Linear Momentum

Linear momentum is related to inertia (or inertial mass), but it is not the same.

If the inertia (inertial mass) of a truck is large, then it has a large resistance to the change in its motion. It would require a large force to accelerate it.

It is easier to stop the truck if it is travelling at a lower velocity.

Momentum is a quantity that depends on both the inertial mass of a body and its velocity.

Linear momentum $\vec{p}$

The linear momentum $\vec{p}$ of a body of mass m moving with a velocity $\vec{v}$ is the product of its mass and its velocity.

$$\vec{p} = m\vec{v}$$

- a vector quantity
- units: $kg\ m\ s^{-1}$

Example

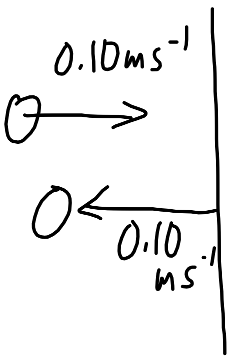
Determine the linear momentum of a truck of mass 10 000 kg moving east with a velocity of $20\ m s^{-1}$.

$m = 10000\ kg$
$\vec{v} = 20\ m s^{-1}$ [E]
$\vec{p} = ?$

$\vec{p} = m\vec{v}$
$\vec{p} = (10000\ kg)(20\ m s^{-1}\ [E])$
$\vec{p} = 2 \times 10^5\ kg \cdot m s^{-1}$ [E]

Example

A ball of mass 0.25kg moving east at $0.10 ms^{-1}$ bounces off a wall rebounding with the same speed. Calculate the change in momentum of the ball.



$$\Delta \vec{p} = \vec{p}_2 - \vec{p}_1$$

$$\Delta \vec{p} = m\vec{v}_2 - m\vec{v}_1$$

$$\Delta \vec{p} = m(\vec{v}_2 - \vec{v}_1) \leftarrow \Delta\vec{v}$$

$$\Delta \vec{p} = 0.25kg\,(0.10ms^{-1}[W] - 0.10ms^{-1}[E])$$

$$\Delta \vec{p} = 0.25kg\,(-0.10ms^{-1}[E] - 0.10ms^{-1}[E])$$

$$\Delta \vec{p} = 0.25kg\,(-0.20ms^{-1}[E])$$

$$\Delta \vec{p} = -0.050\ kg\cdot m/s\ [E]$$

$$\boxed{\Delta \vec{p} = 0.050 kg\cdot m/s\ [W]}$$

OR

$V_1 = +0.10ms^{-1}$

$V_2 = -0.10ms^{-1}$

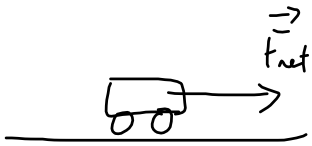
$\Delta V = -0.20ms^{-1}$

[W]

Relationship between force and change of momentum

If a net force $\vec{F}_{net}$ acts on a body of mass m, it accelerates, and according to Newton's second law:

$$\vec{F}_{net} = m\vec{a}$$



Recall $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$

$$\vec{F}_{net} = m\left(\frac{\Delta \vec{v}}{\Delta t}\right)$$

$$\vec{F}_{net} = m\left(\frac{\vec{v}_2 - \vec{v}_1}{\Delta t}\right)$$

$$\vec{F}_{net} = \frac{m\vec{v}_2 - m\vec{v}_1}{\Delta t} \quad \text{OR} \quad \frac{mv - mu}{\Delta t}$$

$$\vec{F}_{net} = \frac{\vec{p}_2 - \vec{p}_1}{\Delta t}$$

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$$

← Really just another form of Newton's second law.

Newton's Second Law (Version 2!)

The net force acting on a body is equal to the rate of change of momentum that it produces in the body.

Two formats for Newton's Second Law:

1. $\vec{F}_{net} = m\vec{a}$
2. $\vec{F}_{net} = \dfrac{\Delta \vec{p}}{\Delta t}$

Impulse of a force $\vec{I}$

The impulse $\vec{I}$ of a force $\vec{F}$ is defined as the product of the force acting and the time Δt for which it acts

$$\vec{I} = \vec{F}\Delta t$$

- vector quantity
- N s

$kg \cdot m\,s^{-2} \cdot s = kg \cdot m\,s^{-1}$ (momentum)

Relationship between Impulse + Change in momentum.

Recall: $\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$

$$\vec{F}_{net}\Delta t = \Delta \vec{p}$$

$$\vec{I}_{net} = \Delta \vec{p}$$

OR

$$\boxed{\vec{F}\Delta t = m\Delta \vec{v}}$$

Another form of Newton's Second Law

(Impulse-Momentum Theorem)

The impulse of the net force is equal to the change in momentum that it causes

Impulse and change of momentum

$$\vec{I}_{net} = \Delta \vec{p}$$

$\vec{J}$

This is really just another form of Newton's Second Law.

Newton's Second Law: The impulse of a net force acting on a body is equal to the change in momentum of the body that it produces

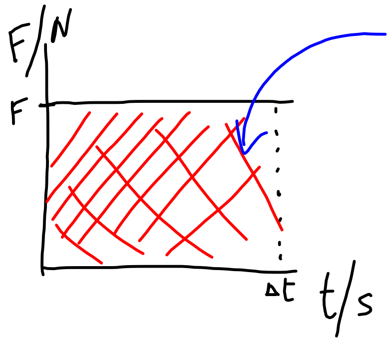
Impulse: units are N s $\rightarrow$ $kg\,m\cancel{s^{-2}}\,\cancel{s}$ = $kg\,m s^{-1}$

momentum: units are $kg\,m s^{-1}$

Same unit!

Momentum can be expressed as $kg\,m s^{-1}$ or N s

Force - time graph



area of a rectangle $= l \times w$

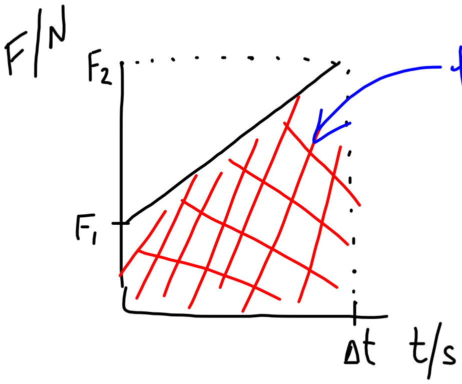
area $= F\Delta t$

$\therefore$ area = Impulse!

(or change in momentum)

The area under a Force - time graph gives the impulse of F_{net} during time Δt.

What happens when the force is not constant?

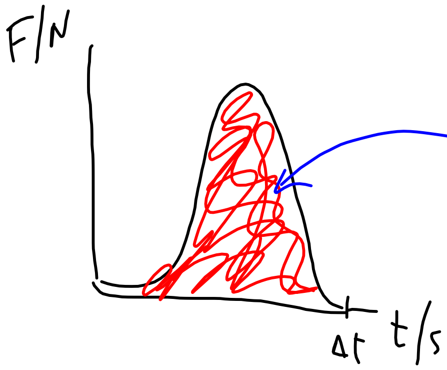


the area still represents the impulse,

but

$$\text{area} = \frac{1}{2}(h_1 + h_2)b$$

$$\text{area} = \frac{1}{2}(F_1 + F_2)\Delta t$$

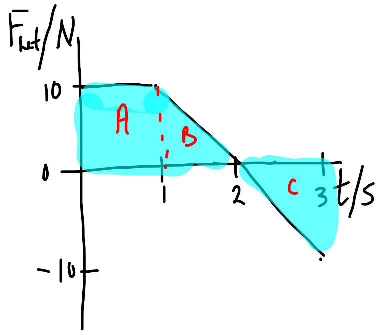


area = ??

1. count squares on the grid on the graph
2. use Calculus (if you know the equation for the curve)
3. use technology (i.e. Logger Pro)

Example

The graph shows the way in which a net force acting on a body of mass 3.0 kg varies with the time for which it acts



Determine:

a) the total impulse of the force

b) the change in momentum of the body as a result of the force

c) the final velocity of the body if its initial velocity was $-5.0\,ms^{-1}$

a) The area is simply the Area of A (B and C cancel out)

$\vec{I}_{net}$ = area of A

$\vec{I}_{net} = (10N)(1s)$

$\boxed{\vec{I}_{net} = 10\,Ns}$

b) $\Delta\vec{p} = \vec{I}_{net}$

$\therefore \Delta\vec{p} = 10\,N\cdot s$ or $10\,kg\,ms^{-1}$

c) $v = ?$

$u = -5.0\,ms^{-1}$

$m = 3.0\,kg$

$\Delta p = 10\,N\cdot s$

$\Delta\vec{p} = m\Delta\vec{v}$ or $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$

$\Delta\vec{p} = m(v-u)$

$\Delta\vec{p} = mv - mu$

$\Delta\vec{p} = m(v-u)$

$\Delta\vec{p} = m\Delta\vec{v}$

$v - u = \frac{\Delta\vec{p}}{m}$

$v = \frac{\Delta\vec{p}}{m} + u$

$v = \frac{10\,Ns}{3.0\,kg} + (-5.0\,ms^{-1})$

$v = 3.3\,ms^{-1} - 5.0\,ms^{-1}$

$\boxed{v \doteq -1.7\,ms^{-1}}$

What happens when two bodies collide?