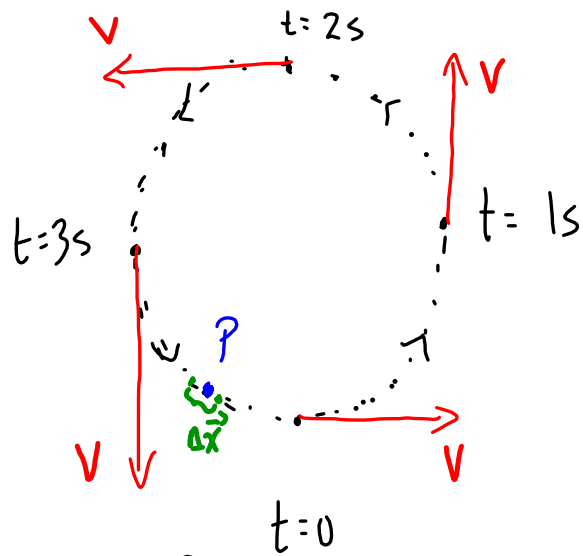


Instantaneous Velocity

The instantaneous velocity of the object at a point P is determined by measuring the displacement $\Delta \vec{x}$ over a small time interval when it passes P.



$$\vec{v}_{inst} = \frac{\Delta \vec{x}}{\Delta t} \quad \text{when } \Delta t \text{ is very small (almost zero)}$$

$$\vec{v}_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t}$$

The instantaneous velocity of an object at a point P is its velocity at the instant that it passes P and it is the average velocity of the object calculated over an infinitely small (but not zero) displacement around P.

Acceleration

We have already defined acceleration:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

A better way (more accurate) is to define acceleration as the rate of change of instantaneous velocity

$$\vec{a} = \frac{\Delta \vec{v}_{inst}}{\Delta t}$$

Acceleration can be the average acceleration in a range or the instantaneous acceleration at a point

average acceleration: $\vec{a}_{av} = \frac{\Delta \vec{v}_{inst}}{\Delta t}$

instantaneous acceleration: $\vec{a}_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}_{inst}}{\Delta t}$

So instantaneous acceleration is like finding the average acceleration but over a very small (but not zero) time interval.

Instantaneous velocity ^{acceleration} can be found:

- use calculus (if an equation is known for the position with respect to time)
 - use a graph (draw a tangent line at time t)
- experimental {
- ticker tape
 - multi-image photography / video analysis
 - motion detector

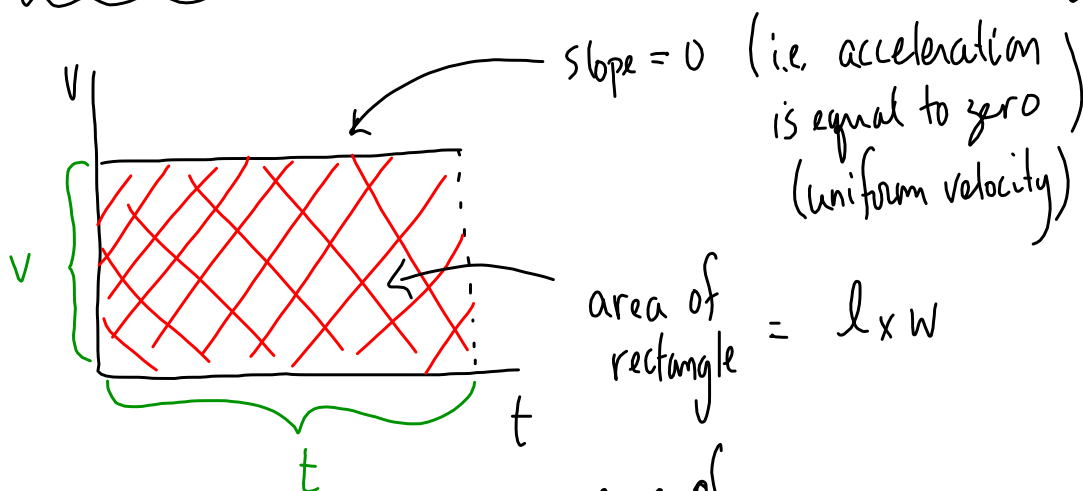
Equations for uniformly accelerated motion

AKA: "The kinematics equations" or the "suvat" equations

These are a set of 5 equations that can be used to solve kinematics problems involving uniformly accelerated motion

You need to be able to derive the equations. Some are not in the data booklet \Rightarrow must memorize them.

Consider a velocity-time graph for motion with uniform velocity



① slope = acceleration
v-t graph

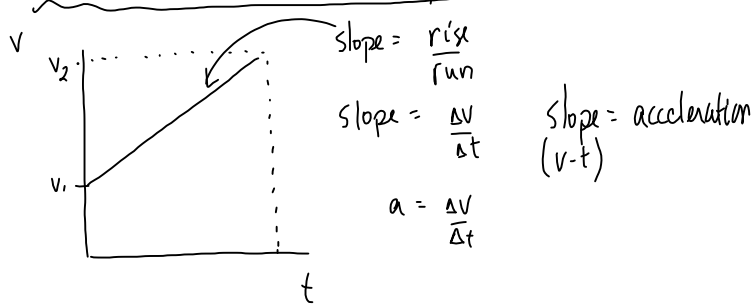
area of rectangle = $v t$

$$d = v t$$

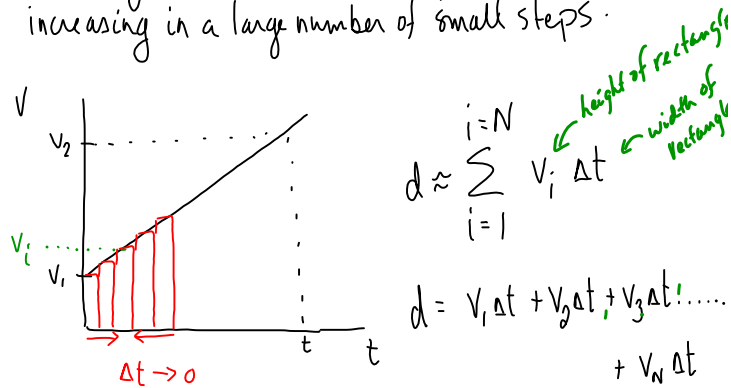
② area under v-t graph = distance

(area = distance travelled)
under v-t graph

Velocity-Time graph for Uniformly Accelerated Motion



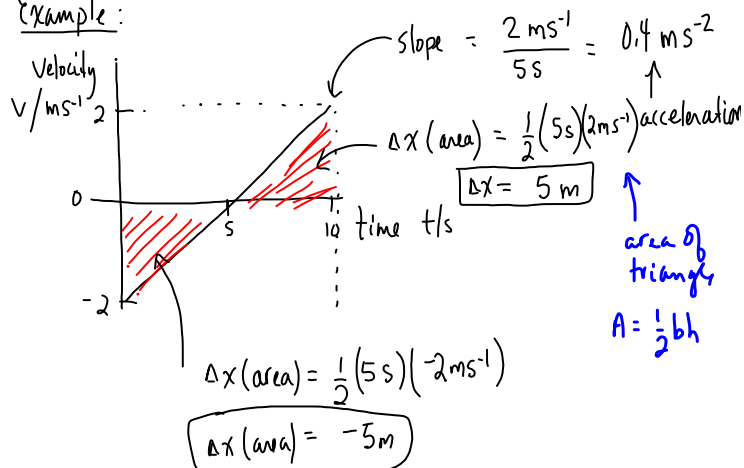
To determine the distance travelled when the velocity is changing, imagine that the slope of the line is increasing in a large number of small steps.



We could think about this shape having an area that is equal to the area of an infinite number of rectangles with a width of almost zero.

- ① slope = acceleration (v-t)
- ② area (v-t) = distance (or displacement)

Example:



Overall displacement (over 0 to 10s) = 0

Equation 1

Acceleration: $a = \frac{\Delta v}{\Delta t}$

v is the final velocity

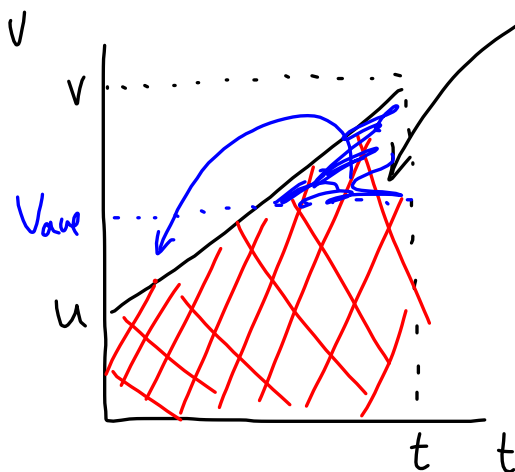
Suvat

$$a = \frac{v - u}{t}$$

u is the initial velocity.

$$at = v - u$$

$$\boxed{v = u + at} \quad (1)$$

Equation 2

area (v-t)
displacement = $\frac{1}{2}(h_1 + h_2) b$

$$s = \frac{1}{2}(u + v) t$$

$$\boxed{s = \left(\frac{u + v}{2}\right) t} \quad (2)$$

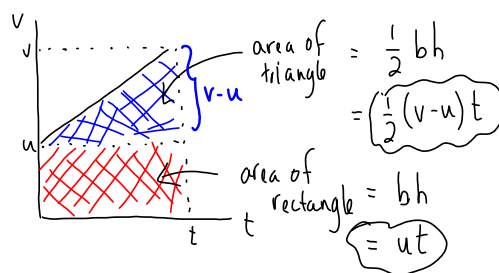
assumed
that there
is constant
acceleration

$$s = v_{\text{ave}} t$$

Equation 3

Recall Equation 1: $v = u + at$
 $v - u = at$ (rearrange for t)
 $t = \frac{v-u}{a}$

Recall Equation 2: $s = \left(\frac{u+v}{2}\right)t$
 $s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$
 $s = \frac{v^2 - u^2}{2a}$
 $2as = v^2 - u^2$
 $v^2 = u^2 + 2as$ (3)

Equation 4

$$s = ut + \frac{1}{2}(v-u)t$$

Recall equation 1: $v = u + at$
 $v - u = at$

$$s = ut + \frac{1}{2}(at)t$$

$$s = ut + \frac{1}{2}at^2$$
 (4)

Equation 5