Vector Addition

- draw a scale diagram (include the scale)

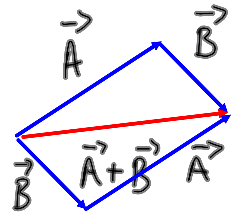
or a reasonable to scale sketch

- use arrows to represent the vectors
- label vectors (and the resultant)
- indicate north or an x-y axis
  ($\overset{N}{\uparrow}$)
- when adding vectors, you join them "head-to-tail"
- the direction of the resultant is determined at the tail of the vector. (measured with respect to a reference direction)

$\vec{A} + \vec{B}$

$\vec{B} + \vec{A}$

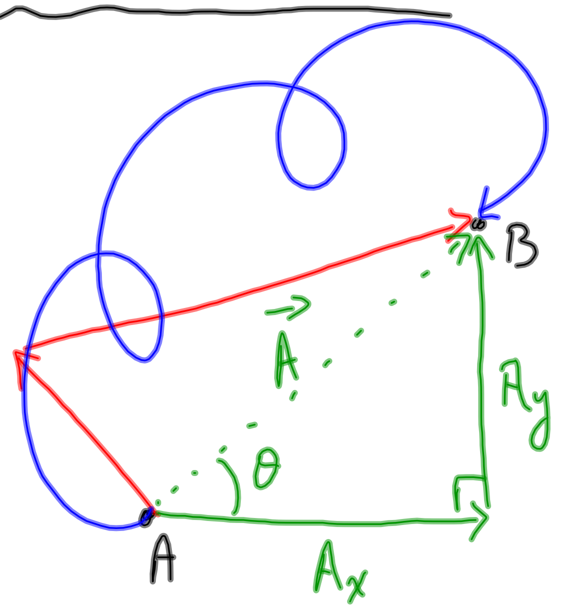
$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

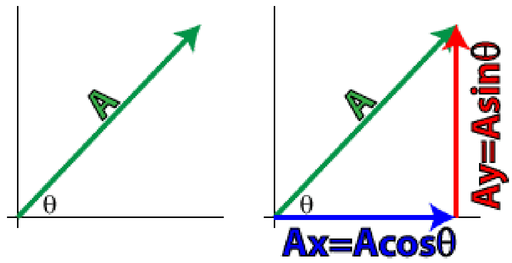


average speed: $v = \frac{\Delta d}{\Delta t}$ ← total distance, ← total time

average velocity: $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ ← overall displacement, ← total time.

Components of Vectors





SOH CAH TOA

$$c^2 = a^2 + b^2$$

Relative Motion Problems

Plane in the Wind

w.r.t ground

plane

wind

$$_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

to an observer on the ground

heading/airspeed

wind

Example

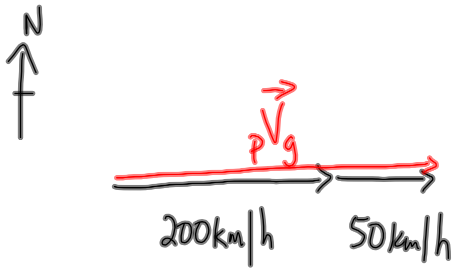
1. $|{}_{p}\vec{V}_{a}| = 200\text{ km/h}$

$${}_{a}\vec{V}_{g} = 50\text{ km/h [E]}$$

$${}_{p}\vec{V}_{g} = ??$$

headings of a) [E] b) [W] c) [N] d) [N40°E

a)

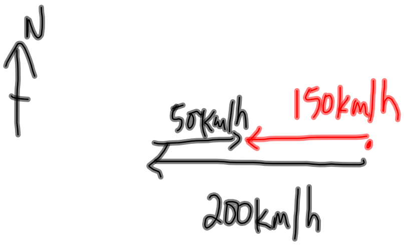


$$_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

$$_{p}\vec{V}_{g} = 200\text{ km/h [E]} + 50\text{ km/h [E]}$$

$$_{p}\vec{V}_{g} = 250\text{ km/h [E]}$$

b)



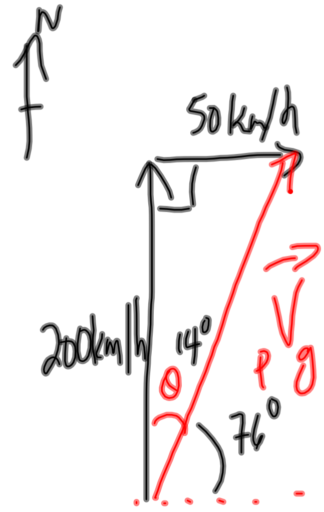
$$_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

$$_{p}\vec{V}_{g} = 200\text{ km/h [W]} + 50\text{ km/h [E]}$$

$$_{p}\vec{V}_{g} = 200\text{ km/h [W]} + -50\text{ km/h [W]}$$

$$_{p}\vec{V}_{g} = 150\text{ km/h [W]}$$

c)



$$_p\vec{V}_g = {_p\vec{V}_a} + {_a\vec{V}_g}$$

$$_p\vec{V}_g = 200\text{ km/h [N]} + 50\text{ km/h [E]}$$

This is a 2-Dimensional Problem..... need a vector addition diagram.

$$c^2 = a^2 + b^2$$

$$c^2 = 200^2 + 50^2$$

$$c = 206\text{ km/h}$$

$$\tan\theta = \frac{50}{200}$$

$$\theta = \tan^{-1}\left(\frac{50}{200}\right)$$

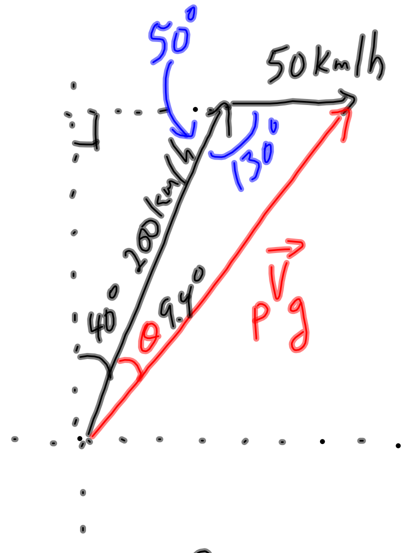
$$\theta = 14^\circ$$

The velocity of the plane wrt the ground is 206 km/h [N 14° E]

[14° E of N]

[E 76° N]

d) N



$$_p\vec{V}_g = {_p\vec{V}_a} + {_a\vec{V}_g}$$

$$_p\vec{V}_g = 200\ \text{km/h}\ [N40°E] + 50\ \text{km/h}\ [E]$$

A 2D problem....
draw a vector addition diagram.

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 200^2 + 50^2 - 2(200)(50)\cos 130°$$

$$c = 235\ \text{km/h}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{235}{\sin 130°} = \frac{50}{\sin\theta}$$

$$235\sin\theta = 50\sin 130°$$

$$\sin\theta = \frac{50\sin 130°}{235}$$

$$\theta = 9.4°$$

The velocity of the plane with respect to the ground is 235 km/h [N49°E]

2. $_s\vec{V}_w = 1.80\text{m/s}\,[N]$

$_w\vec{V}_g = 1.00\text{m/s}\,[E]$

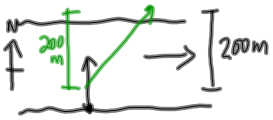
$_s\vec{V}_g = ??$

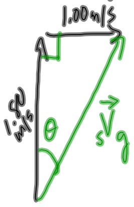
$_s\vec{V}_g = {}_s\vec{V}_w + {}_w\vec{V}_g$

$_s\vec{V}_g = 1.80\text{m/s}[N] + 1.00\text{m/s}[E]$

2D Problem!

a)





$\tan\theta = \frac{1.00}{1.80}$

$\theta = 29.1°$

The velocity of the swimmer wrt the riverbank is $2.06\text{m/s}\,[N\,29.1°\,E]$

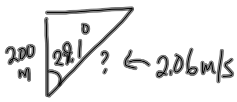
$c^2 = a^2 + b^2$

$c^2 = \left(1.80\frac{m}{s}\right)^2 + \left(1.00\frac{m}{s}\right)^2$

$c = 2.06\text{m/s}$

directions must match.

b) How long to cross:



$\vec{V} = \frac{\Delta\vec{d}}{\Delta t}$

$\Delta t = \frac{\Delta\vec{d}}{\vec{V}}$

$\Delta t = \frac{200\text{ m }[N]}{1.80\frac{m}{s}\,[N]}$ must match

$\Delta t = 111\text{s}$

c) How far downstream? (i.e. East)

$\vec{V} = \frac{\Delta\vec{d}}{\Delta t}$

$\Delta\vec{d} = \vec{V}\,\Delta t$

$\Delta\vec{d} = \left(1.00\frac{m}{s}[E]\right)(111\text{s})$

$\Delta\vec{d} = 111\text{ m }[E]$

Think about: In what direction should the swimmer head in order to finish directly across from her starting point?