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In Summary

Key Ideas

- The degree of all quadratic functions is 2.
- The standard form of a quadratic function is

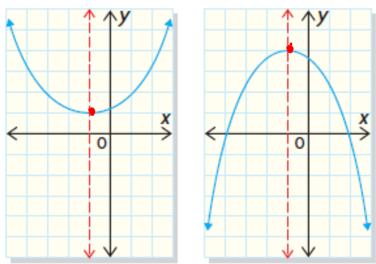
$$y = ax^2 + bx + c$$

  where $a \neq 0$.
- The graph of any quadratic function is a parabola with a single vertical line of symmetry.

Need to Know

- A quadratic function that is written in standard form, $y = ax^2 + bx + c$, has the following characteristics:
  - The highest or lowest point on the graph of the quadratic function lies on its vertical line of symmetry. vertex
  - If a is positive, the parabola opens up. If a is negative, the parabola opens down.

$a > 0$ $a < 0$



  - Changing the value of b changes the location of the parabola's line of symmetry. (location of the vertex)
  - The constant term, c, is the value of the parabola's y-intercept.

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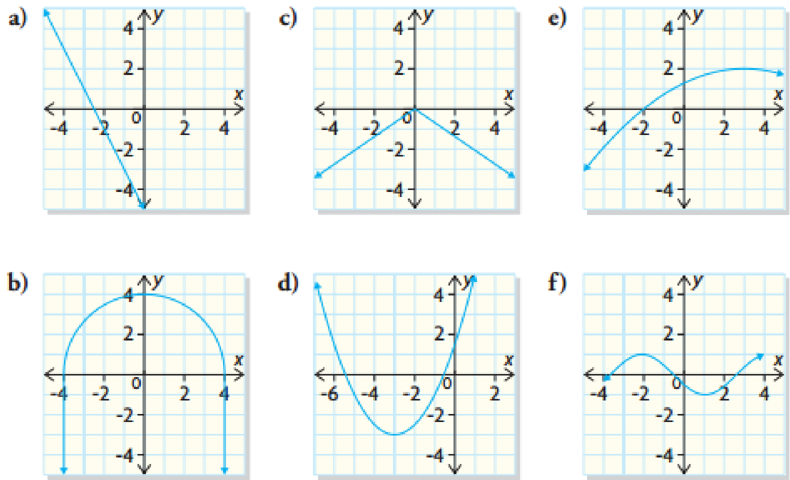
FURTHER Your Understanding

1. Which graphs appear to represent quadratic relations? Explain.



2. Which of the following relations are quadratic? Explain.

a) $y = 2x - 7$
b) $y = 2x(x + 3)$
c) $y = (x + 4)^2 + 1$
d) $y = x^2 - 5x - 6$
e) $y = 4x^3 + x^2 - x$
f) $y = x(x + 1)^2 - 7$

3. State the y-intercept for each quadratic relation in question 2.

4. Explain why the condition $a \neq 0$ must be stated when defining the standard form, $y = ax^2 + bx + c$.

5. Each of the following quadratic functions can be represented by a parabola. Does the parabola open up or down? Explain how you know.

a) $y = x^2 - 4$
b) $y = -2x^2 + 6x$
c) $y = 9 - x + 3x^2$
d) $y = -\frac{2}{3}x^2 - 6x + 1$

Properties of Graphs of Quadratics

Standard form: $y = ax^2 + bx + c \quad (a \neq 0)$

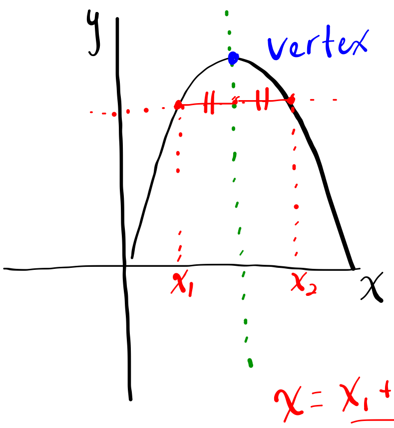
$a \rightarrow$ if $a > 0$, the parabola opens up
$a < 0$, the parabola opens down

$b \rightarrow$ affects the location of the line of symmetry or vertex

$c \rightarrow$ y-intercept.

The symmetry of the parabola can be used to solve problems.

axis of symmetry (line of symmetry)



$x = \frac{x_1 + x_2}{2}$

vertex is located on the axis of symmetry which is exactly halfway between two points that have the same y-value.

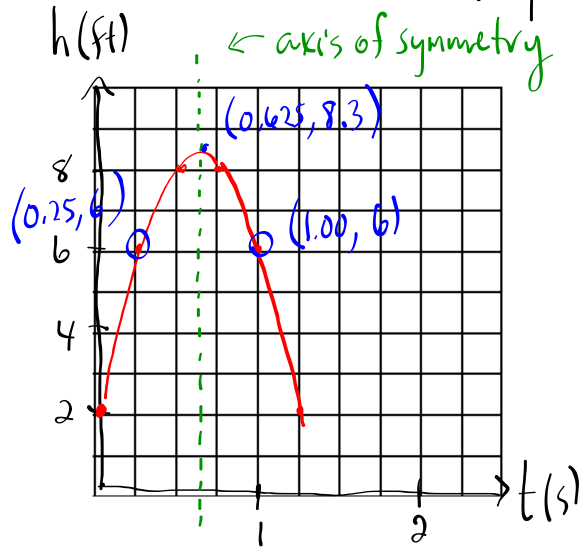
Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$, but you have to consider to location of the vertex

§7.2 → Properties of Graphs of Quadratics (p361)

Example 1 → When did the volley ball reach its maximum height?

time(s)	height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2



The vertex (highest point) is on the axis of symmetry.

The axis of symmetry is exactly halfway between 2 points with the same y-value.

The axis of symmetry can be found by averaging the two x values:

$$x = \frac{0.25 + 1.00}{2}$$

$$x = \frac{1.25}{2}$$

$$\boxed{x = 0.625}$$

The ball will be at its maximum height at 0.625s.

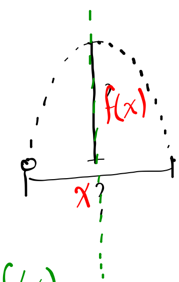
The maximum height is approximately 8.3 ft. (from looking at the graph)

Example 2

y $f(x) = -0.12x^2 + 3x$

height of the water's path

how far from the hole horizontally.



$f(0) = -0.12(0)^2 + 3(0)$

$f(0) = 0$ ft

$(0, 0)$ x, f(x)

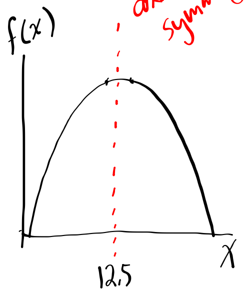
$f(1) = -0.12(1)^2 + 3(1)$

$f(1) = -0.12 + 3$

$f(1) = 2.88$ ft

$(1, 2.88)$

x	0	1	2	.....	12	13
$f(x)$	0	2.88	5.52		18.72	18.72



The axis of symmetry is halfway between 12 and 13

$x = \frac{12 + 13}{2}$

$x = 12.5$ ← where the maximum height occurs

To find the maximum height:

$f(12.5) = -0.12(12.5)^2 + 3(12.5)$

$f(12.5) = 18.75$

The maximum height is 18.75 ft

The water will land at a point 25 ft from the hole
(2×12.5)

Example 3

$y = x^2 + x - 2$ Sketch the graph.

Determine the y-intercept, any x-intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range.

$y = ax^2 + bx + c$

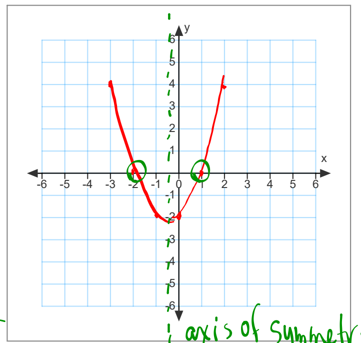
$a = 1$ ← since a is positive the graph opens up.

$b = 1$

$c = -2$ ← y-intercept.

x	-3	-2	-1	0	1	2
y	4	0	-2	-2	0	4

x-int y-int x-int



y-intercept is -2

x-intercepts are -2 and 1 (from the table of values)

axis of symmetry: $x = \frac{-2+1}{2}$

$x = \frac{-1}{2}$

Vertex: $(-0.5, -2.25)$

$y = x^2 + x - 2$

$y = \left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 2$

$y = -2.25$

$\{x \mid x \in R$

$\{y \mid y \in R,\ y \geq -2.25$

Domain and Range:

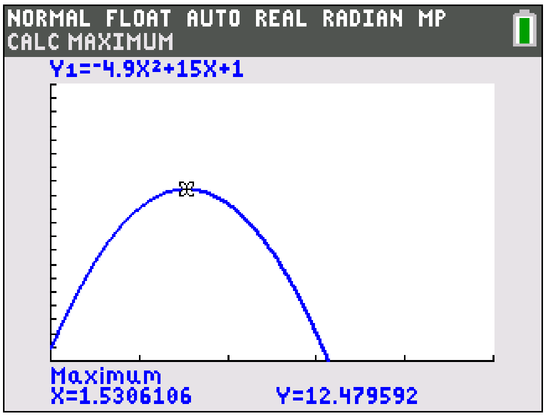
$\{(x,y) \mid x \in R,\ y \geq -2.25, y \in R\}$

Example 4

$$y = -4.9x^2 + 15x + 1$$

Find the maximum height by graphing.

What is the range?



The maximum height is 12.5 m which occured at 1.5 s after take off

Range: $\{y \mid y \in R,\ 0 \leq y \leq 12.5 \text{ m}\}$

TO DO:

① Read over Summary (p 368)

② CYU (p 368-369)

③ p369 | 4

④ p370 | 5-16 (#8, set up table of values + graph by hand)