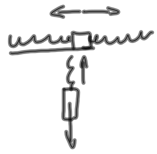
Chapter 13 - Simple Harmonic Motion (SHM)

Periodic Motion → motion that repeats itself.

Simple Harmonic Motion → periodic motion that is generated by a linear restoring force.



Period of a Mass on a Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where T is the period (s)
m is the mass (kg)
k is the spring or force constant (N/m)

Recall Law of Conservation of Mechanical Energy

$$E_{total} = E'_{total}$$

$$E_e + E_k = E_e' + E_k'$$

Recall: $E_k = \frac{1}{2}mv^2$

$E_e = \frac{1}{2}kx^2$ (x is the displacement from equilibrium)

$F_a = kx$ (Hooke's Law)

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$x = +12.0\text{ cm}$

$m = 125\text{ g}$

20.0 cycles in 15.5 s

a) $T = ?$
b) $k = ?$
c) E_{total}
d) $v_{max} = ?$
e) $v = ?$ ($x = 10.0\text{ cm}$)

a) $T = \frac{15.5\text{ s}}{20.0\text{ cycles}} = 0.775\text{ s}$

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$$T^2 = 4\pi^2\frac{m}{k}$$

$$k = \frac{4\pi^2 m}{T^2}$$

$$k = \frac{4\pi^2(0.125\text{ kg})}{(0.775\text{ s})^2}$$

$$k \doteq 8.22\ \frac{\text{N}}{\text{m}}$$

$\frac{\text{kg}\cdot\text{m}}{\text{s}^2\,\text{m}}$

c) $E_{total} = E_e + \cancel{E_k}^{\,0}$

$$E_{total} = \frac{1}{2}kx^2$$

$$E_{total} = \frac{1}{2}\left(8.22\frac{\text{N}}{\text{m}}\right)(0.120\text{ m})^2$$

$$E_{total} = 0.0592\text{ J}$$

d) The mass has its maximum velocity (← max E_k) when it passes through the equilibrium position and $E_e = 0$.

$$E'_{total} = \cancel{E_e'}^{\,0} + E_k'$$

$$0.0592\text{ J} = \frac{1}{2}mv^2$$

$$v^2 = \frac{2(0.0592\text{ J})}{(0.125\text{ kg})}$$

$$v \doteq \pm 0.973\text{ m/s}$$

e) $v = ?$ when $x = 10.0\text{ cm}$

$$E'_{total} = E_e' + E_k'$$

$$0.0592\text{ J} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$0.0592\text{ J} = \frac{1}{2}\left(8.22\frac{\text{N}}{\text{m}}\right)(0.100\text{ m})^2 + \frac{1}{2}(0.125\text{ kg})v^2$$

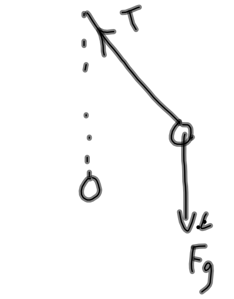
$$0.0592\text{ J} = 0.0411\text{ J} + \frac{1}{2}(0.125\text{ kg})v^2$$

$$0.0181\text{ J} = \frac{1}{2}(0.125\text{ kg})v^2$$

$$v^2 = \frac{2(0.0181\text{ J})}{(0.125\text{ kg})}$$

$$v \doteq \pm 0.538\text{ m/s}$$

Period of a Pendulum



$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is the period (s)
l is the length (m)
g is $9.81\,m/s^2$

There is a restoring force due to F_g

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$m = 2.45\,kg$
$l = 1.36\,m$

a) $T = ?$
b) $2T$, $l = ?$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{1.36\,m}{9.81\,m/s^2}}$$

$$T = 2.34\,s$$

you need to increase the length:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = \frac{4\pi^2 l}{g}$$

$$l = \frac{gT^2}{4\pi^2}$$

$$l' = \frac{g(2T)^2}{4\pi^2}$$

$$l' = 4\left(\frac{gT^2}{4\pi^2}\right)$$

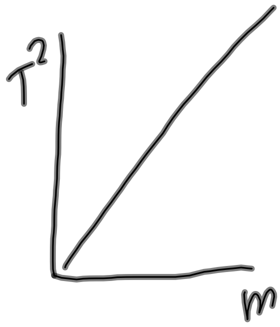
$$l' = 4l$$

The length must be increased by a factor of 4 so $4(1.36\,m) = 5.44\,m$

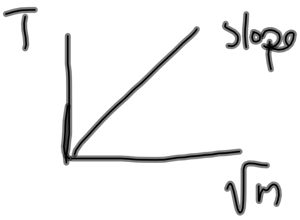
Consider: $T = 2\pi\sqrt{\frac{m}{k}}$ → $T^2 = \frac{4\pi^2}{k} m$ → $T^2 \propto m$

↓

$T \propto \sqrt{m}$



Slope $= \frac{4\pi^2}{k}$



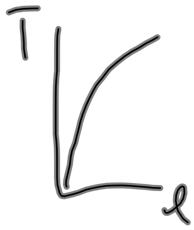
slope $= \frac{2\pi}{\sqrt{k}}$

Consider: $T = 2\pi\sqrt{\frac{l}{g}}$ → $T^2 = \frac{4\pi^2}{g} l$

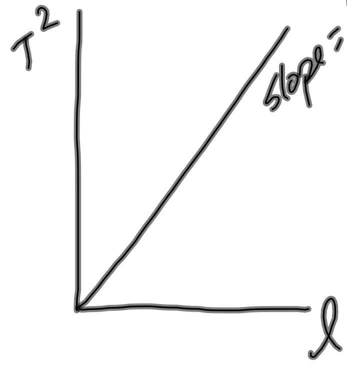
$T^2 \propto l$

↓

$T \propto \sqrt{l}$



slope: $\frac{2\pi}{\sqrt{g}}$



slope: $\frac{4\pi^2}{g}$

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