Trig Functions + Uncertainties

No simple rule .... the uncertainty is half of the difference between the highest and lowest values.

$\cos\theta = ?$ if $\theta = (60 \pm 5)^\circ$

$\cos 60^\circ = 0.500$

$\cos 65^\circ = 0.423$

$\cos 55^\circ = 0.574$

$\frac{0.574 - 0.423}{2} = 0.076$

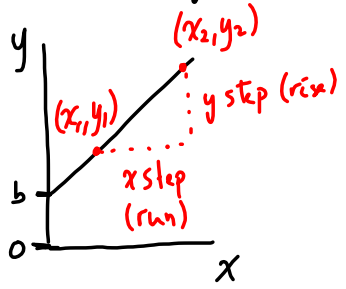
$\cos(60 \pm 5)^\circ = 0.50 \pm 0.08$

NOTE: (in general)

If one uncertainty is very large large compared to another, then you may choose to only work with the very large uncertainty

Interpretation of Linear Graphs

Consider a linear graph (where $b \neq 0$)



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

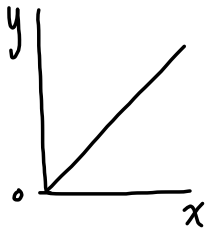
$$\text{slope} = \frac{y \text{ step}}{x \text{ step}}$$

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Recall: $y = mx + b$

What if you have a linear relationship with $b = 0$?



This is called a direct proportionality between y and x

"y is directly proportional to x"

"y varies directly with x"

a proportionality statement → $y \propto x$

$\propto$

"is proportional to"

If you double x
then y is doubled as well

$y \propto x$ (proportionality statement)

$y = kx$ (general equation)

k is the proportionality constant.

$(y = mx + b)$

A graph of y vs x will be linear with a slope of k and a y-intercept of zero!

Not all data you plot will give you a linear graph!

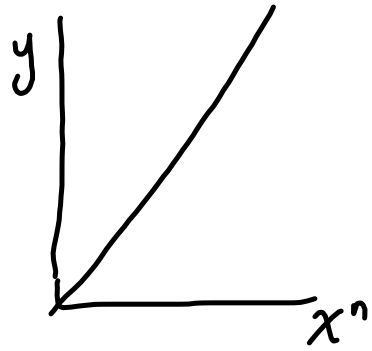
Power Curve

$y \propto x^n$

$y = k(x^n) + 0$

$(y = m x + b)$

← a linear equation "in disguise"



A graph of y vs x^n will be linear with a slope of k and a y-intercept of zero.

↑ linear graph (b=0) $\Rightarrow$ $y \propto x^n$

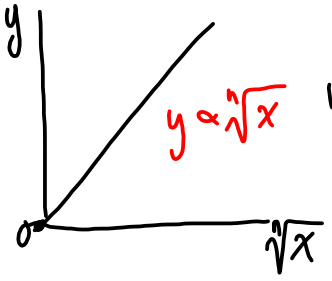
(y is proportional to x^n)

Root Curve

$y \propto \sqrt[n]{x}$

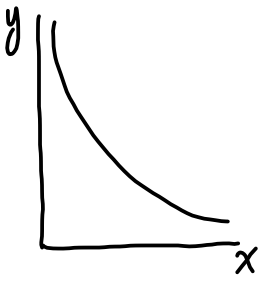
$y = k\sqrt[n]{x}$

$(y = mx + b)$



A graph of y vs $\sqrt[n]{x}$ will be linear with a slope of k and a y-intercept of zero.

Inverse Curve



$y \propto \frac{1}{x^n}$ "y is inversely proportional to x^n"

$y = k\left(\frac{1}{x^n}\right)$ "y is directly proportional to $\frac{1}{x^n}$"

$(y = mx + b)$

A graph of y vs $\frac{1}{x^n}$ will be linear with a slope of k and a y-int of zero.

Summary:

Power: $y \propto x^n$

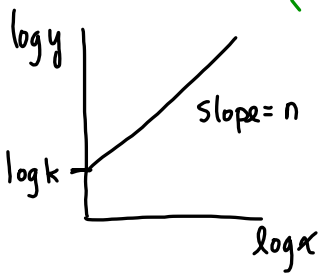
Root: $y \propto \sqrt[n]{x}$ $\left(y \propto x^{\frac{1}{n}}\right)$

Inverse: $y \propto \frac{1}{x^n}$ $\left(y \propto x^{-n}\right)$

all three can be expressed with n in the exponent.

Consider that you need to find the value for n:

$$y \propto x^n$$
$$y = kx^n$$
$$\log y = \log(kx^n)$$
$$\log y = \log k + \log x^n$$
$$\log y = \log k + n \log x$$
$$(y = b + mx)$$



A graph of $\log y$ vs $\log x$ will be linear with a slope of n and a y-intercept of k

$b = \log k$

$k = 10^b$

(inverse log)

Consider $T = 2\pi\sqrt{\frac{l}{g}}$ where g is a constant.

$$T = \frac{2\pi}{1}\frac{\sqrt{l}}{\sqrt{g}}$$

$T \propto \sqrt{l} \rightarrow$ $T = \frac{2\pi}{\sqrt{g}}\frac{\sqrt{l}}{1}$

$y = mx + b$

A graph of T vs $\sqrt{l}$ will be linear with a slope of $\frac{2\pi}{\sqrt{g}}$ and a y-intercept of zero.