Mathematical Equations

Equations in Unit 1, Kinematics

Equation	Variables	Name, if any
$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$	$\Delta\vec{d}$ = displacement $\vec{d}_1$ = initial position $\vec{d}_2$ = final position	displacement
$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$ $\vec{v}_{ave} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$	$\vec{v}_{ave}$ = average velocity $\Delta\vec{d}$ = displacement Δt = time interval	average velocity
$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$	$\vec{a}$ = acceleration $\Delta\vec{v}$ = change in velocity Δt = time interval	acceleration
$a = \frac{v_f - v_i}{\Delta t}$ $v_f = v_i + a\Delta t$ $\Delta d = \left(\frac{v_i + v_f}{2}\right)\Delta t$ $\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$	a = acceleration v_i = initial velocity v_f = final velocity Δt = time interval Δd = displacement	motion under uniform acceleration

Equations in Unit 2, Dynamics

Equation	Variables	Name, if any
$\vec{F}_g = m\vec{g}$	$\vec{F}_g$ = force of gravity (weight) m = mass $\vec{g}$ = acceleration due to gravity (on Earth)	weight
$F_f = \mu F_N$	F_f = force of friction μ = coefficient of friction F_N = normal force	friction
$\vec{F} = m\vec{a}$ $\vec{a} = \frac{\vec{F}}{m}$	$\vec{F}$ = force m = mass $\vec{a}$ = acceleration	Newton's second law
$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$	$\vec{F}_{A \text{ on } B}$ = force of object A acting on object B $\vec{F}_{B \text{ on } A}$ = force of object B acting on object A	Newton's third law
$\vec{p} = m\vec{v}$	$\vec{p}$ = momentum m = mass $\vec{v}$ = velocity	momentum
$\vec{J} = \vec{F}\Delta t$	$\vec{J}$ = impulse $\vec{F}$ = force Δt = time interval	impulse
$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$	$\vec{F}$ = force Δt = time interval m = mass $\vec{v}_1$ = initial velocity $\vec{v}_2$ = final velocity	impulse-momentum theorem

Equations in Unit 3, Momentum and Energy

Equation	Variables	Name, if any
$W = F_{\parallel}\Delta d$ $W = \lvert F\rvert\Delta d\cos\theta$	W = work done $F_{\parallel}$ = magnitude of the force (parallel to the displacement) $\lvert F\rvert$ = magnitude of the force (not parallel to the displacement) Δd = magnitude of the displacement θ = angle between force and displacement vectors	work done
$E_k = \frac{1}{2}mv^2$	E_k = kinetic energy m = mass v = velocity	kinetic energy
$E_g = mg\Delta h$	E_g = gravitational potential energy m = mass g = acceleration due to gravity (on Earth) Δh = change in height (from reference position)	gravitational potential energy
$F = -kx$	F = applied force k = spring constant x = length of extension/compression of spring	Hooke's law
$E_e = \frac{1}{2}kx^2$	E_e = elastic energy k = spring constant x = length of extension/compression	elastic potential energy
$P = \frac{W}{\Delta t}$ $P = \frac{E}{\Delta t}$	P = power E = energy transferred W = work done Δt = time interval	power
efficiency $= \frac{E_o}{E_i} \times 100\%$ efficiency $= \frac{W_o}{W_i} \times 100\%$	E_o = useful output energy E_i = input energy W_o = useful output work W_i = input work efficiency	efficiency
$E'_k + E'_g + E'_e = E_k + E_g + E_e$	E'_k = kinetic energy after process E'_g = gravitational potential energy after process E'_e = elastic potential energy after process E_k = kinetic energy before process E_g = gravitational potential energy before process E_e = elastic potential energy before process	law of conservation of mechanical energy
$W_{nc} = E_{final} - E_{initial}$	W_{nc} = work done by non-conservative forces E_{final} = mechanical energy of system after process $E_{initial}$ = mechanical energy of system before process	work done by non-conservative forces
$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$	M_A = mass of object A M_B = mass of object B $\vec{v}_A$ = velocity of object A before collision $\vec{v}_B$ = velocity of object B before collision $\vec{v}'_A$ = velocity of object A after collision $\vec{v}'_B$ = velocity of object B after collision	law of conservation of momentum

Equations in Unit 4, Waves

Equation	Variables	Name, if any
$T = \frac{\Delta t}{N}$ $f = \frac{N}{\Delta t}$ $f = \frac{1}{T}$	T = period f = frequency Δt = time interval N = number of cycles	period and frequency
$v = f\lambda$	v = speed f = frequency λ = wavelength	wave equation
$v = 331 + 0.59T_C$	v = speed of sound T_C = temperature of air	speed of sound in air
$n = \frac{c}{v}$	n = index of refraction c = speed of light in vacuum v = speed of light in a specific medium	index of refraction
$\frac{\sin \theta_i}{\sin \theta_R} = \text{constant}$ $n_i \sin \theta_i = n_R \sin \theta_R$	n_i = index of refraction of the incidence medium θ_i = angle of incidence n_R = index of refraction of the refracting medium θ_R = angle of refraction	Snell's law
$L_n = (2n - 1)\frac{\lambda}{4}$	L_n = resonance lengths n = a positive integer λ = wavelength	resonance lengths in a closed air column
$L_n = \frac{n\lambda}{4}$	L_n = resonance lengths n = a positive integer λ = wavelength	resonance lengths in an open air column
$f_n = nf_1$ $f_1 = \frac{v}{2L}$	f_n = resonance frequencies n = a positive integer f_1 = first resonance frequency v = speed of wave L = air column length	resonance frequencies of a fixed-length open air column
$f_n = (2n - 1)f_1$ $f_1 = \frac{v}{4L}$	f_n = resonance frequencies n = a positive integer f_1 = first resonance frequency v = speed of wave L = air column length	resonance frequencies of a fixed-length closed air column
$f_{beat} = \lvert f_2 - f_1 \rvert$	f_{beat} = beat frequency f_1 = frequency of one component wave f_2 = frequency of other component wave	beat frequency
$n\lambda = d \sin \theta$ where $n = 0, 1, 2, 3, \ldots$	n = integer number of full wavelengths λ = wavelength of light d = distance between slits θ = angle between slit separation and line perpendicular to light rays	constructive interference of light waves (bright fringe)

Equation	Variables	Name, if any
$\left(n - \frac{1}{2}\right)\lambda = d \sin\theta$ where $n = 0, 1, 2, 3, \ldots$	n = integer number of full wavelengths λ = wavelength of light d = distance between slits θ = angle between slit separation and line perpendicular to light rays	destructive interference of light waves (dark fringe)
$\lambda = \frac{\Delta y d}{x}$	λ = wavelength of light Δy = distance separating adjacent fringes d = distance between slits x = distance from source to screen	approximate wavelength of light

Equations in Unit 5, Force, Motion, Work, and Energy

Equation	Variables	Name, if any
$A_x = \lvert\vec{A}\rvert \cos\theta$ $A_y = \lvert\vec{A}\rvert \sin\theta$	$\vec{A}$ = vector making an angle θ with the x-axis A_x = x-component A_y = y-component θ = angle between vector and x-axis	vector components
$\tau = r_{\perp}F$	τ = torque F = magnitude of force $r_{\perp}$ = lever arm	torque
$\sum \vec{F} = 0$ $\sum \tau = 0$	$\sum \vec{F}$ = sum of all forces acting on object $\sum \tau$ = sum of all torques acting on object	condition for static equilibrium
$\vec{p} = m\vec{v}$	$\vec{p}$ = momentum m = mass $\vec{v}$ = velocity	momentum
$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$	m_A = mass of object A m_B = mass of object B $\vec{v}_A$ = velocity of object A before collision $\vec{v}_B$ = velocity of object B before collision $\vec{v}_A'$ = velocity of object A after collision $\vec{v}_B'$ = velocity of object B after collision	law of conservation of momentum
$a_c = \frac{v^2}{r}$	a_c = centripetal acceleration v = velocity (magnitude) r = radius (of circle)	centripetal acceleration
$F_c = \frac{mv^2}{r}$	F_c = centripetal force m = mass v = velocity r = radius of circular path	centripetal force
$\frac{r^3}{T^2} = k$ $\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_A^2}$ where A and B are two planets	r = distance from the Sun T = period of planet's revolution k = constant	Kepler's laws

Equation	Variables	Name, if any
$F_g = G\frac{m_1 m_2}{r_2}$	F_g = force of gravity G = universal gravitational constant m_1 = first mass m_2 = second mass r = distance between centres of two masses	Newton's law of universal gravitation
$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$	E_T = total energy of system m = mass v = speed of the mass at position x k = spring constant x = distance of mass from equilibrium position	total energy of mass and spring system
$T = 2\pi\sqrt{\frac{m}{k}}$	T = period m = mass k = spring constant	period of mass on spring
$T = 2\pi\sqrt{\frac{\ell}{g}}$	T = period of pendulum ℓ = length of pendulum g = acceleration due to gravity	period of pendulum

Equations in Unit 6, Electric, Gravitational, and Magnetic Fields

Equation	Variables	Name, if any
$F_Q = k\frac{q_1 q_2}{r^2}$	F_Q = electrostatic force between charges k = Coulomb's constant q_1 = electric charge on object 1 q_2 = electric charge on object 2 r = distance between object centres	Coulomb's law
$\vec{E} = \frac{\vec{F}_Q}{q}$	$\vec{E}$ = electric field intensity $\vec{F}_Q$ = electric force q = electric charge	electric field intensity
$\vec{g} = \frac{\vec{F}_g}{m}$	$\vec{g}$ = graviatational field intensity $\vec{F}_g$ = graviatational force m = mass	gravitational field intensity
$\lvert\vec{E}\rvert = k\frac{q}{r^2}$	$\lvert\vec{E}\rvert$ = electric field intensity k = Coulomb's constant q = source charge r = distance from centre of source	electric field intensity near a point charge
$\lvert\vec{g}\rvert = G\frac{m_s}{r^2}$	$\lvert\vec{g}\rvert$ = gravitational field intensity G = universal gravitational constant m = mass of source of field r = distance from centre of source	gravitational field intensity near a point mass
$E_g = -G\frac{Mm}{r}$	E_g = gravitational potential energy G = universal gravitational constant M = mass of planet or celestial body m = mass of object r = distance from centre of planet or celestial body	gravitational potential energy

Equation	Variables	Name, if any
$V = k\frac{q}{r}$	V = electric potential difference k = Coulomb's constant q = electric charge r = distance from centre of charge to point charge	electric potential difference due to a point charge
$V = \frac{\Delta E_Q}{Q}$	V = electric potential difference ΔE_Q = change in electrical potential energy Q = quantity of charge	electric potential difference between two points in a circuit
$I = \frac{Q}{\Delta t}$	I = current Q = amount of charge Δt = time interval	electric current
$Q = Ne$	Q = amount of charge N = number of elementary charges e = elementary charge	amount of charge
$R = \rho\frac{L}{A}$	R = resistance ρ = resistivity L = length of conductor A = cross-sectional area	resistance
$V = IR$	V = potential difference I = current R = resistance	Ohm's law
$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_N$	R_{eq} = equivalent resistance $R_1, R_2, R_3, \ldots R_N$ = resistance of individual loads	equivalent resistance of loads in series
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}$	R_{eq} = equivalent resistance $R_1, R_2, R_3, \ldots R_N$ = resistance of individual loads	equivalent resistance of resistors in parallel
$V_S = \xi - V_{int}$	V_S = terminal voltage ξ = electromotive force V_{int} = internal potential drop of battery	terminal voltage
$P = IV$ $P = \frac{V^2}{R}$ $P = I^2R$	P = power I = current V = potential difference R = resistance	electric power
$B_\perp = \frac{F}{IL}$ $L = n\ell$	B = magnetic field strength $\perp$ = perpendicular to F = motor force I = current L = length of conductor n = number of coil turns ℓ = length of each turn	magnetic field strength

Equations in Unit 7, Waves and Modern Physics

Equation	Variables	Name, if any
$\Delta t = \dfrac{\Delta t_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	Δt = dilated time Δt_o = proper time v = velocity of moving reference frame c = speed of light	time dilation
$L = L_o\sqrt{1 - \dfrac{v^2}{c^2}}$	L = contracted length L_o = proper length v = velocity of moving reference frame c = speed of light	length contraction
$m = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	m = relativistic mass m_o = rest mass v = speed of mass relative to observer c = speed of light	relativistic mass
$mc^2 = m_oc^2 + E_k$	m = relativistic mass m_o = rest mass c = speed of light E_k = kinetic energy	total energy
$E_{k(max)} = hf - W$	$E_{k(max)}$ = maximum kinetic energy of photoelectron h = Planck's constant f = frequency of electromagnetic radiation W = work function of metal	photoelectric effect
$p = \dfrac{h}{\lambda}$	p = momentum h = Planck's constant λ = wavelength	momentum of photon
$\lambda = \dfrac{h}{mv}$	λ = wavelength (of matter wave) h = Planck's constant m = mass v = velocity	de Broglie wavelength

Equations in Unit 8, Nuclear Physics

Equation	Variables	Name, if any
$N = N_o\left(\dfrac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$	N = quantity of sample remaining N_o = quantity of original sample Δt = elapsed time $T_{\frac{1}{2}}$ = half-life	radioactive decay