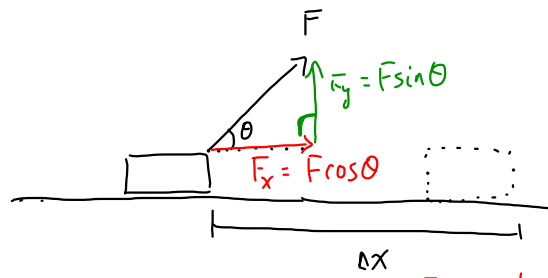


How do we calculate work when the force + displacement are not in the same direction?



$$\Delta W = \overset{\leftarrow}{F} \Delta x \quad \leftarrow F \text{ must be in the same direction as } \Delta x$$

$$\Delta W = (F \cos \theta) \Delta x$$

$$\Delta W = F \Delta x \cos \theta$$

$$\Delta W = F s \cos \theta \quad \leftarrow \text{Data Booklet}$$

If  $\begin{matrix} \vec{F} \\ \vec{\Delta x} \end{matrix} \quad \theta = 0^\circ \text{ and } \Delta W = F s \text{ (maximum work)}$

If  $\begin{matrix} \vec{F} \\ \vec{\Delta x} \end{matrix} \quad \theta = 90^\circ \text{ and } \Delta W = 0 \text{ (no work)}$

If  $\begin{matrix} \vec{F} \\ \vec{\Delta x} \end{matrix} \quad \theta = 180^\circ \text{ and } \Delta W = -F s \text{ (negative work)}$

Looking back at:

Calculate the work done by a net force of 10N which slows a moving object from  $50 \text{ m s}^{-1}$  to rest over a distance of 25m.

$$\Delta W = F s \cos \theta$$

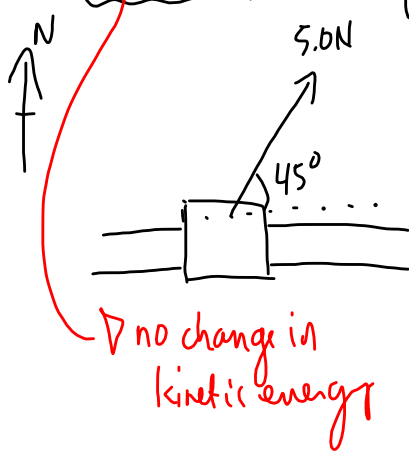
$$\Delta W = (10 \text{ N})(25 \text{ m}) \cos 180^\circ$$

$$\Delta W = -250 \text{ J}$$

The kinetic energy decreased by 250J. This 250J was transferred to internal energy.

Calculate the work done and describe the energy changes in each of the following:

- ① A force of 5.0N northeast moves an object a distance of 3.0m at a constant speed along an east-west track



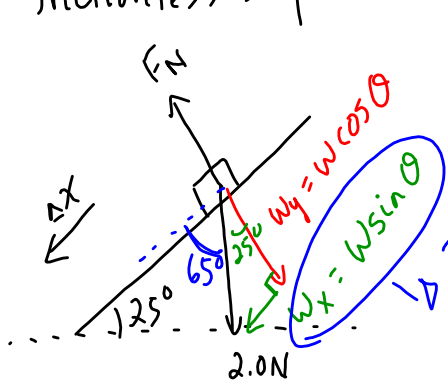
$$\Delta W = F s \cos \theta$$

$$\Delta W = (5.0\text{N})(3.0\text{m}) \cos 45^\circ$$

$$\Delta W = 11\text{J}$$

internal energy

- ② A body whose weight is 2.0N slides 10m down a frictionless slope inclined at 25° to the horizontal.



This component of the weight does the work.

the component of the force parallel to incline.

$$\Delta W = F s$$

$$\Delta W = (2.0\text{N})(\sin 25^\circ)(10\text{m})$$

$$\Delta W = 8.5\text{J}$$

gravitational potential → kinetic

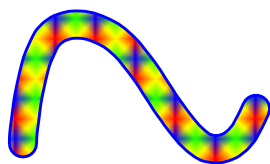
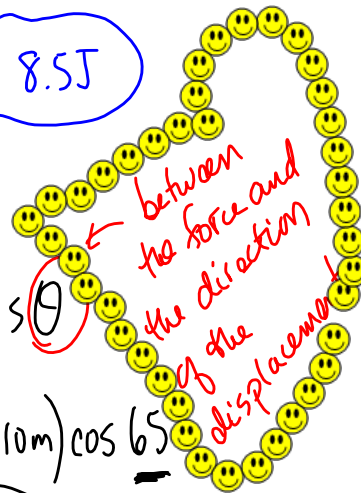
(loses 8.5J) (gains 8.5J)

OR:

$$\Delta W = F s \cos \theta$$

$$\Delta W = (2.0\text{N})(10\text{m}) \cos 65^\circ$$

$$\Delta W = 8.5\text{J}$$



③ A net force of  $2.0 \times 10^3 \text{ N}$  acts for  $5.0 \text{ s}$  on a body of mass  $2.0 \times 10^2 \text{ kg}$  which is initially at rest.

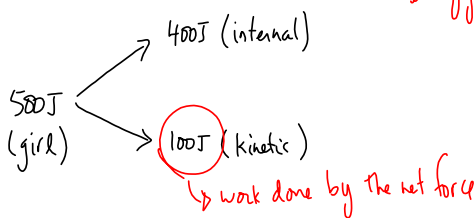
$\begin{matrix} \xrightarrow{F} \\ \xrightarrow{\Delta x} \end{matrix}$ 
 $\Delta W = F \Delta s$  ?? we need to find  $s$ !  
 $F_{\text{net}} = ma$   
 $a = \frac{F_{\text{net}}}{m}$   
 $a = \frac{2.0 \times 10^3 \text{ N}}{2.0 \times 10^2 \text{ kg}}$   
 $a = 10 \text{ m/s}^2$   
 $s = \cancel{vt} + \frac{1}{2} a t^2$   
 $s = \frac{1}{2} (10 \text{ m/s}^2) (5.0 \text{ s})^2$   
 $s = 125 \text{ m}$

work  $\rightarrow$  kinetic energy  
 $\Delta W = F s$   
 $\Delta W = (2.0 \times 10^3 \text{ N})(125 \text{ m})$   
 $\Delta W = 2.5 \times 10^5 \text{ J}$

④ A girl pushes an object  $5.0 \text{ m}$  along a horizontal road with a force of  $100 \text{ N}$  against a friction force of  $80 \text{ N}$ .

girl:  $\Delta W = F s$   
 $\Delta W = (100 \text{ N})(5.0 \text{ m})$   
 $\Delta W = 500 \text{ J}$   
 girl's chemical energy  $\downarrow$  work  $\downarrow$  kinetic energy.

friction:  $\Delta W = F s \cos \theta$   
 $\Delta W = (80 \text{ N})(5.0 \text{ m}) \cos 180^\circ$   
 $\Delta W = -400 \text{ J}$   
 friction  $\downarrow$  work  $\downarrow$  internal energy



(If there were no friction, the object would have gained  $500 \text{ J}$  of kinetic energy)  
 the object gains  $100 \text{ J}$  of kinetic energy  
 $\Delta W = F_{\text{net}} s$   
 $\Delta W = (20 \text{ N})(5.0 \text{ m})$   
 $\Delta W = 100 \text{ J}$

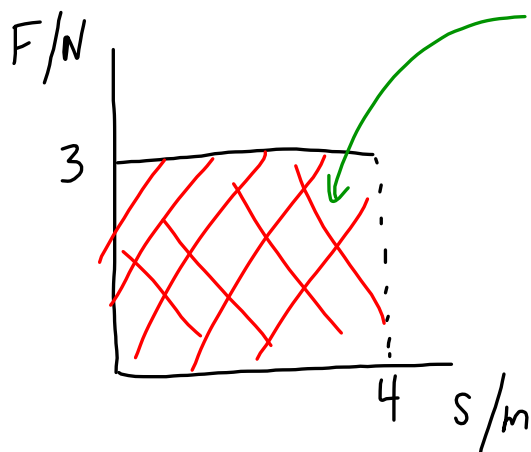
⑤ A man holds a load of  $1000 \text{ N}$  stationary above his head.

$\Delta W = 0 \text{ J}$

man's chemical potential energy  $\rightarrow$  internal

## Force-Distance Graph

Constant Force :



Area =  $l \times w$   
of  
rectangle

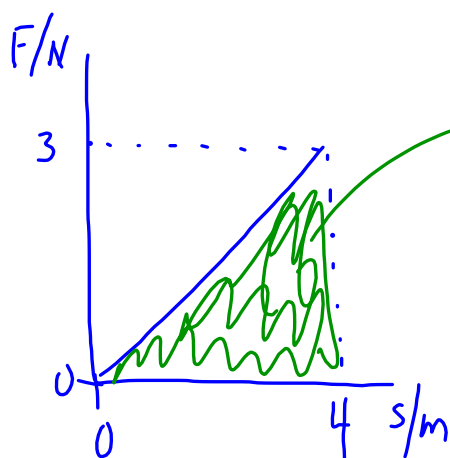
Area =  $Fs$  ← work!

The area under a Force distance graph is Work!

$$\Delta W = (3N)(4m)$$

$$\Delta W = 12J$$

Consider the graph for a force that is constantly changing:

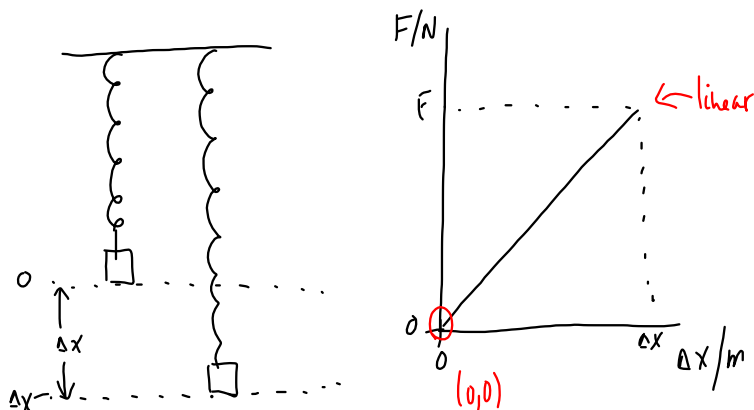


Area =  $\Delta W$

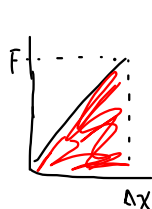
$$\Delta W = \frac{1}{2}(3N)(4m)$$

$$\Delta W = 6J$$

## Hookes Law: Extension of a Spring



Hookes Law  $\rightarrow F \propto \Delta x$   
 $F = k \Delta x$



$k$  is called the force constant (units:  $\text{N m}^{-1}$ )

$k$  is the slope on the  $F - \Delta x$  graph

$\Delta W$  is the area under the graph

$$\Delta W = \text{Area}$$

$$\Delta W = \frac{1}{2} F \Delta x \quad \text{but } F = k \Delta x$$

(this is the force to hold the spring at a distance of  $\Delta x$  from eq)

$$\Delta W = \frac{1}{2} (k \Delta x) (\Delta x)$$

$$\Delta W = \frac{1}{2} k (\Delta x)^2$$

This work is done to give the spring elastic potential energy

$$E_p = \frac{1}{2} k (\Delta x)^2$$

\*\* NOT in the Data Booklet

\*\* you must be able to derive or memorize!

Example

The force constant of a spring in a car suspension is  $5.0 \times 10^4 \text{ N m}^{-1}$

How much work is done by a force in compressing the spring by  $4.0 \text{ cm}$ ? How much energy is stored in the spring?

$0.040 \text{ m}$

$$\Delta W = E_p = \frac{1}{2} k (\Delta x)^2$$

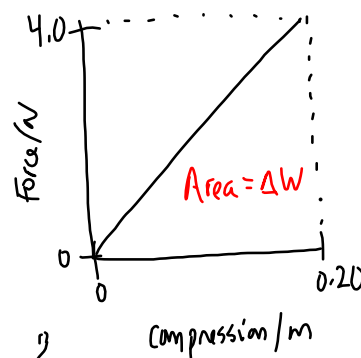
$$E_p = \frac{1}{2} (5.0 \times 10^4 \text{ N m}^{-1}) (0.040 \text{ m})^2$$

$$E_p = 40 \text{ J}$$

If there is  $40 \text{ J}$  stored in the spring as elastic potential energy, then  $40 \text{ J}$  of work was done in compressing the spring.

Example

Graph shows how the compression of a spring depends on the force applied to it.



- How much work is done in compressing the spring by  $0.20 \text{ m}$ ?

$$\Delta W = \text{Area}$$

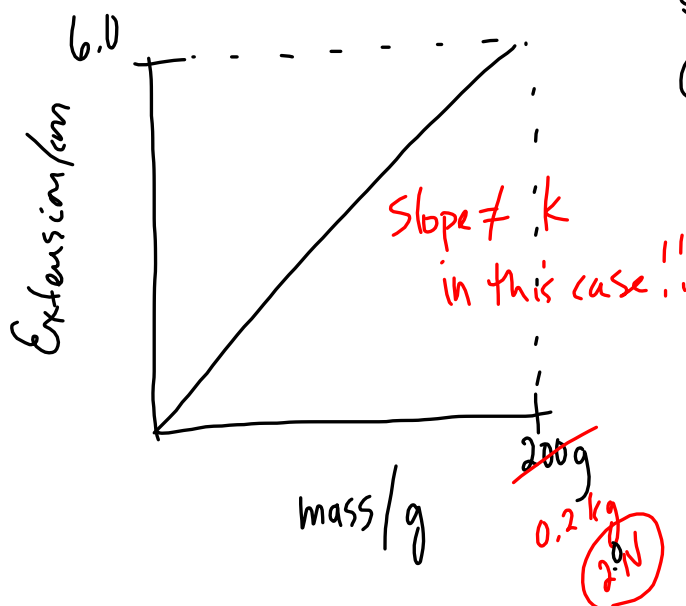
$$\Delta W = \frac{1}{2} (4.0 \text{ N}) (0.20 \text{ m})$$

$$\Delta W = 0.40 \text{ J}$$

you did  $0.40 \text{ J}$  to compress the spring.

What is the potential energy stored in the compressed spring?

$0.40 \text{ J}$

Exampledifferent masses  $\rightarrow$  stretch of spring.

Calculate:

- ① the force constant
- ② the work done to stretch 6.0 cm

(take  $g = 10 \text{ m s}^{-2}$ )

$$\textcircled{1} \quad F = k \Delta x$$

$$k = \frac{F}{\Delta x}$$

$$k = \frac{2.0 \text{ N}}{0.060 \text{ m}}$$

$$k = 33 \text{ N m}^{-1}$$

$$\textcircled{2} \quad \Delta W = \frac{1}{2} k (\Delta x)^2$$

$$\Delta W = \frac{1}{2} (33 \text{ N m}^{-1}) (0.060 \text{ m})^2$$

$$\Delta W = 0.060 \text{ J}$$