15x + 13 = 0$

10. Find the mean, median, and mode of each data set.

(a) 25, 38, 55, 58, 60, 61, 61, 65, 70, 74, 74, 74, 78, 79, 82, 85, 90

(b) 13, 14, 16, 17, 18, 20, 20, 22, 26, 30, 31, 32, 32, 35