§13.1 Describing Simple Harmonic Motion (SHM)

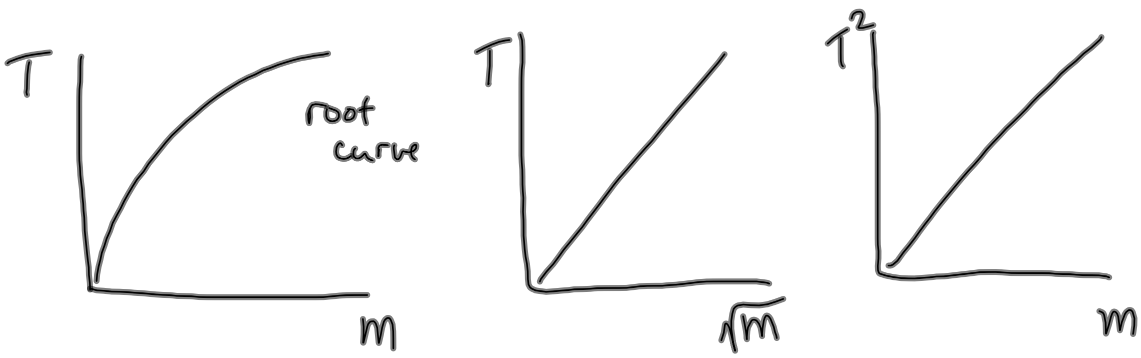
Mass on a Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where T is the period (s)
m is the mass (kg)
k is the spring constant (N/m)

In order to double the period, you need to quadruple the mass

$$T \propto \sqrt{m}$$



$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = \frac{(2\pi)^2}{k} m$$

$$T^2 = \frac{4\pi^2}{k} m$$

$$y = m x + b$$

On a graph of T^2 vs m

slope $= \frac{4\pi^2}{k}$

MP/606

$m = 125 g$

$x = 12.0 cm$

20.0 cycles in 15.5 s

a) $T = ?$

b) $k = ?$

c) $E_{total} = ?$

d) $V_{max} = ?$

e) $v = ?$ ($x = 10.0 cm$)

a) $T = \frac{15.5 s}{20.0 \text{ cycles}}$

$T = 0.775 s$

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 = \frac{4\pi^2 m}{k}$

$k = \frac{4\pi^2 m}{T^2}$

$k = \frac{4\pi^2(0.125 kg)}{(0.775 s)^2}$

$k = 8.22 \frac{N}{m}$

c) $E_{total} = E_e$ (at the max displacement from equilibrium)

Recall $E_e = \frac{1}{2}kx^2$

$\therefore \quad E_{total} = \frac{1}{2}kx^2$

$E_{total} = \frac{1}{2}(8.22 \frac{N}{m})(0.120 m)^2$

$E_{total} = 0.0592 J$

d) At the equilibrium position, $E_e = 0$ and $E_k = 0.0592 J$ (all the energy is kinetic energy)

Recall: $E_k = \frac{1}{2}mv^2$

$v^2 = \frac{2E_k}{m}$

$v^2 = \frac{2(0.0592 J)}{0.125 kg}$

$v = 0.973 m/s$

e) $v = ?$, when $x = 10.0 cm$

$E_{total} = 0.0592 J$ (made up of both $E_e + E_k$)

$E_{total} = E_e + E_k$

$E_{total} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$0.0592 J = \frac{1}{2}(8.22 N/m)(0.100 m)^2 + \frac{1}{2}(0.125 kg)v^2$

$0.0592 J = 0.0411 J + \frac{1}{2}(0.125 kg)v^2$

$0.0181 J = \frac{1}{2}(0.125 kg)v^2$

$\frac{2(0.0181 J)}{0.125 kg} = v^2$

$v = 0.538 m/s$

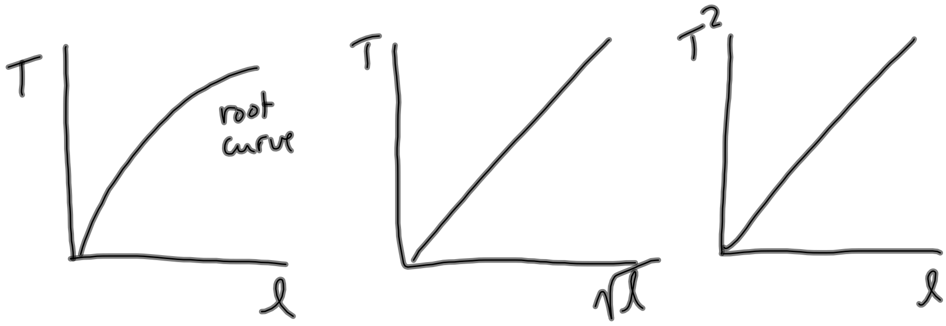
Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is the period (s)
l is the length (m)
g is the acc of gravity $\left(\frac{m}{s^2}\right)$

In order to double the period, you need to quadruple the length

$$T \propto \sqrt{l}$$



A graph of T^2 vs l is linear:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = \frac{4\pi^2}{g} l$$

← slope = $\frac{4\pi^2}{g}$

Energy

$$E_{TOTAL} = E_g + E_K$$

Recall: $E_g = mgh$

$E_K = \frac{1}{2}mv^2$

E_g is greatest at top of swing, E_K is zero

E_K is greatest at bottom of swing (equilibrium), E_g is zero

TO DO:

① PP|608

② MP|613 + PP|614