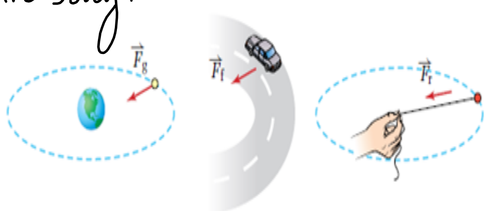
Centripetal Force

FBDs are VERY important when solving centripetal force problems !!!

Since a body travelling a circular path is continuously accelerating, there must be a net force (unbalanced force) acting on the body.



F_c is the the net force or unbalanced force!

↑centripetal.

$$\vec{F}_{net} = m\vec{a}$$

$$F_c = \frac{mv^2}{r}$$

F_{net}

recall

$v = \omega r$

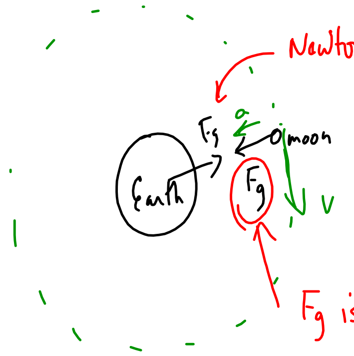
$v^2 = \omega^2 r^2$

$F_c = \frac{m\omega^2 r^{\cancel{2}}}{\cancel{r}} = m\omega^2 r$

(there may be more than one force that results in F_{net} (F_c))

Consider the moon orbiting the Earth:

Newton's 3rd law -> the moon exerts a force on the Earth equal to the force that the Earth exerts on the moon but in the opposite direction.



F_g is the centripetal force on the moon ← holds the moon in its orbit

it is directed to the centre of the Earth (in the same direction of the acceleration)

Example:
The speed of the Earth in its orbit about the Sun is about 3.0 x 10^4 m s^{-1} and the radius of its orbit is 1.5 x 10^{11} m. Calculate the centripetal acceleration of the Earth.

$$a_c = \frac{v^2}{r}$$ magnitude

$$a_c = \frac{(3.0\times10^4\,ms^{-1})^2}{1.5\times10^{11}\,m}$$

$$a_c = 6.0\times10^{-3}\,m\,s^{-2}$$

direction: towards the Sun.

Example:

The period or rotation of a bicycle wheel is 0.25 s. What is its rotation frequency?

$$f = \frac{1}{T}$$

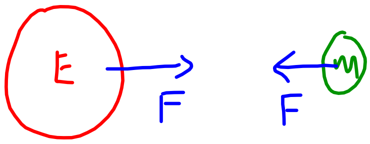
$$f = \frac{1}{0.25\,s}$$

$$f = 4.0\ Hz$$

$$1\,Hz = 1\,s^{-1} = 1/s$$

Uniform circular motion (continued)

What is the Newton's third law pair of the gravitational force of the Earth acting on the Moon?



equal + opposite

Why doesn't the Earth orbit around the Moon under the action of this force? Earth has a very large mass compared to the Moon

Why doesn't the Moon fall down? or does it? it is falling, but never reaches the Earth.

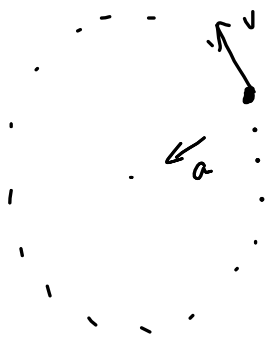
What if we could turn off the Earth's gravity? What would happen to the moon? inertia moon would go in a straight line (tangent to its orbit)

Why does a satellite stay in orbit?

Why does an astronaut feels weightless? (cutting elevator cable) (constant free fall, they actually have weight

When you are inside a car that is turning to the left, why do you feel like you are going to the right? inertia

Work done by a centripetal force



acceleration is to the centre ∴ the net force is to the centre which is perpendicular to the motion (i.e. velocity vector).

$\Delta W = Fs\cos\theta$ 90°

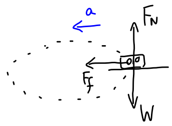
$\boxed{\Delta W = 0\,J}$

What is the consequence of that for the Moon in its orbit around the Earth?

No work done so there is no change in the Moon's Energy.

FBDs and Centripetal Force

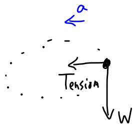
For a car going around a curved level track:



$\vec{F}_{net} = m\vec{a}$

$F_f = \frac{mv^2}{r}$

Twirling a ball attached to a string in horizontal circle:

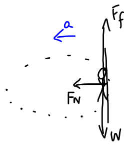


$\vec{F}_{net} = m\vec{a}$

$T = \frac{mv^2}{r}$

(tension)

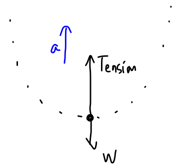
Gravitron:



$\vec{F}_{net} = m\vec{a}$

$F_N = \frac{mv^2}{r}$

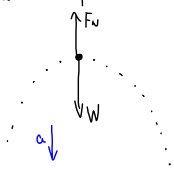
Spiderman at the bottom of his swing on his web:



$\vec{F}_{net} = m\vec{a}$

$\text{Tension} - W = \frac{mv^2}{r}$

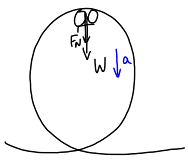
You are at the top of a ferris wheel:



$\vec{F}_{net} = m\vec{a}$

$W - F_N = \frac{mv^2}{r}$

Loop-de-loop in your motorcycle:

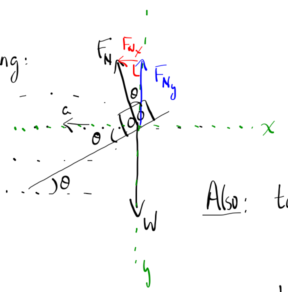


$\vec{F}_{net} = m\vec{a}$

$F_N + W = \frac{mv^2}{r}$

zero at the minimum speed

Curve banking:
(frictionless)



$\vec{F}_{net} = m\vec{a}$

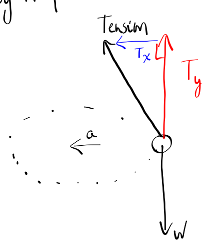
$F_{N_x} = \frac{mv^2}{r}$

Also: $\tan\theta = \frac{F_{Nx}}{F_{Ny}}$

$\tan\theta = \frac{F_{Nx}}{mg}$

$F_{N_x} = mg\tan\theta$

Toy Airplane:



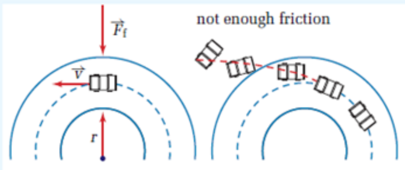
$\vec{F}_{net} = m\vec{a}$

$T_x = \frac{mv^2}{r}$

Centripetal Force in a Horizontal and a Vertical Plane

1. A car with a mass of 2135 kg is rounding a curve on a level road. If the radius of curvature of the road is 52 m and the coefficient of friction between the tires and the road is 0.70, what is the maximum speed at which the car can make the curve without skidding off the road?

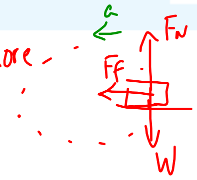
μ



continued ▶

You need to know more about friction:

$F_f = \mu F_N$



$\vec{F}_{net} = m\vec{a}$

$F_f = m\frac{v^2}{r}$

$\mu F_N = m\frac{v^2}{r}$

$\mu \cancel{m}g = \cancel{m}\frac{v^2}{r}$

$v^2 = \mu g r$

$F_f = 0.70(2135\,kg)(9.81\,m/s^2)$

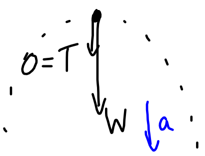
$F_f = 14661.045\,N$

$v^2 = 0.70(9.81\,m/s^2)(52\,m)$

$v = 19\,m\,s^{-1}$

2. You are playing with a yo-yo with a mass of 225 g. The full length of the string is 1.2 m. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.
   (a) Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.
   (b) At the speed that you determine in part (a), find the tension in the string when the yo-yo is at the side and at the bottom of its swing.

a) at the top (T = 0 when it is at its minimum speed)



$\vec{F}_{net} = m\vec{a}$

$W + \cancel{T} = m\frac{v^2}{r}$ ($T = 0$)

$W = m\frac{v^2}{r}$

$\cancel{m}g = \cancel{m}\frac{v^2}{r}$

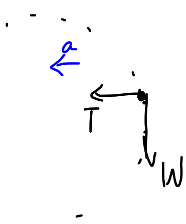
$v^2 = gr$

$v^2 = (9.81\,m/s^2)(1.2\,m)$

$v = 3.4\,m\,s^{-1}$

↑ If you swing faster, then there will be tension

b) at the side:



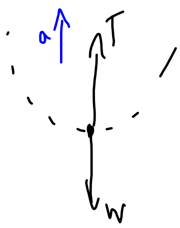
$\vec{F}_{net} = m\vec{a}$

$T = m\frac{v^2}{r}$

$T = \frac{(0.225\,kg)(3.4\,m\,s^{-1})^2}{1.2\,m}$

$T = 2.2\,N$

at the bottom:



$\vec{F}_{net} = m\vec{a}$

$T - W = m\frac{v^2}{r}$

$T = m\frac{v^2}{r} + W$

$T = 2.2\,N + (0.225\,kg)(9.81\,m\,s^{-2})$

$T = 4.4\,N$

Summary ... if $v = 3.4\,m\,s^{-1}$

the $T = 0$ (at top)

$T = 2.2\,N$ (at side)

$T = 4.4\,N$ (at bottom)