Chapter 13 - Simple Harmonic Motion

Simple Harmonic Motion is a special type of periodic motion. It is a motion that repeats at regular time intervals. The acceleration (and the restoring force) is in a direction that opposes the displacement of an object from its rest position and the acceleration is directly proportional to the displacement.

Period of a Mass on a Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where T is the period of oscillation (s)
m is the mass (kg)
k is spring constant (N/m)
(force constant)

Recall from last year: $(F_a = kx)$

Hooke's Law $F = -kx$

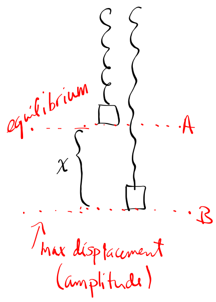
the restoring force

displacement from equilibrium

Elastic Potential Energy: $E_e = \frac{1}{2}kx^2$

Conservation of Energy:

$$E_{total} = E'_{total}$$



$$E_{k_A} + \cancel{E_{e_A}} = \cancel{E'_{k_B}} + E'_{e_B}$$
$$\qquad\quad 0 \qquad\quad 0$$

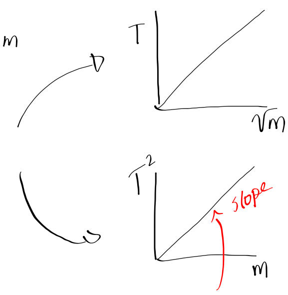
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

A note about the equation:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad T^2 = \frac{4\pi^2 m}{k}$$

$$T \propto \sqrt{m} \qquad T^2 \propto m$$

A Graph of T vs m is a root curve:



$$T^2 = \frac{4\pi^2}{k}m$$

MP|606

$x = 12.0\,cm$

$m = 125\,g$

20.0 cycles in 15.5 s

a) $T = ?$

b) $k = ?$

c) $E_{tot} = ?$

d) $v_{max} = ?$

e) $v_{10cm} = ?$

a) $T = \frac{15.5\,s}{20.0\,cycles}$

$\boxed{T = 0.775\,s}$

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 = \frac{4\pi^2 m}{k}$

$k = \frac{4\pi^2 m}{T^2}$

$k = \frac{4\pi^2 (0.125\,kg)}{(0.775\,s)^2}$

$\boxed{k = 8.22\,N/m}$ $\frac{kg \cdot \cancel{m}}{s^2 \cancel{m}}$

c) Total energy: (at maximum displacement)

$E_{TOT} = \cancelto{0}{E_k} + E_e$

$E_{TOT} = E_e$

$E_{TOT} = \frac{1}{2}kx^2$

$E_{TOT} = \frac{1}{2}(8.22\,N/m)(0.120\,m)^2$

$\boxed{E_{TOT} = 0.0592\,J}$

d) $v_{max} = ?$ Max speed occurs when passing through the equilibrium position (i.e. $E_e = 0$)

(at eq) $E_{TOT} = \cancelto{0}{E_e} + E_k$

$E_{TOT} = E_k$

$E_{TOT} = \frac{1}{2}mv^2$

$0.0592\,J = \frac{1}{2}(0.125\,kg)\,v^2$

$v^2 = \frac{2(0.0592\,J)}{0.125\,kg}$

$v = \pm 0.973\,m/s$

The speed will be 0.973 m/s

e) At 10 cm from equilibrium:

$E_{TOT} = E_e + E_k$

$0.0592\,J = \frac{1}{2}(8.22\,N/m)(0.100\,m)^2 + \frac{1}{2}(0.125\,kg)\,v^2$

$0.0592\,J = 0.0411\,J + \frac{1}{2}(0.125\,kg)\,v^2$

$0.0181\,J = \frac{1}{2}(0.125\,kg)\,v^2$

$\boxed{v = \pm 0.538\,m/s}$

The speed will be 0.538 m/s at 10.0 cm from equilibrium.

Period of a Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is the period of oscillation (s)

l is the length of the pendulum (m)

g is $9.81\,m/s^2$ near the Earth's Surface

Energy Conservation:

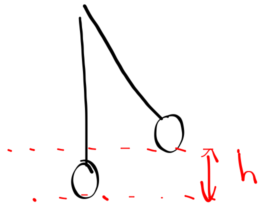
$$E_{TOT} = E'_{TOT}$$

$$\cancel{E_k}^{0} + E_g = E_k' + \cancel{E_g'}^{0}$$

(top) (bottom)

Recall $E_g = mgh$

$$mgh = \frac{1}{2}mv^2$$

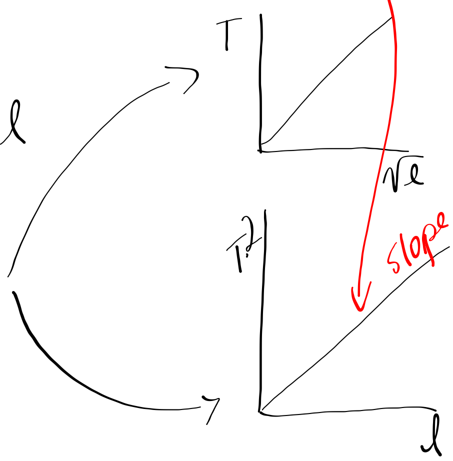


A note about the equation:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \underline{\underline{OR}} \quad T^2 = \frac{4\pi^2 l}{g}$$

$$T \propto \sqrt{l}$$

A graph of T vs l will be a root curve



TO DO

① PP/608

② MP/613 + PP/614