Projectiles - Review

Horizontally - velocity is constant.

$$v = \frac{\Delta d}{\Delta t}$$

Vertically - Velocity is changing; constant acceleration

$$a = \frac{\Delta v}{\Delta t} \quad \text{and} \quad v_{ave} = \frac{\Delta d}{\Delta t}$$

Maybe useful:

$$\Delta d = v_1 t + \frac{1}{2}at^2$$

$$\Delta d = v_2 t - \frac{1}{2}at^2$$

$$v_2^2 = v_1^2 + 2a\Delta d$$

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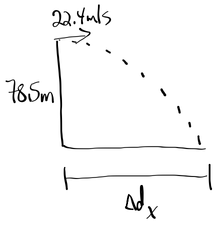
3.



Vertically

$v_1 = 0$

$a = -9.81 m/s^2$

$\Delta d = -78.5m$

$\Delta t = ?$

$$\Delta d = \overset{0}{\cancel{v_1 t}} + \frac{1}{2}at^2$$

$$\Delta d = \frac{1}{2}at^2$$

$$-78.5 = -4.905\, t^2$$

$$t^2 = \frac{78.5}{4.905}$$

$$\boxed{t = 4.00s}$$

Horizontally - velocity constant

a)

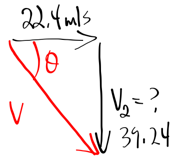
$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = v\Delta t$$

$$\Delta d = (22.4m/s)(4.00s)$$

$$\boxed{\Delta d = 88.0m}$$

b) Velocity when landing:



$$v^2 = 39.4^2 + 22.4^2$$

$$v = 45.2 m/s$$

$$\tan\theta = \frac{39.24}{22.4}$$

$$\boxed{\theta = 60.3^\circ}$$

Vertically - constant acc

$V_1 = 0$

$\Delta d = -78.5m$

$a = -9.81 m/s^2$

$\Delta t = 4.00s$

$V_2 = ?$

$$V_2^2 = \cancel{V_1^2} + 2a\Delta d$$

$$a = \frac{\Delta V}{\Delta t}$$

$$a = \frac{V_2 - V_1}{\Delta t}$$

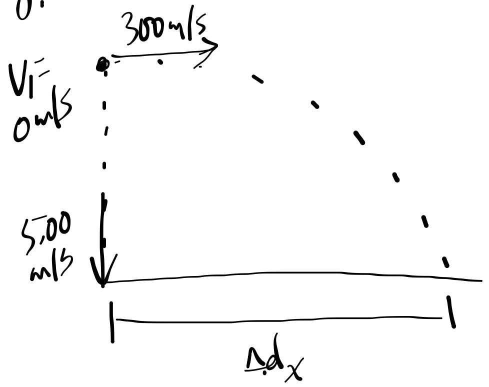
$$V_2 = \overset{0}{\cancel{V_1}} + a\Delta t$$

$$V_2 = (-9.81)(4.00s)$$

$$V_2 = -39.24 m/s$$

The velocity when landing is 45.2 m/s [60.3° below horiz]

8.



Vertically

$v_1 = 0$

$v_2 = -5.00\,m/s$

$a = -9.81\,m/s^2$

$\Delta t = ?$

$a = \frac{\Delta v}{\Delta t}$

$\Delta t = \frac{\Delta v}{a}$

$\Delta t = \frac{-5.00\,m/s}{-9.81\,m/s^2}$

$\Delta t = 0.510\,s$

Horizontally (velocity is constant)

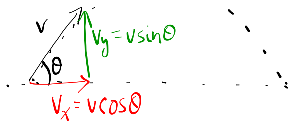
a) $\Delta d = (300\,m/s)(0.510\,s)$

$\Delta d = 153\,m$

b) The vertical velocity of the bullet when it hits the ground is the same as that for the bullet casing when it hits the ground. (i.e. 5.00 m/s) Both objects "fall" the same distance so spend the same time in the air. (Since they had the same initial vertical velocity).

Projectiles Returning to the same level (Symmetrical Trajectory)

Consider a projectile that returns to the same level:



Vertically

$v_1 = v\sin\theta$

$a = -g$

$\Delta d = 0$

$\Delta t = ?$

$$\Delta d = v_1 t + \frac{1}{2}at^2$$

$$0 = (v\sin\theta)t - \frac{g}{2}t^2$$

$$t\left(v\sin\theta - \frac{g}{2}t\right) = 0$$

so $t=0$ and $v\sin\theta - \frac{g}{2}t = 0$

$$v\sin\theta = \frac{g}{2}t$$

$$\boxed{t = \frac{2v\sin\theta}{g}}$$

Horizontally - velocity is constant.

$$\Delta d_x = v_x \Delta t$$

$$\Delta d_x = v\cos\theta\left(\frac{2v\sin\theta}{g}\right)$$

$$\Delta d_x = \frac{v^2\,\boxed{2\sin\theta\cos\theta}}{g} \rightarrow \text{IDENTITY}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

RANGE

$$\boxed{\cancel{\Delta d_x}\; R = \frac{v^2\sin 2\theta}{g}}$$

Maximum Height

$v_1 = v\sin\theta$

$a = -g$

$\Delta d_y = ?$

$v_2 = 0$

$$\cancel{v_2^2} = v_1^2 + 2a\Delta d$$

$$0 \underset{H}{=} v^2\sin^2\theta - 2g\Delta d_y$$

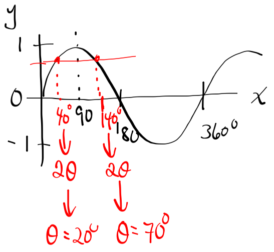
$$2g\boxed{\Delta d_y} = v^2\sin^2\theta$$

$$\boxed{H = \frac{v^2\sin^2\theta}{2g}}$$

Looking at the range equation in more detail:

$$R = \frac{v^2\sin 2\theta}{g}$$

Maximum range for $\theta = 45°$



TO DO

① Look over MP/547

② PP/549