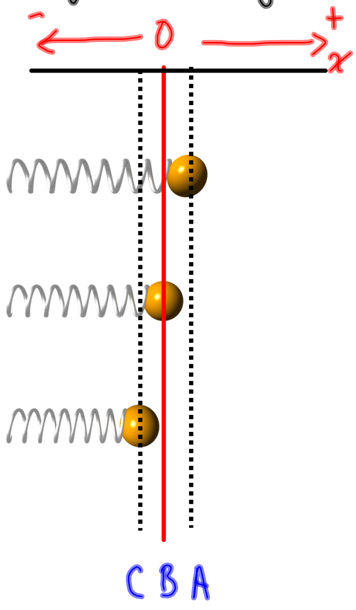
4.2 Energy Changes during SHM

Interchange of energy



at A → max displacement from equilibrium
→ max elastic potential energy
→ velocity is zero
→ Kinetic energy is zero

at B → at the equilibrium position
→ elastic potential energy is zer
→ velocity is a maximum
→ Kinetic energy is a maximum

at C → Same as A, but the displacement is negative.

There is a continuous exchange between elastic potential energy and kinetic energy.

Law of Conservation of Energy applies SHM (neglecting any damping forces).

$$E_{total\ at\ A} = E_{total\ at\ B} = E_{total\ at\ C}$$

or anywhere in between.

The total mechanical energy is constant

$$E_T = E_k + E_p$$

Energy Analysis: Potential Energy

$$E_p = \frac{1}{2}kx^2$$

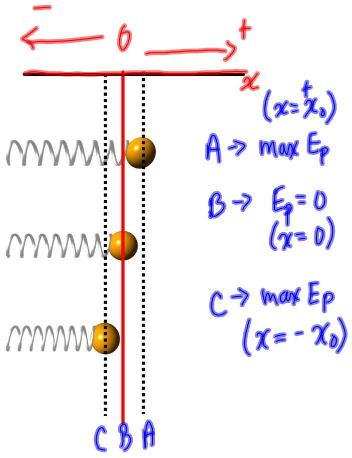
where k is the force constant

$$F_{net} = -kx$$

$$ma = -kx$$

$$a = \frac{-k}{m}x = -\omega^2 x$$

defining equation



$$\therefore \frac{k}{m} = \omega^2$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$k = m\omega^2$$

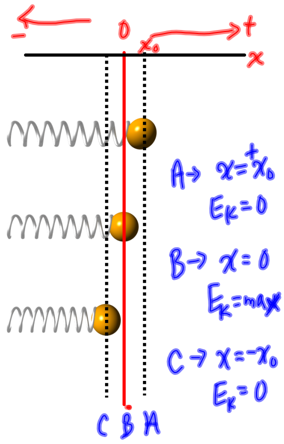
$$\therefore \boxed{E_p = \frac{1}{2}m\omega^2 x^2}$$

Energy Analysis: Kinetic Energy

$$E_k = \frac{1}{2}mv^2$$

$$v^2 = \omega^2(x_0^2 - x^2)$$

$$\therefore \boxed{E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)}$$



Energy Analysis: Total Energy

$$E_p = \frac{1}{2}m\omega^2 x^2$$

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2) \qquad \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x^2$$

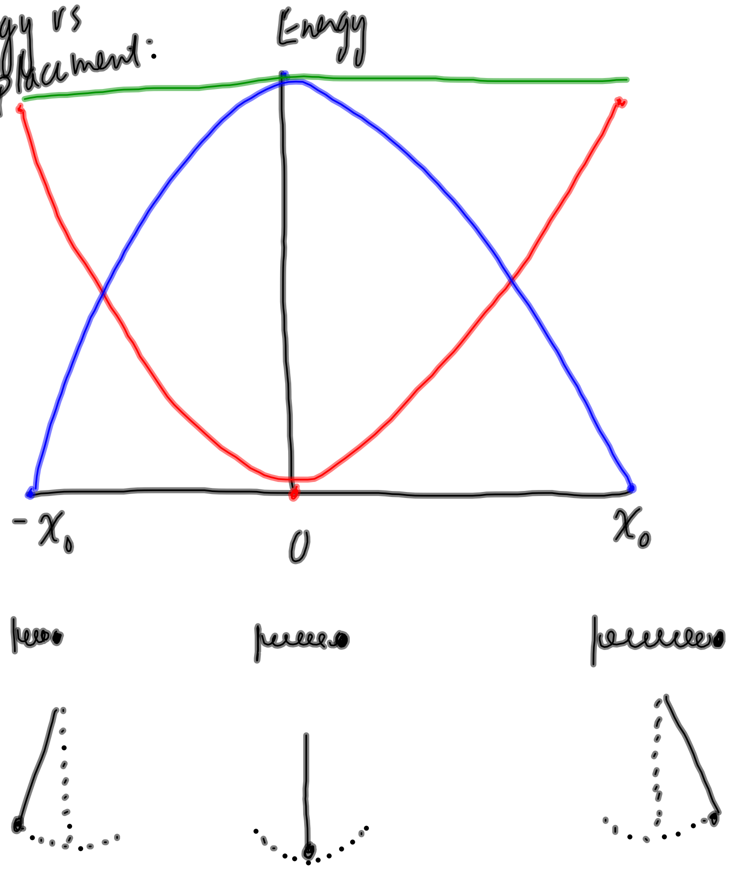
$$E_T = \frac{1}{2}m\omega^2 x_0^2$$

Total mechanical energy remains constant in the absence of damping forces

This total energy is essentially the elastic potential energy given to the mass-spring system by doing work.

Graphical Representation of Energy in SHM:

Energy vs Displacement:



$E_p = \frac{1}{2} m \omega^2 x^2$

$E_k = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$

$E_T = \frac{1}{2} m \omega^2 x_0^2$

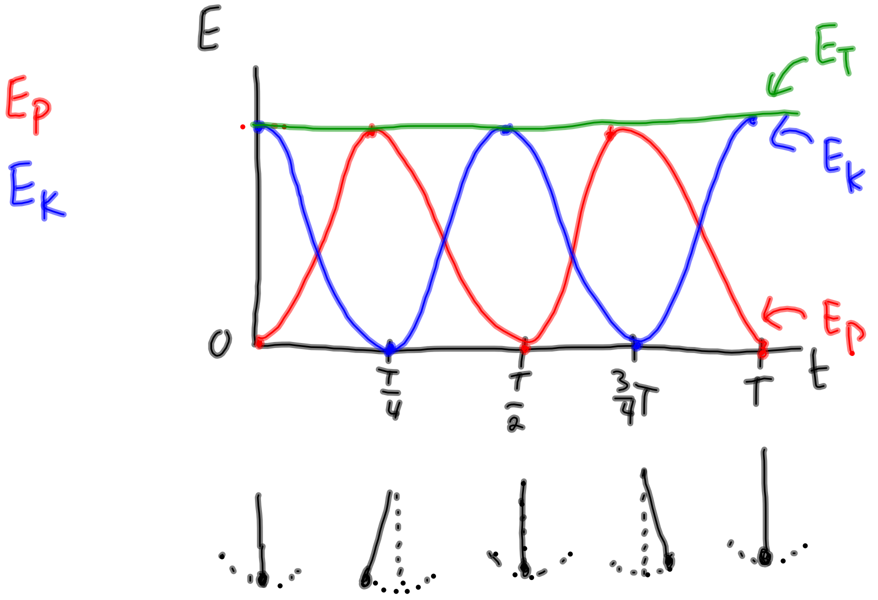
Graphical Representation of energy in SHM:

Energy versus time

Potential Energy: $E_p = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 x_0^2 \sin^2 \omega t$

Kinetic Energy: $E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$

Total Energy: $E_T = \frac{1}{2} m \omega^2 x_0^2$ (constant!)



Data Booklet

$E_k = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

$E_{K(max)} = \frac{1}{2} m\omega^2 x_0^2$ $(E_p = 0)$

$E_T = \frac{1}{2} m\omega^2 x_0^2$

MassSpring[1].galleryitem

D01501[1].galleryitem