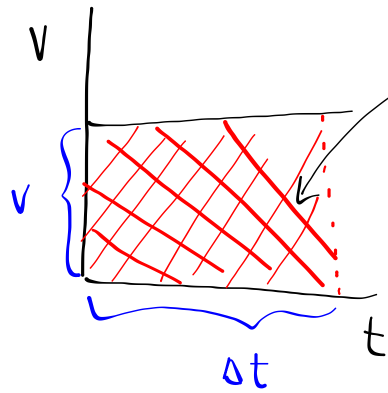
Acceleration & Displacement

Consider an object moving with constant velocity:



Area of Rectangle = $l \times w$

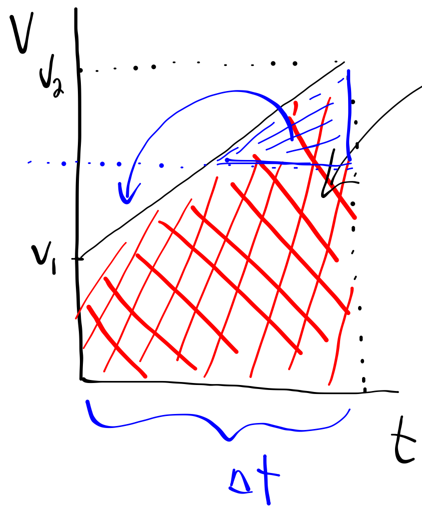
Area = $v\Delta t$

Recall: $v = \frac{\Delta d}{\Delta t}$

$\Delta d = v\Delta t$

The area under the v-t graph = Δd (displacement)

What if we have an object moving with constant acceleration?

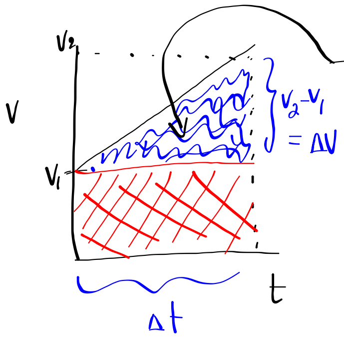


Area of Trapezoid = $\frac{1}{2}(h_1 + h_2)\, b$

$\Delta d = \frac{1}{2}(v_1 + v_2)\Delta t$ ← V_{ave}

$\Delta d = V_{ave}\,\Delta t$

$\left(V_{ave} = \frac{\Delta d}{\Delta t}\right)$



Area of Trapezoid = Area of (red rectangle) + Area of (blue triangle)

$$\Delta d = v_1 \Delta t + \frac{1}{2}(\Delta t)(\Delta v)$$

Recall: $a = \frac{\Delta v}{\Delta t}$

$$\Delta v = a\,\Delta t$$

$$\Delta d = v_1 \Delta t + \frac{1}{2}(\Delta t)(a\,\Delta t)$$

"Maybe Useful Equations"

① $\Delta d = v_1 \Delta t + \frac{1}{2} a (\Delta t)^2$

② $\Delta d = v_2 \Delta t - \frac{1}{2} a (\Delta t)^2$

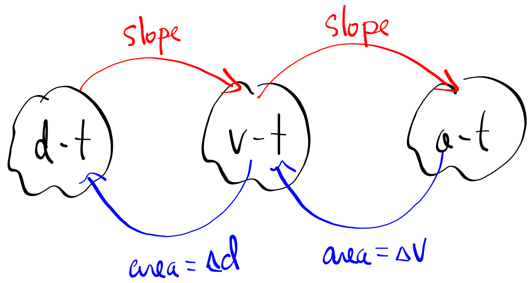
③ $v_2^2 = v_1^2 + 2a\Delta d$

* For any acceleration problem, there are 5 kinematics variables ($v_1, v_2, a, \Delta t, \Delta d$). If you know any 3, you can find the other 2.

* You can get by with:

$v_{ave} = \frac{\Delta d}{\Delta t}$ where $v_{ave} = \frac{(v_1 + v_2)}{2}$

$a = \frac{\Delta v}{\Delta t}$ where $\Delta v = v_2 - v_1$



MP/84

$\vec{v}_1 = 8.3\,m/s\ [down]$ ($-8.3\,m/s$)

$\Delta t = 6.9\,s$

$\vec{\Delta d} = ?$

$\vec{a} = 9.81\,m/s^2\ [down]$ ($-9.81\,m/s^2$)

$$\Delta d = v_1 \Delta t + \frac{1}{2}a(\Delta t)^2$$

$$\Delta d = (-8.3\,m/s)(6.9\,s) + \frac{1}{2}(-9.81\,m/s^2)(6.9\,s)^2$$

$$\Delta d = -290.8\,m$$

↑ down

$$\vec{\Delta d} = 2.9 \times 10^2\,m\ [down]$$

(the displacement)

So the cliff is $2.9 \times 10^2\,m$ high

Example

What is the displacement of a train as it is accelerated uniformly from 11 m/s to 33 m/s in a 20.0 s time interval?

$v_1 = 11\,m/s$

$v_2 = 33\,m/s$

$\Delta t = 20.0\,s$

$\Delta d = ?$

$$v_{ave} = \frac{\Delta d}{\Delta t}$$

$$\Delta d = v_{ave} \Delta t$$

$$\Delta d = \left(\frac{v_1 + v_2}{2}\right)\Delta t$$

$$\Delta d = \left(\frac{11\,m/s + 33\,m/s}{2}\right)20.0\,s$$

$$\Delta d = (22\,m/s)(20.0\,s)$$

v_{ave}

$$\Delta d = 4.4 \times 10^2\,m$$

Example

An airplane must reach a velocity of 71 m/s for take-off. If the runway is 1.0 km long, what must the constant acceleration be?

$v_1 = 0$

$v_2 = 71\,m/s$

$\Delta d = 1.0 \times 10^3\,m$

$a = ?$

$$v_2^2 = v_1^2 + 2a\Delta d$$

$$v_2^2 - v_1^2 = 2a\Delta d$$

$$a = \frac{(v_2^2 - v_1^2)}{(2\Delta d)}$$

($v_1^2 = 0$)

$$a = \frac{(71\,m/s)^2}{2(1.0 \times 10^3\,m)}$$

$$a = 2.5\,m/s^2$$