Multiplying & Dividing

When multiplying & dividing, add the relative uncertainties to get the relative uncertainty of the result.

$$\text{If } y = \frac{ab}{c} \text{ then } \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

relative uncertainty

Example:

$(9.7 \pm 0.5)\,m \times (4.3 \pm 0.2)\,m = 41.7\,m^2 \pm ??$

add the relative uncertainties: $\frac{0.5}{9.7} + \frac{0.2}{4.3} = 0.052 + 0.047 = 0.099$

relative uncertainty of $41.7\,m^2$

absolute uncertainty for $41.7\,m^2$: $0.099(41.7\,m^2) = 4.1283\,m^2$

relative uncert. absolute uncertainty

Final answer: $(41.7 \pm 4)\,m^2$

same place value

$(42 \pm 4)\,m^2$

Example: Determine the answer with its absolute uncertainty:

$$\frac{(9.7 \pm 0.5)\,m}{(4.3 \pm 0.2)\,m} = 2.2558 \pm ??$$

Add relative uncertainties: $0.052 + 0.047 = 0.099$ ← relative uncertainty for 2.2558

absolute uncertainty in final answer: $0.099(2.2558) = 0.2233242$

can only have 1 sd

Final answer:

(2.2558 ± 0.2)

(2.3 ± 0.2)

Powers (and roots)

When raising a value to the power of n, multiply the relative uncertainty by n to give the relative uncertainty of the result.

Example: $[(9.7 \pm 0.5)m]^3 = 912.673 m^3 \pm ??$

relative uncertainty for 9.7: $\frac{0.5}{9.7} = 0.052$

relative uncertainty for $(9.7)^3$: $3(0.052) = 0.156$ ← relative uncertainty for the final answer. (912.673)

absolute uncertainty in $(9.7)^3$: $0.156(912.673) = 141.135$

$(912.673 \pm 141.135)m^3$

$(900 \pm 100)m^3$

$(9 \pm 1) \times 10^2 \ m^3$

Example The radius of a sphere is measured to be (8.5 ± 0.2)cm. Determine its volume with its absolute uncertainty.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (8.5)^3$

$\boxed{V = 2572.4 \ cm^3}$

relative uncertainty 8.5 cm: $\frac{0.2}{8.5} = 0.0235$

relative uncertainty $(8.5 cm)^3$: $3(0.0235) = 0.07059$

absolute uncertainty for $(8.5cm)^3$ = $0.07059(8.5 cm)^3 = 43.351....$

$V = \frac{4}{3}\pi(614.125 \pm 43.351) \ cm^3$

$V = (2572.4 \pm 181.59) \ cm^3$

$V = (2.6 \pm 0.2) \times 10^3 \ cm^3$

EASIER to find the absolute uncertainty for the final answer by:

$0.07059(2572.4)$

$= 181.59$ (the same)

Example:

The surface area of a square swimming pool is found to be $12 m^2$ with an absolute uncertainty of $2m^2$. Determine the length of each side of the pool with its absolute error.

$$\text{Area} = (\text{length})^2$$

$$\text{length} = \sqrt{\text{Area}}$$

$$\text{length} = \text{Area}^{1/2}$$

$$\text{length} = \sqrt{12 m^2}$$

$$\text{length} = 3.464\ldots$$

relative uncertainty area: $\frac{2}{12} = 0.17$

relative uncertainty side length: $\frac{1}{2}(0.17) = 0.0833$

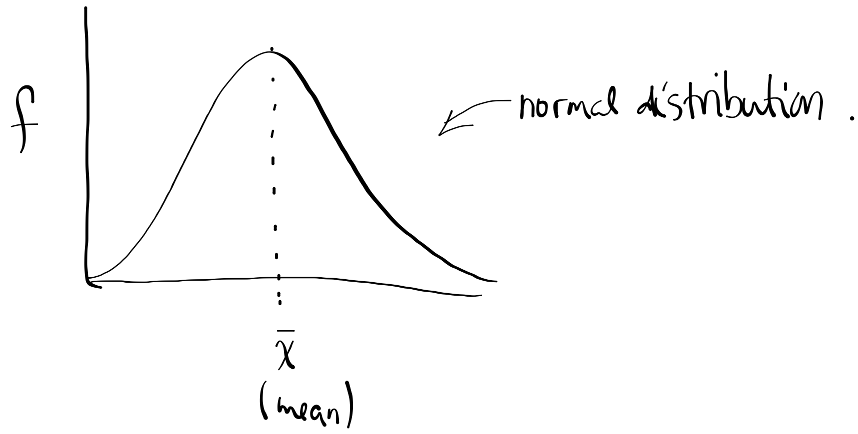
final answer

absolute uncertainty side: $0.0833(3.464) = 0.2847\ldots$

$$(3.464 \pm 0.2847)\,m$$

$$(3.5 \pm 0.3)\,m$$

If you take repeated measurements of the same thing, the measurements will follow a normal distribution.



If we have a large sample size, the uncertainty is basically the standard deviation.

Usually, in the lab, we might only take a sample of 5. Instead of using the standard deviation as a measure of the uncertainty, we can use $\frac{1}{2}$ of the range.

$$\bar{x} \pm \frac{(x_{max} - x_{min})}{2}$$

$$\text{mean} \pm \frac{\text{range}}{2}$$