Order of Magnitude

The order of magnitude of a number is the number expressed to the nearest whole number power of 10.

6.9×10^9 people => order of magnitude is 10 (10^{10})

$10^9 \rightarrow 10^{10}$

acceleration of gravity $9.81 ms^{-2}$ order of magnitude is 1 $(10^1)\ ms^{-2}$

$10^0 \rightarrow 10^1$

mass of an electron 9.1×10^{-31} kg 10^{-30} kg

10^{-31} and 10^{-30}

Examples

pressure	1.01×10^5 Pa	10^5 Pa
height of person	1.784 m	10^0 m
box of apples	16 kg	10^1 kg
cheese	0.075 kg 7.5×10^{-2}	10^{-1} kg

When dealing with order magnitude, the cut-off is 3.16

3.0×10^2 => 10^2

4.0×10^2 => 10^3

3.16×10^2 => 10^2 or 10^3

Example

Which of the following is the same as 3.8×10^{-3} to the nearest order of magnitude? $10^{-3} \rightarrow 1$

a) 3.0×10^{-3}

b) 4.0×10^{-2}

Very large + Very small numbers.

lab table (~ 1m) or 10^0 m (order of magnitude)

width of a strand of hair (0.1mm) 10^{-4} m

1×10^{-4} m

distance to the moon 10^9 m

Can you visualize the speed of light?

$c = 3.00 \times 10^8 \, ms^{-1}$ $(10^8 \, ms^{-1})$

- light would travel back + forth across this room a million times in 1 second.
- light takes 8 min to reach the Earth from the sun.
- LHC => 27 km long -> protons can travel around the collider 10,000 times per second
- it takes light 4 years to get to the Earth from our nearest star and 100,000 years to cross the Milky Way.

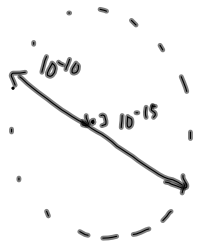
Here are some smallest and largest distances, masses & times that you need to know:

Distance:	diameter of a proton	10^{-15} m
	extent of the visible universe	10^{25} m
Masses:	mass of an electron	10^{-30} kg
	mass of the universe	10^{50} kg
Times:	time for light to cross a nucleus	10^{-23} s
	age of the universe	10^{18} s

Ratios are often used to compare orders of magnitude:

diameter of a hydrogen atom is about 10^{-10} m
diameter of a hydrogen nucleus is about 10^{-15} m

the ratio: $\frac{\text{diameter of H atom}}{\text{diameter of H nuc}} = \frac{10^{-10}}{10^{-15}} = 10^5$



The diameter of the hydrogen atom is 5 orders of magnitude larger than its nucleus.

Example
The diameter of a proton is about 10^{-15} m and the diameter of a hydrogen atom is about 10^{-10} m.

How many orders of magnitude is the volume of a hydrogen atom greater than the volume of its nucleus?

$V = \frac{4}{3}\pi r^3$

$$\frac{\text{volume of H atom}}{\text{volume of H nucleus}} = \frac{\frac{4}{3}\pi\left(\frac{d_{atom}}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{d_{nuc}}{2}\right)^3} \qquad \frac{d_{atom}^3}{2^3} \cdot \frac{2^3}{d_{nuc}^3}$$

$$= \left(\frac{d_{atom}}{d_{nuc}}\right)^3$$

$$= \frac{d_{atom}^3}{d_{nuc}^3}$$

$$= \left(\frac{10^{-10}}{10^{-15}}\right)^3$$

$$= (10^5)^3$$

$$= 10^{15}$$

The volume of the atom would be 15 orders of magnitude bigger than nuc.

Estimate to 1 or 2 significant digits the size of everyday objects.

- nearest order of magnitude.
- estimate familiar lengths, masses, weights + times.
- estimate based on a scale diagram
- rough estimates for calculations.
- trace any error between the estimated and calculate.

Examples

- dimensions of physics book in cm (to 1 sf)
- mass of an apple in kg (to 1 sf) 10^{-1} 10^{0}
- period of a heart beat in s (to 1 sf)
- the quantity of milk you drink in a year in cm^3 (to 1 sf)