Projectiles

$v = \frac{\Delta x}{t}$ (horizontal)

horizontally → velocity is constant

vertically → constant acceleration due to gravity ($-9.81\ ms^{-2}$)

suvat equation

data booklet

① $v = u + at$ ← comes from $a = \frac{\Delta v}{\Delta t}$, $a = \frac{v-u}{t}$

② $s = \left(\frac{u+v}{2}\right)t$

③ $v^2 = u^2 + 2as$

④ $s = ut + \frac{1}{2}at^2$

⑤ $s = vt - \frac{1}{2}at^2$

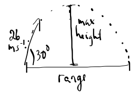
* The link between the horizontal and vertical motion is the time!

Example

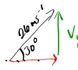
A projectile is launched with a speed of $26\,ms^{-1}$ at an angle of $30°$ above the horizontal. Neglecting air resistance, determine:

a) the height reached by the projectile

b) the range of the projectile (the horizontal displacement)



Components of the initial velocity:



$V_y = (26\,ms^{-1})\sin 30° = 13\,ms^{-1}$ (changes)

$V_x = (26\,ms^{-1})\cos 30°$

$V_x = 22.5\,ms^{-1}$

(stays the same)

a) to find max height:

$u = 13\,ms^{-1}$

$v = 0$

$a = -9.81\,ms^{-2}$

$s = ?$

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0 - (13\,ms^{-1})^2}{2(-9.81\,ms^{-2})}$$

$$s = \frac{-(13\,ms^{-1})^2}{2(-9.81\,ms^{-2})}$$

$$\boxed{s = 8.6\,m}$$

The maximum height is 8.6m

b) to find the range (horizontal displacement), we need to know the time and the horizontal velocity ($22.5\,ms^{-1}$)

?

find the time it is in the air by analyzing the vertical motion

Vertically (half trip/going up)

$u = 13\,ms^{-1}$

$v = 0$

$s = 8.6\,m$

$a = -9.81\,ms^{-1}$

$t = ?$

$$a = \frac{\Delta v}{t}$$

$$t = \frac{\Delta v}{a}$$

$$t = \frac{v - u}{a}$$

$$t = \frac{0 - 13\,ms^{-1}}{-9.81\,ms^{-2}}$$

$t = 1.3\,s$ (half the trip)

$t_{total} = 2(1.3\,s) = \boxed{2.7\,s}$

horizontally

$$v = \frac{\Delta x}{t}$$

$$\Delta x = vt$$

$$\Delta x = (22.5\,ms^{-1})(2.7\,s)$$

$\boxed{\Delta x = 60\,m}$ ← the range of the projectile.

Another way to get the time for the whole trip:

Vertically

$s = 0$ (since it returns to same level)

$a = -9.81\,ms^{-2}$

$u = 13\,ms^{-1}$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = ut + \frac{1}{2}at^2$$

$$0 = 13t - \frac{9.81}{2}t^2$$

$$0 = t\left(13 - \frac{9.81}{2}t\right)$$

Set each factor equal to zero:

$t = 0$ $\qquad$ $13 - \frac{9.81}{2}t = 0$

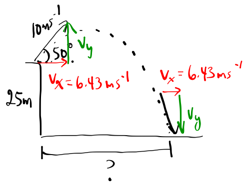
$$13 = +4.905\,t$$

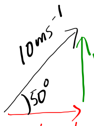
$$t = \frac{13\,ms^{-1}}{+4.905\,ms^{-2}}$$

$$\boxed{t = 2.7\,s}$$

Example

A projectile is launched from a cliff of height 25m with a speed of $10\,ms^{-1}$ and at an angle of 50° above the horizontal. Neglecting air resistance, determine the velocity just before it hits the ground and how far it lands from the base of the cliff.





(changes)

$V_y = (10ms^{-1})(\sin 50^\circ) = 7.66\,ms^{-1}$

$V_x = (10ms^{-1})(\cos 50^\circ) = 6.43\,ms^{-1}$

(stays the same)

Vertically

$u = 7.66\,ms^{-1}$

$a = -9.81\,ms^{-2}$

$s = -25m$

$t = ?$

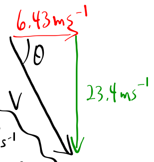
$v = ?$

$v^2 = u^2 + 2as$

$v^2 = (7.66)^2 - 2(9.81)(-25)$

$v = \pm 23.4\,ms^{-1}$

$v = -23.4\,ms^{-1}$ ← use this since it is going down



$\tan\theta = \frac{23.4}{6.43}$

$\theta = 75^\circ$

$24.3\,ms^{-1}$ at 75° below horizontal

to find the horizontal displacement we need to find the time

horizontally the velocity is constant:

$V = \frac{\Delta x}{t}$

$\Delta x = vt$

$\Delta x = (6.43ms^{-1})(3.17s)$

$\Delta x = 20m$

↑ the projectile would land 20 m from base of the cliff.

$a = \frac{\Delta V}{t}$

$t = \frac{\Delta V}{a}$

$t = \frac{v-u}{a}$

$t = \frac{-23.4 - 7.66}{-9.81}$

$t = 3.17s$

Another way to find the time:

$u = 7.66\,ms^{-1}$

$a = -9.81\,ms^{-2}$

$s = -25m$

$t = ?$

$s = ut + \frac{1}{2}at^2$

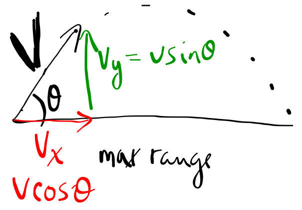
$-25 = 7.66t - 4.905t^2$

$\underset{a}{4.905t^2} - \underset{b}{7.66t} - \underset{c}{25} = 0$

use the quadratic formula:

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

What angle will give the maximum range for a projectile that returns to the same level?



Vertically

$s = 0$ (returns to same level)

$a = -g$

$u = V\sin\theta$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = (V\sin\theta)t - \frac{g}{2}t^2$$

$$0 = t\left(V\sin\theta - \frac{g}{2}t\right)$$

So $t = 0$ and $V\sin\theta - \frac{g}{2}t = 0$

$$V\sin\theta = \frac{g}{2}t$$

$$t = \frac{2V\sin\theta}{g}$$

Horizontally

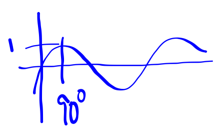
$$v = \frac{\Delta x}{t}$$

$$\Delta x = vt$$

range: $\Delta x = (V\cos\theta)\left(\frac{2V\sin\theta}{g}\right)$

$$\Delta x = \frac{V^2\,2\sin\theta\cos\theta}{g} \longrightarrow \sin 2\theta = 2\sin\theta\cos\theta$$

$$\Delta x = \frac{V^2\sin 2\theta}{g} \quad 90^\circ$$



The maximum range occurs when $\sin 2\theta = 1$

So: $2\theta = 90^\circ$

$$\theta = 45^\circ$$