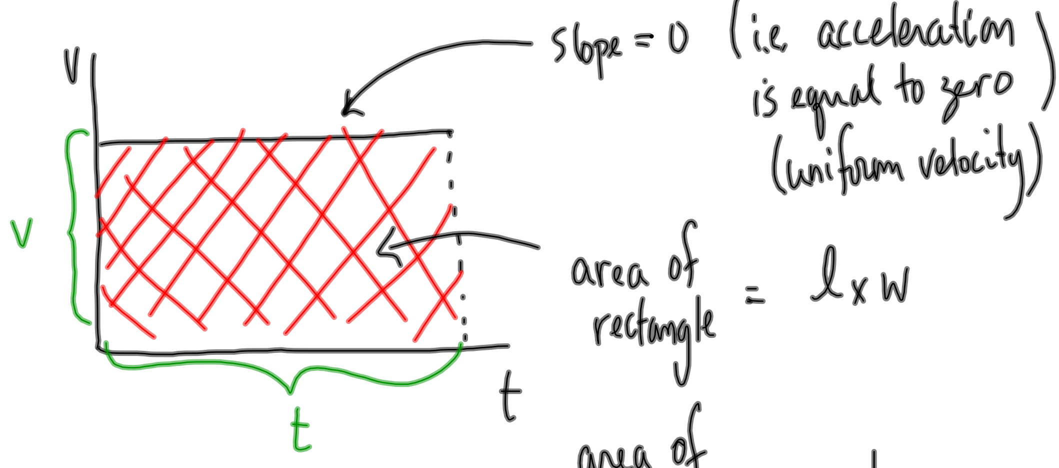
Equations for uniformly accelerated motion

AKA: "The kinematics equations" or the "suvat" equations

These are a set of 5 equations that can be used to solve kinematics problems involving uniformly accelerated motion

You need to be able to derive the equations. Some are not in the data booklet ⇒ must memorize them.

Consider a velocity - time graph for motion with uniform velocity



area of rectangle = l x w

area of rectangle = v t

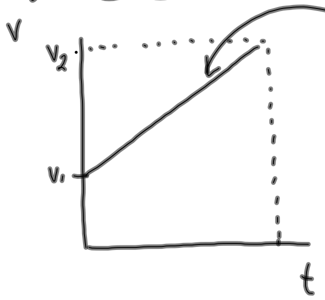
$d = vt$

(area under v-t graph = distance travelled)

① slope (v-t graph) = acceleration

② area under v-t graph = distance

Velocity-Time Graph for Uniformly Accelerated Motion



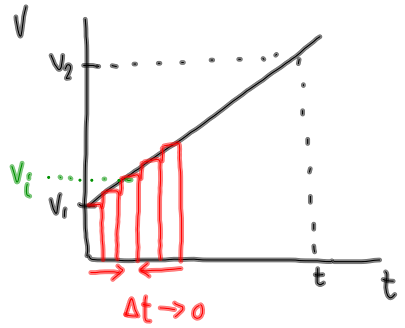
slope = $\frac{rise}{run}$

slope = $\frac{\Delta v}{\Delta t}$

$a = \frac{\Delta v}{\Delta t}$

$\text{slope}_{(v-t)}$ = acceleration

To determine the distance travelled when the velocity is changing, imagine that the slope of the line is increasing in a large number of small steps.



$$d \approx \sum_{i=1}^{i=N} v_i \Delta t$$

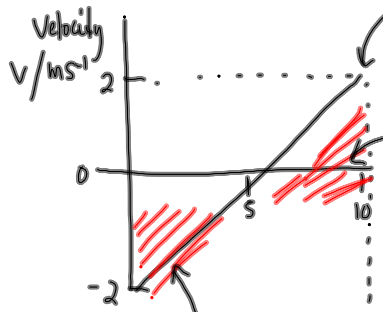
$$d = v_1\Delta t + v_2\Delta t + v_3\Delta t \ldots + v_N\Delta t$$

We could think about this shape having an area that is equal to the area of an infinite number of rectangles with a width of almost zero.

① $\text{slope}_{(v-t)}$ = acceleration

② $\text{area}_{(v-t)}$ = distance (or displacement)

Example:



slope = $\frac{2\,ms^{-1}}{5\,s}$ = 0.4 m… ↑ acc…

$\Delta x\,(\text{area}) = \frac{1}{2}(5s)(2ms^{-1})$

$\Delta x = 5\,m$

$\Delta x\,(\text{area}) = \frac{1}{2}(5s)(-2ms^{-1})$

Equation 1

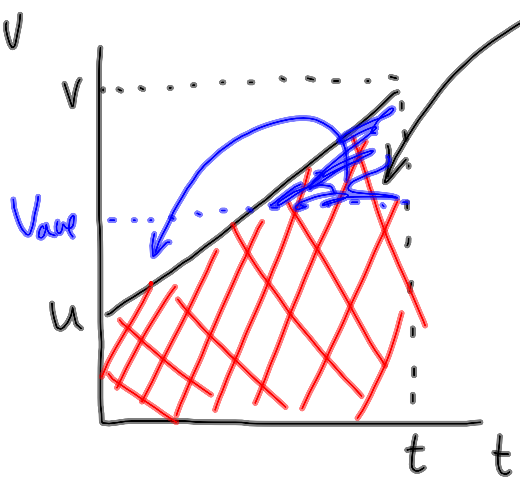
Acceleration: $a = \frac{\Delta V}{\Delta t}$

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$\boxed{V = u + at} \quad (1)$$

Equation 2



area (v-t) = area of a trapezoid

displacement = $\frac{1}{2}(h_1 + h_2)\, b$

$$s = \frac{1}{2}(u + v)\, t$$

$$s = \left(\frac{u + v}{2}\right) t \quad (2)$$

assumed that there is constant acceleration

$$s = V_{ave}\, t$$

Equation 3

Recall Equation 1: $v = u + at$

$v - u = at$ (rearrange for t)

$t = \frac{v-u}{a}$

Recall Equation 2: $s = \left(\frac{u+v}{2}\right)t$

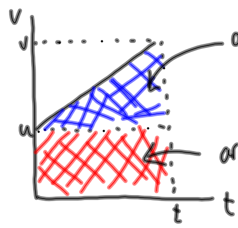
$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$

$s = \frac{v^2 - u^2}{2a}$

$2as = v^2 - u^2$

$v^2 = u^2 + 2as$ (3)

Equation 4



area of triangle $= \frac{1}{2}bh$

$= \frac{1}{2}(v-u)t$

area of rectangle $= bh$

$= ut$

$s = ut + \frac{1}{2}(v-u)t$

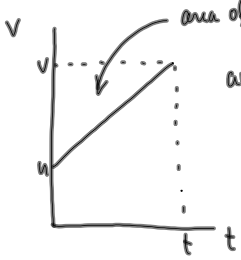
Recall equation 1: $v = u + at$

$v - u = at$

$s = ut + \frac{1}{2}(at)t$

$s = ut + \frac{1}{2}at^2$ (4)

Equation 5



area of triangle $= \frac{1}{2}(v-u)t$

area of big rectangle $= vt$

$s = vt - \frac{1}{2}(v-u)t$ (at)

$s = vt - \frac{1}{2}at^2$ (5)