TOPIC 6 · ~~Fields + Forces~~ Circular Motion + Gravitation

6.1 Gravitational Force + Field

Newton → all masses exert an attractive force.
→ based on ~~astronomical~~ astronomical evidence collected by Kepler

$F \propto m_1$
$F \propto m_2$
$F \propto \frac{1}{r^2}$

Combine:

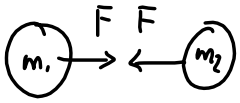
$F \propto \frac{m_1 m_2}{r^2}$

Newton's Law of Universal Gravitation →

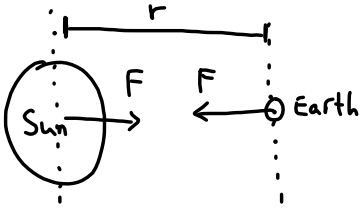
$$F = \frac{G m_1 m_2}{r^2}$$

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Note that r is the separation between the two masses (measured centre to centre)
We consider the masses to be point masses meaning that the radius of the mass is negligible compared to the separation distance.



↑ Newton's third Law.



The value of G was determined by Cavendish using a torsion balance. (1797)

Example

Determine the force between two point masses, one 2.5kg and the other 5.0kg, placed 25cm apart.

$m_1 = 2.5\,kg$

$m_2 = 5.0\,kg$

$r = 0.25\,m$

$F = ?$

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{(6.67\times10^{-11}\,Nm^2kg^{-2})(2.5kg)(5.0kg)}{(0.25m)^2}$$

$$F = 1.3\times10^{-8}\,N$$

(attractive)

Example

What is the gravitational force between the Sun and the Earth?

$m_{sun} = 2.0\times10^{30}\,kg$

$m_{earth} = 6.0\times10^{24}\,kg$

distance between = $1.5\times10^{11}\,m$ (centre to centre)

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{(6.67\times10^{-11}\,Nm^2kg^{-2})(2.0\times10^{30}kg)(6.0\times10^{24}kg)}{(1.5\times10^{11}m)^2}$$

$$F = 3.6\times10^{22}\,N$$

Example

What is the force of gravity on a 250 g apple at the surface of the Earth at the equator? ($m_{Earth} = 5.97 \times 10^{24}$ kg; radius of Earth = 6.378×10^{6} m at equator)

$$F = \frac{Gm_1m_2}{r^2}$$

$g = 9.79\ m/s^2$

$$F = \frac{G(0.250\text{kg})(5.97 \times 10^{24}\text{kg})}{(6.378 \times 10^{6}\text{m})^2} = 2.45\text{N}$$

What is the acceleration due to gravity of the apple towards the Earth?

$9.79\ ms^{-2}$

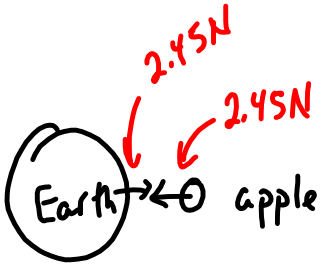
$F_{net} = ma$

$a = \frac{F_{net}}{m}$

$a = \frac{2.45\text{N}}{0.250\text{kg}}$

$a = 9.79\ m/s^2$

What is the acceleration of the Earth towards the apple?



$F_{net} = ma$

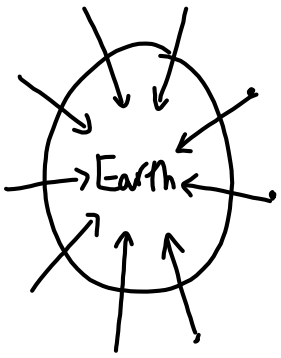
$a = \frac{F_{net}}{m}$

$a = \frac{2.45\text{N}}{5.97 \times 10^{24}\text{kg}}$

$a = 4.10 \times 10^{-25}\ ms^{-2}$

Gravitational Field

- a region of space throughout which a test mass experiences a gravitational force.



- test mass → hypothetical
  → small

Gravitational Field Strength ($\vec{g}$)

$$\vec{g} = \frac{\vec{F}}{m}$$ (depends on location)

units: $N\,kg^{-1}$

Example

The force acting on a body whose mass is 0.10 kg at a point in space due to the Earth's gravitational field is 0.50N towards the Earth.

What is the Earth's gravitational field strength at that point?

$$\vec{g} = \frac{\vec{F}}{m} \qquad \vec{g} = \frac{0.50N\ [\text{towards Earth}]}{0.10kg}$$

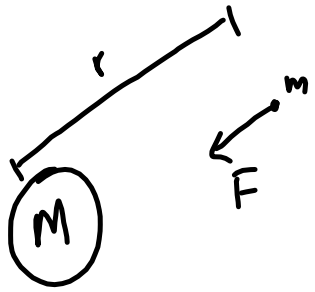
$(F = mg)$ $\qquad \vec{g} = 5.0\,N\,kg^{-1}\ [\text{towards Earth}]$

The acc due to gravity: $5.0\,ms^{-2}$ [towards the Earth]

Gravitational Field Strength due to a point mass (M)

Consider the force between a test mass (m) and point mass (M)

at separation of r:



What is the force experienced by m?

$$F = \frac{G m M}{r^2}$$

So the gravitational field strength at r is given by:

$$g = \frac{F}{m}$$

$$g = \frac{\frac{G m M}{r^2}}{m}$$

Not in your data booklet (memorize or derive)

$$g = \frac{GM}{r^2}$$

← magnitude

direction is towards the centre of M

Example

Determine the strength of the Sun's gravitational field at the position of the Earth. Justify any assumptions.

mass of Sun = 2.0×10^{30} kg

distance to the Sun from Earth = 1.5×10^{11} m

Use this information to determine:

a) the centripetal acceleration of the Earth in its orbit about the Sun.

b) the speed of the Earth in its orbit

c) the period of the Earth's orbit