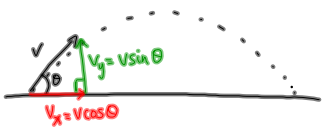
Symmetrical Trajectories (A projectile that returns to the same level)

A special case since the vertical displacement is ZERO!



Find expressions for:

a) Δt

b) Δd_x

c) H (max height)

a) To find the time the projectile is in the air, then we need to look at the vertical motion.

Vertically

$v_i = v\sin\theta$

$a = -g$

$\Delta d = 0$

$\Delta t = ?$

$$\Delta d = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$0 = v\Delta t\sin\theta - \frac{g}{2}(\Delta t)^2$$

$$0 = \Delta t\left(v\sin\theta - \frac{g}{2}\Delta t\right)$$

$\Delta t = 0$ $\quad v\sin\theta - \frac{g}{2}\Delta t = 0$

$$v\sin\theta = \frac{g}{2}\Delta t$$

$$\Delta t = \frac{2v\sin\theta}{g}$$

b) To find the horizontal displacement, we need to use the horizontal velocity and the Δt.

Horizontally $\quad v_x = \frac{\Delta d_x}{\Delta t}$

$$\Delta d_x = v_x\Delta t$$

$$\Delta d_x = (v\cos\theta)\left(\frac{2v\sin\theta}{g}\right)$$

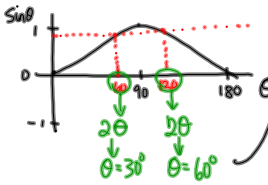
$$\Delta d_x = \frac{v^2(2\cos\theta\sin\theta)}{g}$$

$2\cos\theta\sin\theta = \sin 2\theta$ (trig identity)

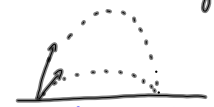
range of projectile →

$$\Delta d_x = \frac{v^2\sin 2\theta}{g}$$

A launch angle of 45° gives you the maximum range.



A launch angle of 30° and 60° will have the same range.



(complementary launch angles give the same range)

c) Maximum height:

$\Delta d_y = ?$ (max height)

$a = -g$

$v_i = v\sin\theta$

$v_2 = 0$ (at max height)

$$v_2^2 = v_i^2 + 2a\Delta d$$

$$0 = v^2\sin^2\theta + 2(-g)(\Delta d_y)$$

$$2g\Delta d_y = v^2\sin^2\theta$$

max height → $\Delta d_y = H = \frac{v^2\sin^2\theta}{2g}$

Maybe useful projectile equations:

$$\Delta t = \frac{2v\sin\theta}{g} \qquad \Delta d_h = R = \frac{v^2\sin 2\theta}{g} \qquad H = \frac{v^2\sin^2\theta}{2g}$$

To DO:

① Look over MP|547

② PP|549

③ Assignment: p570|15-20