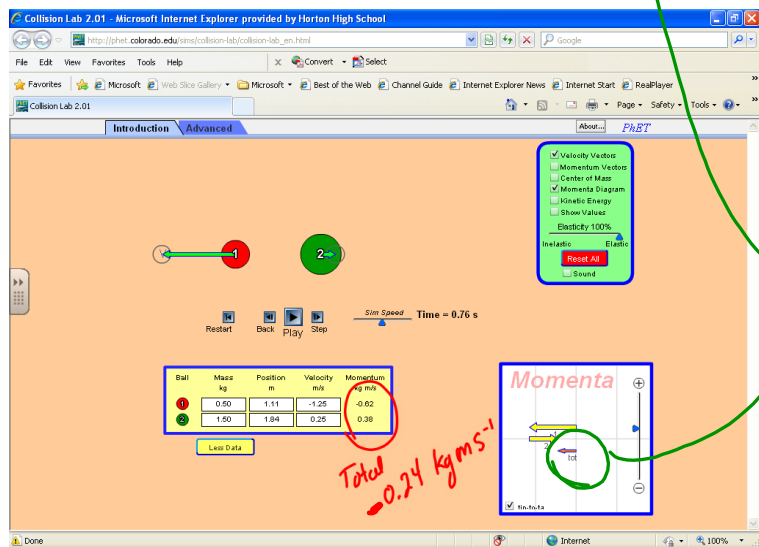


BEFORE collision  
 $p_{total} = -0.25 \text{ kg m s}^{-1}$



AFTER collision  
 $p_{total} = -0.24 \text{ kg m s}^{-1}$

ALSO note the momenta diagram

the total momentum vector is the sum

In all 8 of the collisions you investigated, the total momentum was the same both before and after the collision (true even when the bodies stuck together or even when a 2D collision or even when more than 2 bodies involved)

We can express this mathematically

$$\vec{P}_{\text{total (before)}} = \vec{P}_{\text{total (after)}} \quad \left( \sum \vec{p} \right)_{\text{total}}$$

$$m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

- This is a particular case (i.e. two bodies) for the more general form of the Law of Conservation of Momentum.
- This applies only when the collision occurs in an "isolated system".

### Conservation laws in Physics

- many types of quantities are conserved in nature.
- these quantities are called "conserved quantities" and their conservation is expressed in "conservation laws"
  - mass
  - energy
  - electric charge
  - linear momentum.
- within a closed system, these quantities can be transferred from one body to another, but the total remains the same.

### Closed System (Isolated System)

The conserved quantities are only conserved (constant) provided there are no outside influences.

### Law of Conservation of Linear Momentum

In any closed system of interacting bodies (i.e. one in which there are no external forces acting) the total momentum (like friction/air resistance) of the bodies is constant in both magnitude and direction regardless of the interaction of its parts.

**Example:** A 2.0 kg skateboard is rolling across a smooth flat floor when a small girl kicks it, causing it to speed up to 4.5 m s<sup>-1</sup> in 0.50 s without changing its direction. If the average force exerted by the girl on the skateboard in its direction of motion was 6.0 N, with what initial velocity was it moving?

$$m = 2.0 \text{ kg}$$

$$v = 4.5 \text{ m s}^{-1}$$

$$\Delta t = 0.50 \text{ s}$$

$$F_{\text{net}} = 6.0 \text{ N}$$

$$u = ?$$

$$\vec{F}_{\text{net}} \Delta t = m \Delta \vec{v} \quad (\text{impulse-momentum})$$

$$\vec{F}_{\text{net}} \Delta t = m (\vec{v} - \vec{u})$$

$$\vec{v} - \vec{u} = \frac{\vec{F}_{\text{net}} \Delta t}{m}$$

$$-\vec{u} = \frac{\vec{F}_{\text{net}} \Delta t}{m} - \vec{v}$$

$$\vec{u} = \vec{v} - \frac{\vec{F}_{\text{net}} \Delta t}{m}$$

$$\vec{u} = 4.5 \text{ m s}^{-1} - \frac{(6.0 \text{ N})(0.50 \text{ s})}{(2.0 \text{ kg})}$$

$$\vec{u} = 4.5 \text{ m s}^{-1} - 1.5 \text{ m s}^{-1}$$

$$\vec{u} = 3.0 \text{ m s}^{-1} \quad [\text{forward}]$$

**EXAMPLE:** A loaded railway car of mass 6000 kg is rolling to the right at 2.0 m s<sup>-1</sup> when it collides and couples with an empty freight car of mass 3000 kg, rolling to the left on the same track at 3.0 m s<sup>-1</sup>. What is the velocity of the pair after the collision?

railway car:

$$m_1 = 6000 \text{ kg}$$

$$u_1 = +2.0 \text{ m/s}$$

↑ right

freight car:

$$m_2 = 3000 \text{ kg}$$

$$u_2 = -3.0 \text{ m/s}$$

↑ left

$$\vec{P}_{\text{total (before)}} = \vec{P}_{\text{total (after)}}$$

$$\vec{P}_1 + \vec{P}_2 = \vec{P}_{12}$$

$$m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_{12} \vec{v}_{12}$$

$$(6000 \text{ kg})(2.0 \text{ m/s}) + (3000 \text{ kg})(-3.0 \text{ m/s}) = (9000 \text{ kg}) \vec{v}_{12}$$

$$12000 \text{ kg} \cdot \text{m/s} - 9000 \text{ kg} \cdot \text{m/s} = (9000 \text{ kg}) \vec{v}_{12}$$

$$3000 \text{ kg} \cdot \text{m/s} = (9000 \text{ kg}) \vec{v}_{12}$$

$$\vec{v}_{12} = \frac{3000 \text{ kg} \cdot \text{m/s}}{9000 \text{ kg}}$$

$$\vec{v}_{12} = 0.33 \text{ m/s} \text{ [R]}$$

**EXAMPLE:** Calculate the recoil velocity of an unconstrained rifle of mass 5.0 kg after it shoots a 50 g bullet at a speed of 300 m s<sup>-1</sup>, with respect to the Earth

$$\vec{P}_{\text{total (before)}} = \vec{P}_{\text{total (after)}}$$

$$\vec{P}_{\text{gun+bullet}} = \vec{P}_{\text{gun}} + \vec{P}_{\text{bullet}}$$

$$m_{gb} \vec{V}_{gb} = m_g \vec{V}_g + m_b \vec{V}_b$$

$$0 = (5.0 \text{ kg}) \vec{V}_g + (0.050 \text{ kg})(300 \text{ m s}^{-1})$$

$$-(5.0 \text{ kg}) \vec{V}_g = (0.050 \text{ kg})(300 \text{ m s}^{-1})$$

$$\vec{V}_g = \frac{(0.050 \text{ kg})(300 \text{ m s}^{-1})}{-5.0 \text{ kg}}$$

$$\vec{V}_g = \ominus 3.0 \text{ m s}^{-1}$$

$$\vec{V}_g = 3.0 \text{ m s}^{-1} \quad \text{[opposite the motion of the bullet]}$$

**EXAMPLE:** A  $1.0 \text{ kg}$  ball moving with a velocity of  $2.0 \text{ m s}^{-1}$  to the right collides straight-on with a stationary  $2.0 \text{ kg}$  ball. After the collision, the  $2.0 \text{ kg}$  ball moves off to the right with a velocity of  $1.2 \text{ m s}^{-1}$ . What is the velocity of the  $1.0 \text{ kg}$  ball after the collision?

+ right  
- left

$m_A = 1.0 \text{ kg}$   
 $m_B = 2.0 \text{ kg}$

$$\vec{P}_{\text{total (before)}} = \vec{P}_{\text{total (after)}}$$

$$\vec{P}_{A1} + \vec{P}_{B1} = \vec{P}_{A2} + \vec{P}_{B2}$$

$$m_A \vec{u}_A + m_B \vec{u}_B = m_A \vec{v}_A + m_B \vec{v}_B$$

$$m_A \vec{u}_A + \cancel{m_B \vec{u}_B} - m_B \vec{v}_B = \vec{v}_A$$


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$$\frac{m_A}{(1.0 \text{ kg})(+2.0 \text{ m s}^{-1}) - (2.0 \text{ kg})(+1.2 \text{ m s}^{-1})} = \vec{v}_A$$

$$\frac{1.0 \text{ kg}}{2.0 \text{ kg} \cdot \text{m s}^{-1} - 2.4 \text{ kg} \cdot \text{m s}^{-1}} = \vec{v}_A$$

$$-0.40 \text{ m s}^{-1} = \vec{v}_A$$

$$\vec{v}_A = 0.40 \text{ m s}^{-1} \text{ [left]}$$