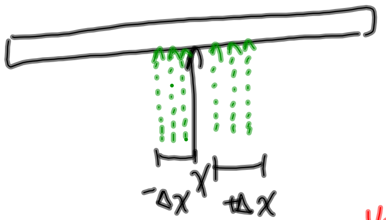
Absolute Uncertainty

The absolute uncertainty of a value is an interval above or below that value such that measurements of the value all lie within this interval.

It is the magnitude of the maximum difference between the value (or an estimate of the value) and a measurement of its value.



$x \pm \Delta x$

↑ value ↑ uncertainty

Relative (or Fractional) Uncertainty

Consider an absolute uncertainty of 1m (Δx)

If $x = 1008$m and $\Delta x = 1$m (precise)

If $x = 5$m and $\Delta x = 1$m (no very precise)

Note: The absolute uncertainty (Δx) expressed with the measurement indicates the precision but NOT when expressed by itself.

The quality of the measurement is better indicated by relative (or fractional) uncertainty by using the ratio: $\dfrac{\Delta x}{x}$ ← The relative (or fractional) uncertainty is the ratio of the absolute uncertainty to the measurement itself

Percentage Uncertainty

The relative uncertainty is expressed as a percentage

Summary:

measurement => x
absolute uncertainty => Δx

$$\left.\begin{array}{l}\text{measurement} \Rightarrow x\\ \text{absolute uncertainty} \Rightarrow \Delta x\end{array}\right\} x \pm \Delta x$$

relative uncertainty => $\frac{\Delta x}{x}$

percent uncertainty => $\frac{\Delta x}{x} \cdot 100\%$

Example The weight of an object is measured to be 2.7N with an absolute uncertainty of 0.1N.

$$\left.\begin{array}{l}\text{measurement} \Rightarrow 2.7\text{N} \quad x\\ \text{absolute uncertainty} \Rightarrow 0.1\text{N} \quad \Delta x\end{array}\right\} (2.7 \pm 0.1)\text{N}$$

relative uncertainty => $\frac{0.1\text{N}}{2.7\text{N}} \doteq 0.04$

percent uncertainty => $0.04 \times 100\% = 4\%$ 1sf

Example A length of 10m and a length of 10mm are each measured with an absolute uncertainty of 2mm. What is the relative uncertainty and percent uncertainty? Which measurement is more precise?

write to same dec. place

$10\text{m} \pm 0.002\text{m}$ → $(10.000 \pm 0.002)\text{m}$

$10\text{mm} \pm 2\text{mm}$ → $(10 \pm 2)\text{mm}$

relative uncertainty: $\frac{0.002}{10.000} = 2 \times 10^{-4}$ | $\frac{2}{10} = 0.2$

percent uncertainty: $(2 \times 10^{-4})(100\%) = 2 \times 10^{-2}\%$ | $(0.2)(100\%) = 20\%$

10m is the more precise measurement.

Adding + Subtracting

When adding or subtracting values, add the absolute uncertainties to give the absolute uncertainty in the result.

If $y = a \pm b$ $\quad \Delta y = \Delta a + \Delta b$

For adding or subtracting

you always add the absolute uncertainties

Example

$(9.7 \pm 0.5)\,m + (4.3 \pm 0.2)\,m = (14.0 \pm 0.7)\,m$

$(9.7 \pm 0.5)\,m - (4.3 \pm 0.2)\,m = (5.4 \pm 0.7)\,m$

Example Determine the perimeter of a square of side (2.4 ± 0.5) cm.

$$
\begin{array}{r}
(2.4 \pm 0.5)\,cm \\
(2.4 \pm 0.5)\,cm \\
(2.4 \pm 0.5)\,cm \\
+\ (2.4 \pm 0.5)\,cm \\
\hline
(9.6 \pm 2.0)\,cm
\end{array}
$$

$(10 \pm 2)\,cm$

OR

$4(2.4 \pm 0.5)\,cm$

$(9.6 \pm 2.0)\,cm$

$(10 \pm 2)\,cm$

Multiplying + Dividing

When multiplying or dividing, add the relative (or fractional) uncertainties to get the relative uncertainty in the result.

If $y = \frac{ab}{c}$ then $\frac{\Delta y}{y} = \underbrace{\frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}}_{\text{relative uncertainties in the values used}}$

relative uncertainty in the result

Example

$(9.7 \pm 0.5)\text{m} \times (4.3 \pm 0.2)\text{m} = 41.7\,\text{m}^2 \pm ???$

Add the relative uncertainties

$\frac{0.5}{9.7} + \frac{0.2}{4.3} = 0.052 + 0.047 = 0.099$

relative uncertainty for the result ($41.7\,\text{m}^2$)

absolute uncertainty for $41.7\,\text{m}^2$

$0.099 \times 41.7\,\text{m}^2 = 4.1283\,\text{m}^2$

(relative unc)

final answer: $(41.7 \pm 4.1283)\,\text{m}^2$

$(42 \pm 4)\,\text{m}^2$

Example Determine the answer with its absolute uncertainty:

$\frac{(9.7 \pm 0.5)\text{m}}{(4.3 \pm 0.2)\text{m}} = 2.2558 \pm ???$

Add the relative uncertainties.

$0.052 + 0.047 = 0.099$ ← relative uncertainty for 2.2558

absolute uncertainty in final answer: $0.099 \times 2.2558 = 0.2233242$

Final answer: 2.2558 ± 0.2233242

(2.3 ± 0.2)

Powers (and Roots)

When raising a value to the power of n, multiply the relative uncertainty value by n to give the relative uncertainty of the result.

Example: $\left[(9.7 \pm 0.5)\,m\right]^3 = 912.673 \pm ???$

relative uncertainty in 9.7 : $\frac{0.5}{9.7} = 0.052$

relative uncertainty in $(9.7)^3$: $3(0.052) = 0.156$ ← relative uncertainty in 912.673

absolute uncertainty in $(9.7)^3$: $0.156(912.673) = 141.135$

$9 \times 10^2 \pm 1 \times 10^2$

$(912.673 \pm 141.135)\,m^3$

$(9 \pm 1) \times 10^2\ m^3$

Example: The radius of a sphere is measured to be (8.5 ± 0.2)cm. Determine its volume with its absolute uncertainty.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8.5)^3 = 2572.4\ cm^3$

relative uncertainty (for 8.5cm): $\frac{0.2}{8.5} = 0.0235$

relative uncertainty (for $(8.5cm)^3$): $3(0.0235) = 0.07059$ Answer here → $(2.6 \pm 0.2) \times 10^3\ cm^3$

absolute uncertainty (for $(8.5cm)^3$): $(0.07059)(2572.4cm^3) = 181.6\ cm^3$ $2 \times 10^2\ cm^3$

Example: The surface area of a square swimming pool is measured and found to be $12m^2$ with an absolute uncertainty of $2m^2$. Determine the length of each side of the pool with its absolute error.

Area $= (\text{side})^2$

$\therefore$ side $= \text{Area}^{1/2}$

relative uncertainty (for area): $\frac{2}{12} = 0.17$

Area $= \text{side}^2$

side $= \sqrt{12m^2}$

side $= 3.464\,m$

relative uncertainty (for side) $\frac{1}{2}(0.17) = 0.0833$

absolute uncertainty (for side) $0.08(3.464m) = 0.28867$

$\therefore$ Side $= (3.5 \pm 0.3)\,m$

Trig Functions

No simple rule.... the uncertainty is half the difference between the highest and lowest values

$\cos\theta = ?$ if $\theta = (60 \pm 5)^\circ$

$\cos 60^\circ = 0.500$

$\cos 55^\circ = 0.423$

$\cos 65^\circ = 0.574$

$\rightarrow \frac{\text{difference}}{2} = 0.076$

0.08

so $\cos(60 \pm 5)^\circ = 0.50 \pm 0.08$

NOTE

If one uncertainty is very large compared to another, then you may use only that uncertainty