Rounding, Scientific Notation, and Significant Digits

When working with experimental data, follow basic rules to ensure that accuracy and precision are not either overstated or compromised. Consider the 100 m sprint race. Several people using different equipment could have timed the winner of the race. The times might not agree, but would all be accurate within the capability of the equipment used.

Sprinter's Time with Different Devices

Time (s)	Estimated error of device (s)	Device
11.356	±0.000 5	photogate timer
11.36	±0.005	digital stopwatch
11.4	±0.05	digital stopwatch
11	±0.5	second hand of a dial watch

Using the example of the 100 m race, you will solidify ideas you need to know about exact numbers, number precision, number accuracy, and significant digits.

Exact Numbers If there were eight competitors in the race, then the number 8 is considered to be an exact number. Whenever objects are counted, number accuracy and significant digits are not involved.

Number Precision If our race winner wants a very precise value of her time, she would want to see the photogate result. The electronic equipment is able to provide a time value accurate to 1/1000th of a second. The time recorded using the second hand on a dial watch is not able to provide nearly as precise a value.

Number Accuracy and Significant Digits The race winner goes home to share the good news. She decides to share the fastest time with her family. What timing method does she share? She would share the 11 s time recorded using the second hand of a dial watch. All of the other methods provide data that has her taking a longer time to cross the finish line. Is the 11 s value accurate?

The 11 s value is accurate to within ±0.5 s, following common scientific practice of estimating error. The 11.356 s time is accurate to within 0.0005 s. The photogate time is simply more precise. It would be inaccurate to write the photogate time as 11.356 00 s. In that case, you would be adding precision that goes beyond the ability of the equipment used to collect the data, as the photogate method can measure time only to the thousandths of a second. Scientists have devised a system to help ensure that number accuracy and number precision are maintained. It is a system of significant digits, which requires that the precision of a value does not exceed either (a) the precision of the equipment used to obtain it or (b) the least precise number used in a calculation to determine the value. The table on the left provides the number of significant digits for each measurement of the sprinter's times.

There are strict rules used to determine the number of significant digits in a given value.

When Digits Are Significant ✔

1. All non-zero digits are significant (159 — three significant digits).
2. Any zeros between two non-zero digits are significant (109 — three significant digits).
3. Any zeros to the right of *both* the decimal point and a non-zero digit are significant (1.900 — four significant digits).
4. All digits (zero or non-zero) used in scientific notation are significant.

When Digits Are Not Significant ✘

1. Any zeros to the right of the decimal point but preceding a non-zero digit are not significant; they are placeholders. For example, 0.000 19 kg = 0.19 g (two significant digits).
2. Ambiguous case: Any zeros to the right of a non-zero digit are not significant; they are placeholders (2500 — two significant digits). If the zeros are intended to be significant, then scientific notation must be used. For example, 2.5×10^3 (two significant digits) and 2.500×10^3 (four significant digits).

Calculations and Accuracy As a general rule, accuracy is maintained though mathematical calculations by ensuring that the final answer has the same number of significant digits as the least precise number used during the calculations.

Example:

Find the product of these lengths.

12.5 m   16 m   15.88 m

Product = 12.5 m × 16 m × 15.88 m

Product = 3176 m^3

Considering each data point, notice that 16 has only two significant digits; therefore the answer must be shown with only two significant digits.

Total length = 3.2×10^3 m (two significant digits)

Rounding to Maintain Accuracy It would seem that rounding numbers would introduce error, but in fact, proper rounding is required to help maintain accuracy. This point can be illustrated by multiplying two values with differing numbers of significant digits. As you know, the right-most digit in any data point contains some uncertainty. It follows that any calculations using these uncertain digits will yield uncertain results.

Multiply 3**2** and 13.5**5**. The last digit, being the most uncertain, is highlighted.

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  13.55
 × 32
  2710 Each digit in this line is obtained using an uncertain digit.
 4065  In this line only the 5 is obtained using uncertain digits.
433.60
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The product 4**33.60** should be rounded so that the last digit shown is the only one with uncertainty. Therefore, 4.3×10^2.

Notice that this value contains two significant digits, which follows the general rule.

Showing results of calculations with every digit obtained actually introduces inaccuracy. The number would be represented as having significantly more precision than it really has. It is necessary to round numbers to the appropriate number of significant digits.

Rounding Rules When extra significant digits exist in a result, rounding is required to maintain accuracy. Rounding is not simply removing the extra digits. There are three distinct rounding rules.

1. **Rounding Down**
   When the digits dropped are less than 5, 50, 500, etc., the remaining digit is left unchanged.

   Example:

   4.123 becomes
   4.12   rounding based on the "3"
   4.1   rounding based on the "23"

2. **Rounding Up**
   When the digits dropped are greater than 5, 50, 500, etc., the remaining digit is increased or rounded up.

   Example:

   4.756 becomes
   4.76   rounding based on the "6"
   4.8   rounding based on the "56"

3. **Rounding with 5, 50, 500, etc.**
   When the digits dropped are exactly equal to 5, 50, 500, etc., the remaining digit is rounded to the *closest even number*.

   Example:

   4.850 becomes
   4.8   rounding based on "50"
   4.750 becomes
   4.8   rounding based on "50"

Always carry extra digits throughout a calculation, rounding only the final answer.

Scientific Notation Numbers in science are sometimes very large or very small. For example, the distance from Earth to the Sun is approximated as 150 000 000 000 m and the wavelength of red light is 0.000 000 65 m. Scientific notation allows a more efficient method of writing these types of numbers.

- Scientific notation requires that a single digit between 1 and 9 be followed by the decimal and all remaining significant digits.
- The number of places the decimal must move determines the exponent.
- Numbers greater than 1 require a positive exponent.
- Numbers less than 1 require a negative exponent.
- Only significant digits are represented in scientific notation.

Example:

1 5 0 0 0 0 0 0 0 0 0 0 .   becomes 1.5×10^{11} m

0.0 0 0 0 0 0 6.5   becomes 6.5×10^{-7} m

continued ▶

Skill Set 2 continued from previous page

SET 2 Skill Review

1. There are a dozen apples in a bowl. In this case, what type of number is 12?
2. Put the following numbers in order from most precise to least precise.
   (a) 3.2, 5.88, 8, 8.965, 1.000 08
   (b) 6.22, 8.5, 4.005, 1.2000×10^{-8}
3. How many significant digits are represented by each value?
   (a) 215
   (b) 31
   (c) 3.25
   (d) 0.56
   (e) 1.06
   (f) 0.002
   (g) 0.006 04
   (h) 1.250 000
   (i) 1×10^6
   (j) 3.8×10^4
   (k) 6.807×10^{58}
   (l) 3.000×10^8
4. Round the following values to two significant digits.
   (a) 1.23
   (b) 2.348
   (c) 5.86
   (d) 6.851
   (e) 6.250
   (f) 4.500
   (g) 5.500
   (h) 9.950
5. Complete the following calculations. Provide the final answer to the correct number of significant digits.
   (a) 2.358×4.1
   (b) $102 \div 0.35$
   (c) $2.1 + 5.88 + 6.0 + 8.526$
   (d) $12.1 - 4.2 - 3$
6. Write each of the following in scientific notation.
   (a) 2.5597
   (b) 1000
   (c) 0.256
   (d) 0.000 050 8
7. Write each value from question 6 in scientific notation accurate to three significant digits.