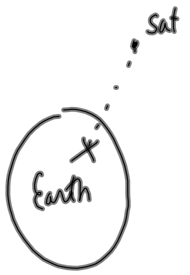
Satellite Motion (§12-2)

Newton imagined amountain that was high enough to launch a cannonball so that it would orbit the Earth. (Newton's Cannon / Newton's Mountain)

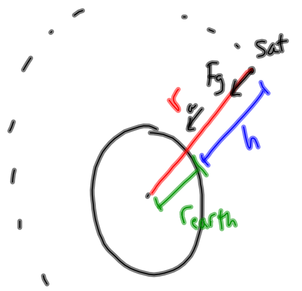


For a satellite to alway be directly above the same spot on the surface of the Earth, it must have the same period of rotation as the Earth on its axis (ie. 1 day)

geosynchronous or geostationary.

MP|589

At what velocity and altitude must a satellite orbit in order to be geostationary?



$T = 24h = 86400s$

$r_{earth} = 6.38 \times 10^6 m$

$M_{earth} = 5.98 \times 10^{24} kg$

According to Newton's Hypothesis:

$$F_g = F_c$$

$a_c = \frac{v^2}{r}$ $v = \frac{2\pi r}{T}$

$$\frac{G m_{sat} M_{earth}}{r^2} = m_{sat} a_c$$

$$\frac{G M_{earth}}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$r^3 = \frac{G M_{earth}}{4\pi^2} T^2$$

↖ Kepler's constant for the Earth.

$$r^3 = \frac{\left(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)\left(5.98 \times 10^{24} kg\right)\left(86400s\right)^2}{4\pi^2}$$

$r \doteq 4.23 \times 10^7 m$

$v = \frac{2\pi r}{T}$

$v = \frac{2\pi (4.23 \times 10^7 m)}{86400s}$

$v = 3.07 \times 10^3 \frac{m}{s}$

$r = r_{earth} + h$

$h = r - r_{earth}$

$h = 4.23 \times 10^7 m - 6.38 \times 10^6 m$

$h = 3.59 \times 10^7 m$ ← altitude

Kepler's Laws

1. elliptical orbits
2. sweep equal areas in equal times.
3. $K = \frac{r^3}{T^2}$ $\left(K_{sun} = 3.35\times10^{18} \frac{m^3}{s^2} \right)$

Newton's Law of Universal Gravitation

$$F_g = \frac{Gm_1m_2}{r^2}$$

Newton's Hypothesis

$$F_g = F_c$$

Recall: $F_c = ma_c$ ← really just Fnet.

where $a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$

Satellite Motion

geosynchronous/geostationary ⇒ $T = 24h$

To Do:

① PP|591

② Assignment p 597|22-33

Assignment hints:

① compare ⇒ with a ratio

② weight of flea ⇒ planck's mass

③ In #32 .... use your mean K to find mass of Saturn.

④ In #33 .... answers in book are wrong.