Period + frequency of Simple Harmonic Motion

Defining equation for SHM: $a = -\omega^2 x$

where ω is used to find the frequency or period of the oscillation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Angular speed or angular frequency.

units: s^{-1}

(recall $T = \frac{1}{f}$ or $f = \frac{1}{T}$)

Example - the period of a pendulum:

Recall: $a = -\frac{g}{\ell}x$ and $a = -\omega^2 x$

$$\omega^2 = \frac{g}{\ell}$$

$$\omega = \sqrt{\frac{g}{\ell}}$$

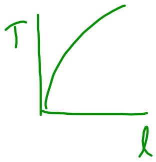
$$\frac{2\pi}{T} = \sqrt{\frac{g}{\ell}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{\ell}{g}}$$

- note that the amplitude doesn't affect period
- mass doesn't affect the period.

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

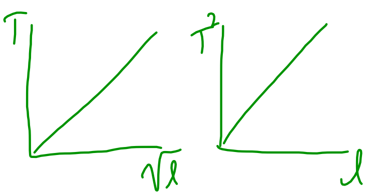
T vs ℓ



$$T^2 = \frac{4\pi^2}{g}\ell$$

a graph of T^2 vs ℓ will linear with a slope $\frac{4\pi^2}{g}$ and a y-int. of zero

$T \propto \sqrt{\ell}$ $T^2 \propto \ell$



The slope could be used to find a value for g:

$$\text{Slope} = \frac{4\pi^2}{g}$$

Data Booklet

$\omega = \frac{2\pi}{T} \rightarrow \omega = 2\pi f$

$x = x_0 \sin \omega t$; $x = x_0 \cos \omega t$

$v = v_0 \cos \omega t$; $v = -v_0 \sin \omega t$

$v = -x_0 \omega \sin \omega t$

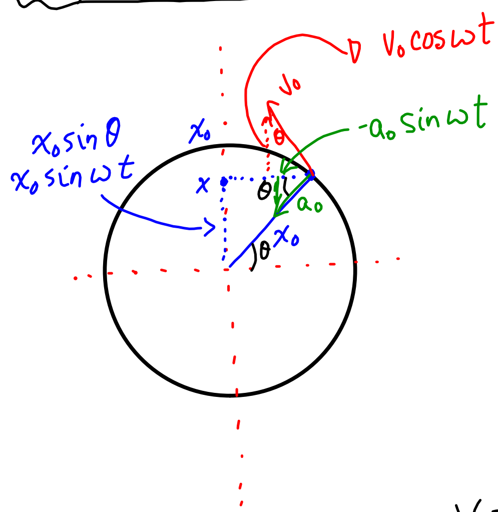
projection on the vertical axis

projection on horizontal axis

$a = -\omega^2 x_0 \cos \omega t$

$a = -\omega^2 x$ ← MUST KNOW!

Projected circular motion - with projection on the vertical axis.



The Solution to the defining equation $a = -\omega^2 x$ and the corresponding values of v and a become: (projecting on the vertical axis)

$x = x_0 \sin \omega t$

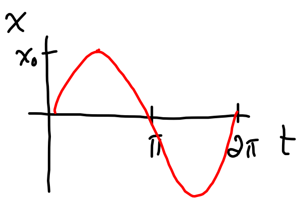
$V = V_0 \cos \omega t$ or $V = x_0 \omega \cos \omega t$

$a = -a_0 \sin \omega t$ or $a = -x_0 \omega^2 \sin \omega t$

$a = -\omega^2 x$

If $x = 0$ at $t = 0$

$x = x_0 \sin \omega t$

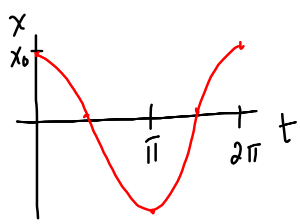


$V = V_0 \cos \omega t$

$(a = -a_0 \sin \omega t)$

If $x = x_0$ at $t = 0$

$x = x_0 \cos \omega t$



$V = -V_0 \sin \omega t$

$(a = -a_0 \cos \omega t)$

Relationship between displacement x and velocity v

$$v = v_0 \cos\omega t$$

$$1 = \sin^2\theta + \cos^2\theta$$

$$v^2 = v_0^2 \cos^2\omega t$$

$$v^2 = v_0^2 (1 - \sin^2\omega t)$$

$$(v_0 = x_0\omega)$$

$$v^2 = x_0^2 \omega^2 (1 - \sin^2\omega t)$$

$$v^2 = \omega^2 (x_0^2 - x_0^2 \sin^2\omega t)$$

x^2

$$(x = x_0 \sin\omega t)$$

$$v^2 = \omega^2 (x_0^2 - x^2)$$

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

← don't need to know t in order to find v.

$\ominus$ left / $\oplus$ right

RADIANS!

If $x = 0$ (ie at equilibrium the velocity will be a maximum

If $x = \pm x_0$ (ie. at min/max) the velocity will be zero.

EXAMPLE:

A pendulum has a period of 1.2 s and an amplitude of 0.10 m. Calculate the displacement, velocity, and acceleration of the pendulum bob 0.70 s after it is released. $t = 0,\ x = x_0$

$T = 1.2\,s$

$x_0 = 0.10\,m$

$t = 0.70\,s$

$x = ?$

$v = ?$

$a = ?$

Since the pendulum is released from its maximum displacement and that is when the timing starts, we use: $x = x_0 \cos \omega t$ etc

$v = -v_0 \sin \omega t$

$a = -a_0 \cos \omega t$ or $a = -\omega^2 x$

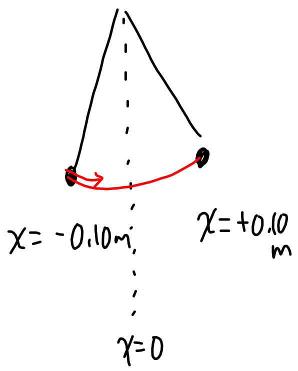
easier!

First find ω:

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{1.2\,s}$$

$$\omega \doteq 5.2\,s^{-1}$$



$$x = x_0 \cos \omega t$$

$$x = (0.10\,m)\cos((5.2\,s^{-1})(0.7\,s))$$

$$x = -0.088\,m$$

$$v = -v_0 \sin \omega t$$

$$v = -x_0 \omega \sin \omega t$$

$$v = -(0.10\,m)(5.2\,s^{-1})\sin((5.2\,s^{-1})(0.7\,s))$$

$$v = +0.25\,m\,s^{-1}$$

going to the right.

OR

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

you will have to choose which is approp.

$$a = -\omega^2 x$$

$$a = -(5.2\,s^{-1})^2(-0.087\,m)$$

$$a = +2.4\,m\,s^{-2}$$

acceleration is to the right (Fnet is to the right) (bob is to the left of equilib)

EXAMPLE:

A pendulum has a period of 1.2 s and an amplitude of 0.10 m. Calculate the displacement, velocity, and acceleration of the pendulum bob 0.70 s after it is released.

T $\quad x_0$

Graph the displacement, velocity and acceleration vs time.
(over 1 full period)

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{1.2s}$$

$$\omega = 5.2\,s^{-1}$$

$$x = x_0 \cos \omega t$$

$$x = (0.10)\cos(5.2t)$$

$\uparrow$ m

$\frac{2\pi}{T} \Rightarrow T = 1.2s$

$$v = -x_0 \omega \sin \omega t$$

$$v = -(0.10)(5.2)\sin(5.2t)$$

$$v = -(0.52)\sin(5.2t)$$

$\uparrow$ ms^{-1}

reflection of sine

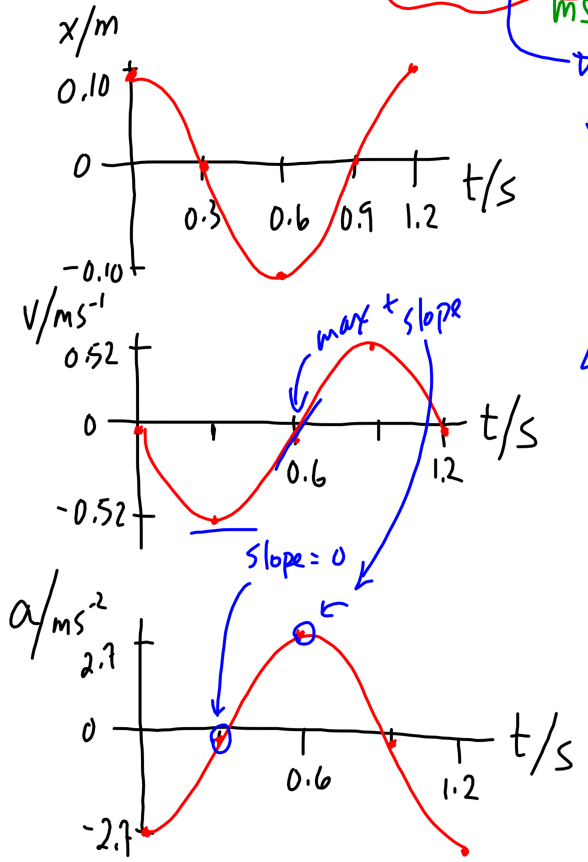
$$a = -x_0 \omega^2 \cos \omega t$$

$$a = -(0.10)(5.2)^2 \cos 5.2t$$

$$a = -2.7 \cos 5.2t$$

ms^{-2}

reflection



Slope of tangents at time t

Slope of the tangent