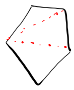
§2-4 Angle Properties in Polygons

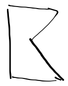
→ Determine properties of angles in polygons and use them to solve problems.

Triangle → 3 sided polygon → sum of interior angles = 180°

Today, focus on convex polygons.



pentagon
(convex)



pentagon
(non convex or concave)

What is the relationship between the sum of interior angles and the number of sides?

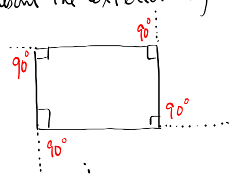
Polygon	Number of Sides (n)	Number of Triangles ($n-2$)	Sum of angles.
triangle	3	1	180°
quadrilateral	4	2	360°
pentagon	5	3	540°
hexagon	6	4	720°
heptagon	7	5	900°
octagon	8	6	1080°
dodecagon	12	10	1800°

$$S = 180°(n-2)$$

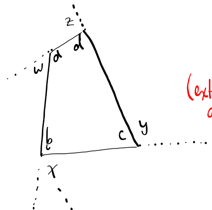
where n is the number of sides in the polygon.

What about the exterior angles?

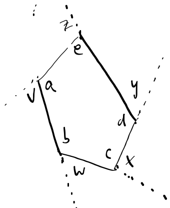
Consider a rectangle



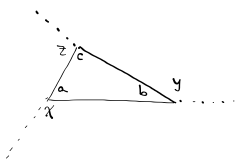
$S = 360°$
(exterior angles)



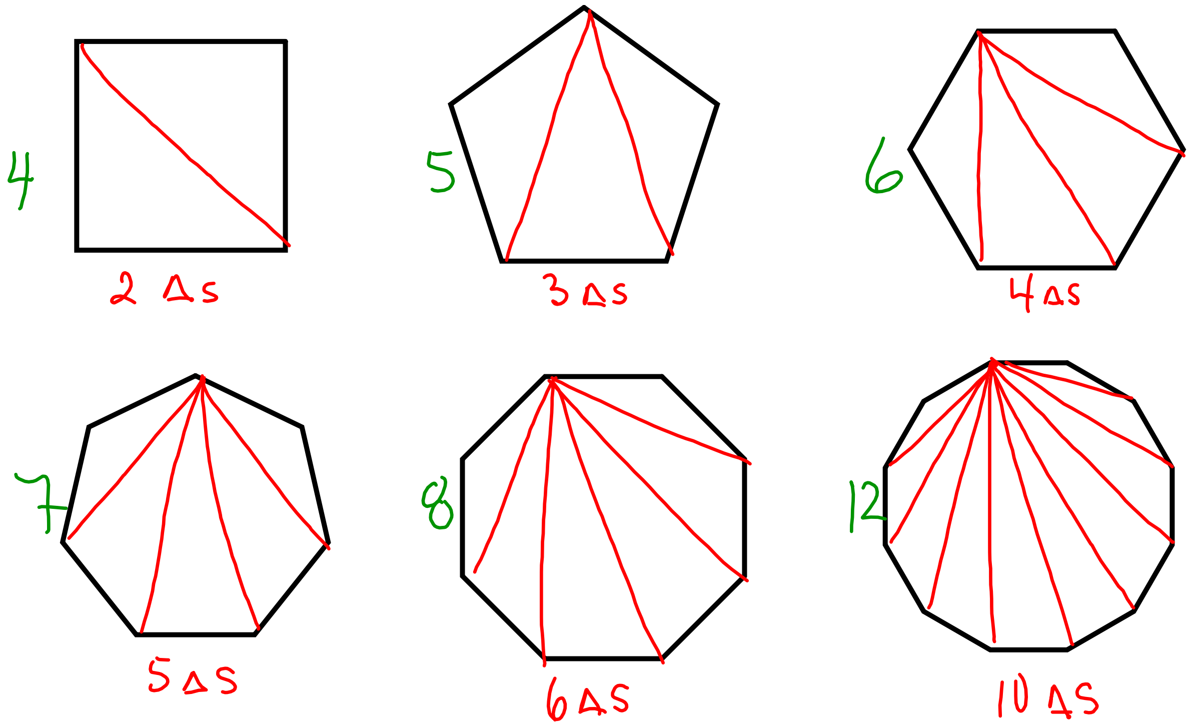
$S = 360°$
(exterior angles)



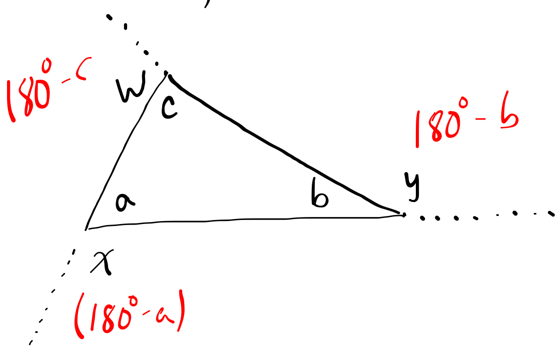
$S = 360°$
(exterior angles)



$S = 360°$
(exterior angles)


4
2 Δs
5
3 Δs
6
4 Δs
7
5 Δs
8
6 Δs
12
10 Δs

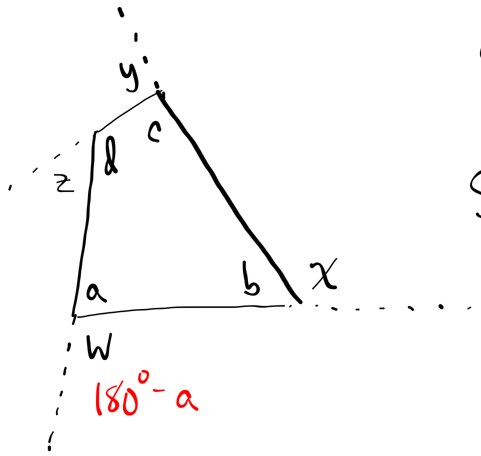
Consider a triangle:



We know that the sum of the interior angles is 180°

so: $a + b + c = 180°$

$$S(\text{exterior angles}) = x + y + w$$
$$= (180° - a) + (180° - b) + (180° - c)$$
$$= 540° - a - b - c$$
$$= 540° - (a + b + c)$$
$$= 540° - 180°$$
$$= 360°$$



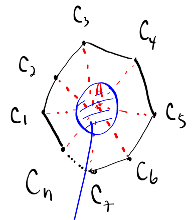
$$S(\text{interior}) = 360° = a + b + c + d$$

$$S(\text{exterior}) = w + x + y + z$$
$$= (180° - a) + (180° - b) + (180° - c) + (180° - d)$$
$$= 720° - a - b - c - d$$
$$= 720° - (a + b + c + d)$$
$$= 720° - 360°$$
$$= 360°$$

The sum of the exterior angles for any polygon is 360°.

Example 1 (p96)

Prove that the sum of the measures of the interior angles of any n-sided convex polygon can be expressed as $180°(n-2)$



Divide the polygon up into n-triangles. The polygon had n-sides.

The sum of the angles in each triangle is $180°$

sum of all the angles in n-triangles

$S = 180°n - 360°$

not included to find the sum of the interior angles

$$S = 180°(n-2)$$

Example 2 (p97)

Determine the interior angles for a regular hexagon.

$n = 6$

$S = 180°(n-2)$

$S = 180°(6-2)$

$S = 180°(4)$

$S = 720°$ ← the sum of 6 equal interior angles

Each angle: $\frac{720°}{6} = 120°$

Determine the interior angles for a regular 15-sided figure (pentadecagon)

$n = 15$

$S = 180°(n-2)$

$S = 180°(15-2)$

$S = 180°(13)$

$S = 2340°$

So each angle: $\frac{2340°}{15} = 156°$

TO DO:

① Look over Example 3 (p98)

② CYU (p99)

③ p99 / 4 - 18

Summary

① Sum of Interior Angles: $S = 180°(n-2)$

② Interior Angle for a regular polygon: $\frac{180°(n-2)}{n}$

③ Sum of Exterior Angles: $360°$