PROJECTED CIRCULAR MOTION

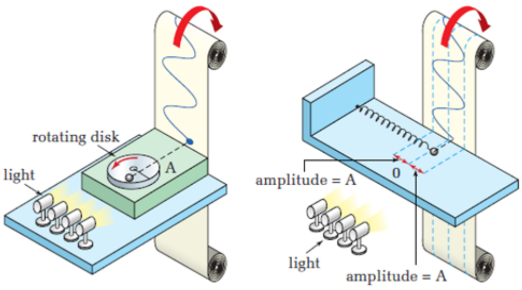


Figure 13.4 The shadows of (**A**) the marker on the edge of a rotating disk and of (**B**) a mass on the end of a spring are recorded on a tape that is moving at a constant speed.

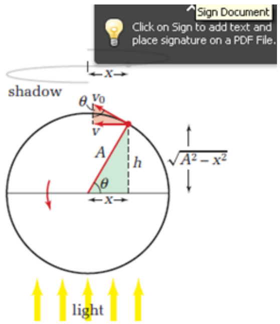


Figure 13.5 The disk of radius A is rotating counterclockwise at a constant speed.

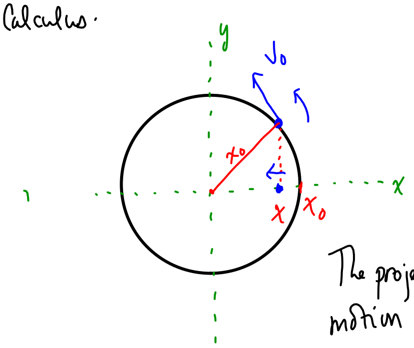
Projected Circular Motion

Recall the defining equation for SHM: $a = -\omega^2 x$

$$\frac{\Delta}{\Delta t}\left(\frac{\Delta x}{\Delta t}\right) = -\omega^2 x$$

to solve for x we would need to use calculus.

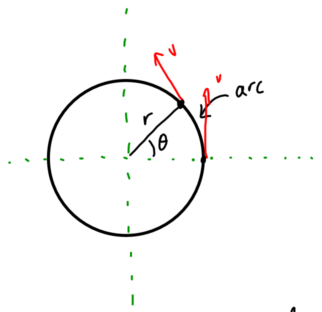
By using projected circular motion, we can develop equations for x without using Calculus.



Body moving with a constant speed v_0 around a circle of radius x_0

The projection of the circular motion onto the x-axis is SHM of amplitude x_0 and a maximum speed of v_0.

Review of Circular Motion:



$\theta = \frac{arc}{r}$ (θ is in radians)

$1^\circ = \frac{\pi}{180}$ radians

or: $180^\circ = \pi$ radians

angular speed: (angular frequency) $\omega = \frac{\theta}{t}$ (definition)

phase angle: $\theta = \omega t$

(for one complete rotation $\theta = 2\pi$ and $t = T$) $\Rightarrow$ $\omega = \frac{2\pi}{T} = 2\pi f$

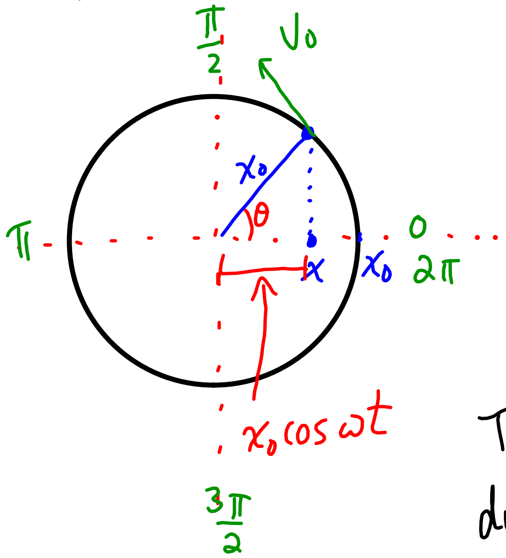
linear speed around the circle: $v = r\omega$

the centripetal acceleration: $a = \frac{v^2}{r}$

$$a = \frac{r^2\omega^2}{r}$$

$$\boxed{a = r\omega^2}$$

Horizontal Components of displacement x of the revolving body:



Consider the object moving counter-clockwise with constant speed (V_0) around the circle of radius x_0.

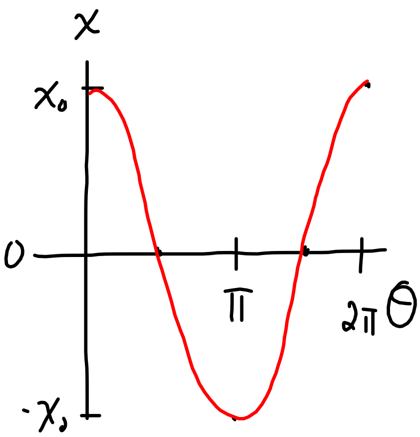
$$\theta = \omega t$$

The horizontal component of the displacement:

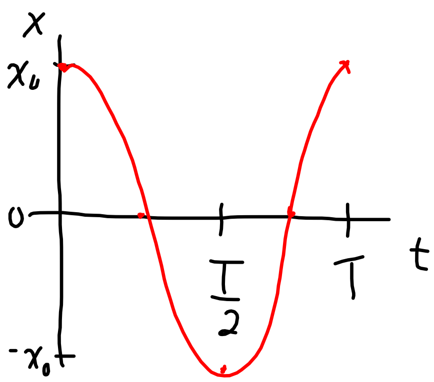
$$x = x_0 \cos \theta$$

$$x = x_0 \cos \omega t \quad \text{where: } \omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f$$

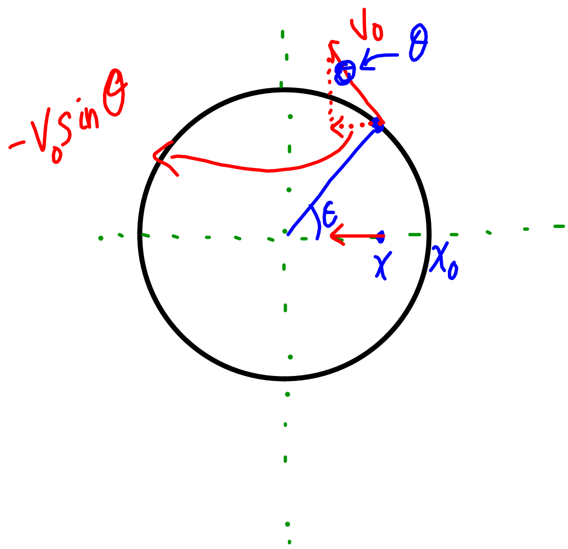


Graph of x vs θ



Graph of x vs t

Horizontal Components of the Velocity v of the revolving body



The horizontal component:

$$v = -v_0 \sin\theta$$

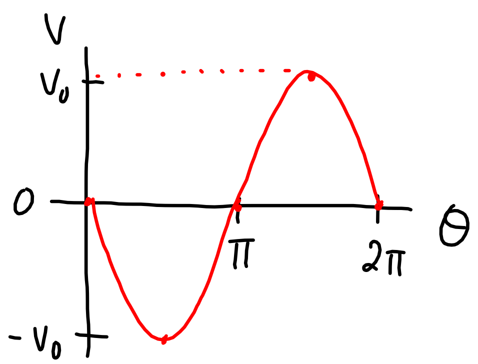
$$v = -v_0 \sin \omega t$$

recall: $v = r\omega$ (general)

so $v_0 = x_0\omega$

$$v = -x_0 \omega \sin \omega t$$

Graph of v vs θ:



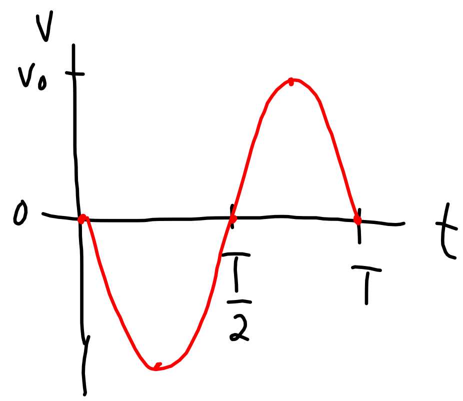
(note: this is a reflection of sin)

$$v = -v_0 \sin\theta$$

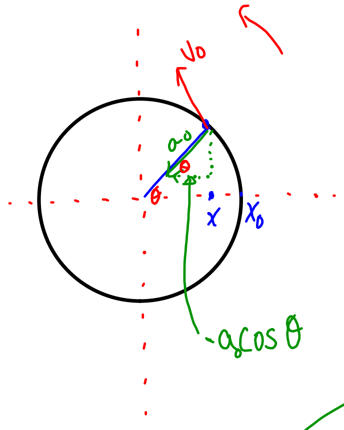
← amplitude of the sinusoidal function (i.e. on the graph)

↑ reflection

Graph of v vs t



Horizontal Components of Acceleration a of the revolving body



recall:
$x = x_0 \cos\omega t$

The horizontal component:

$a = -a_0 \cos\theta$

$a = -a_0 \cos\omega t$

$a = -x_0 \omega^2 \cos\omega t$

(a_0 is the centripetal acceleration)

$a_0 = \frac{v^2}{r}$

$a_0 = \frac{r^2 \omega^2}{r}$

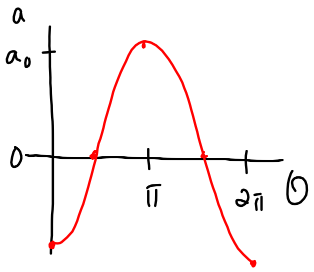
$a_0 = \cancel{r}\,\omega^2$ x_0

$$a = -\omega^2 x$$

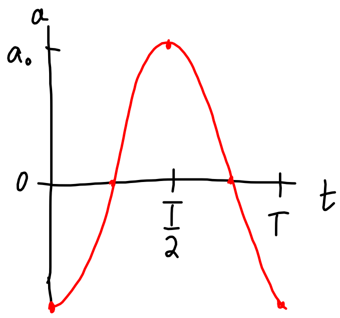
This is the defining equation! Surprise!

This equation proves that projected circular motion is SHM.

Graph of a vs θ



Graph of a vs t



Summary:

Using projected circular motion as SHM, one solution to the defining equation $a = -\omega^2 x$ is:

$x \Rightarrow$ $x = x_0 \cos\omega t$ where $\omega = \frac{2\pi}{T} = 2\pi f$

$v \Rightarrow$ $v = -v_0 \sin\omega t$ or $v = -x_0 \omega \sin\omega t$

$a \Rightarrow$ $a = -a_0 \cos\omega t$ or $a = -x_0 \omega^2 \cos\omega t$

$(a = -\omega^2 x)$