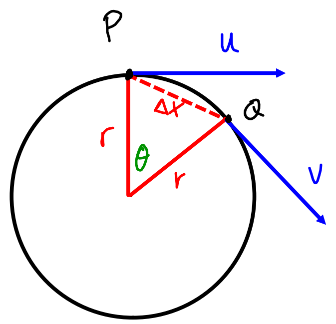
6.1 Uniform circular motion

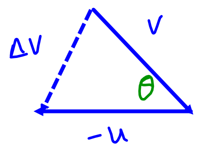
- uniform -> constant speed on a circular path
- the velocity continuously changes since its direction changes.
- since there is a change in velocity, there is acceleration.
- acceleration is the rate of change of velocity

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

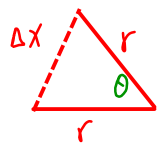
- the object is continuously accelerating since the the velocity is continuously changing

Direction of Acceleration





$|v| = |u| = v$
(uniform motion)



as $\Delta t \to 0$, $\theta \to 0$

Also, the distance travelled along the circular path approaches Δx

Note: $u \perp r$, $v \perp r$ } tangent to curve

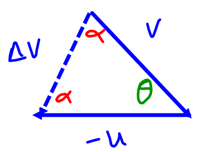
$$\frac{\Delta v}{v} = \frac{\Delta x}{r}$$

$$\Delta v = \frac{\Delta x\, v}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta x}{\Delta t}\,\frac{v}{r}$$

approaches the actual distance when $\Delta t \to 0$

$$\frac{\Delta v}{\Delta t} = v\left(\frac{v}{r}\right)$$



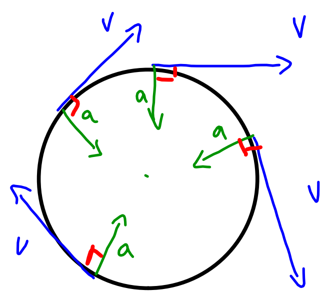
$$\boxed{a_c = \frac{v^2}{r}}$$

Centripetal acceleration

"centreseeking"

As $\Delta t \to 0$, $\theta \to 0$
which means that $\alpha \to 90^\circ$

So $\Delta v \perp v$, the acceleration is in the same direction as Δv and will be $\perp$ to v (i.e. along the radius of curvature)



Period and frequency for circular motion

Period T → time for one rotation/revolution/vibration.

→ unit: s $\frac{\text{time}}{\text{rotations}}$

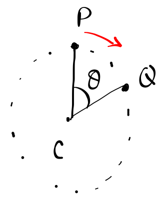
frequency f → how many rotations/revolutions/vibrations in a given time (usually 1 s) $\frac{\text{rotations}}{\text{time}}$

→ s^{-1} or Hz (hertz)

Note that period and frequency are reciprocals of one another.

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

Angular Displacement θ is the angle swept by a line joining the body to the centre.



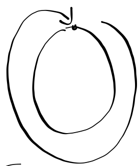
unit: radian

Angular Velocity ω ← omega

The angular velocity is the angular displacement swept out per time unit

$$\omega = \frac{\Delta\theta}{\Delta t}$$

units: radians s^{-1}



For one complete rotation $\Delta\theta = 2\pi$

and $\Delta t = T$

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi\left(\frac{1}{T}\right)$$

$$\omega = 2\pi f$$

Angular velocity and speed.

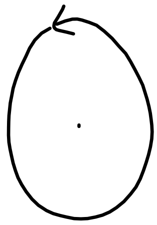
$$v = \underbrace{\frac{2\pi}{T}}_{\omega} r$$

$$\boxed{v = \omega r}$$

Alternative Expressions for Centripetal Acceleration

* $a_c = \dfrac{v^2}{r}$

For 1 complete revolution around a circle:



$v = \dfrac{\Delta x}{\Delta t}$

$v = \dfrac{2\pi r}{T}$

(tangential speed)

Data booklet

$a_c = \dfrac{\left(\dfrac{2\pi r}{T}\right)^2}{r}$

* $\boxed{a_c = \dfrac{4\pi^2 r}{T^2}}$

Not in data booklet. → $\boxed{a_c = 4\pi^2 r f^2}$