

Chapter 12

Kepler's Laws

1. elliptical orbits
2. equal areas in equal times \Rightarrow faster when closer to sun
3. $K = \frac{r^3}{T^2}$ \leftarrow unique for each central body

Newton's Law of Universal Gravitation:

$$F_g = \frac{Gm_1m_2}{r^2} \rightarrow g \text{ where } m_1 \text{ is the central mass}$$

Newton's Hypothesis: $F_g = F_c$ \rightarrow F_c is really F_{net} \rightarrow $F_{net} = ma$ \leftarrow orbiting mass

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

99/586

14.

$$M_{\text{galaxy}} = ?$$

$$v_{\text{star}} = 2.0 \times 10^5 \text{ m/s}$$

$$r_{\text{star orbit}} = 5 \times 10^9 \text{ AU}$$

$$1 \text{ AU} = 1.49 \times 10^8 \text{ km}$$

$$1.49 \times 10^{11} \text{ m}$$

$$F_g = F_c$$

$$\frac{Gm_{\text{galaxy}}m_{\text{star}}}{(r_{\text{star orbit}})^2} = \frac{m_{\text{star}}v_{\text{star}}^2}{r_{\text{star orbit}}}$$

$$\frac{Gm_{\text{galaxy}}}{r_{\text{star orbit}}} = v_{\text{star}}^2$$

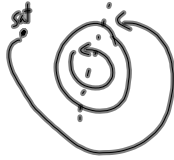
$$M_{\text{galaxy}} = \frac{v_{\text{star}}^2 r_{\text{star orbit}}}{G}$$

$$M_{\text{galaxy}} = \frac{(2.0 \times 10^5 \text{ m/s})^2 (5 \times 10^9 \text{ AU}) \left(\frac{1.49 \times 10^{11} \text{ m}}{1 \text{ AU}} \right)}{6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}}$$

Geosynchronous or Geostationary Satellite

The satellite will always be directly above a specified location on the Earth.

$T_{\text{earth rotation (in its axis)}} = T_{\text{sat}} = 24\text{h} = 86400\text{s}$



MP/589

$T = 86400\text{s}$

$r_{\text{earth}} = 6.38 \times 10^6\text{m}$

$r_{\text{sat orbit}} = ? \quad (r_{\text{earth}} + h)$

$h = ?$

$v = ?$

$m_{\text{earth}} = 5.98 \times 10^{24}\text{kg}$

$F_g = F_c$
 $\frac{G m_{\text{earth}} m_{\text{sat}}}{r_{\text{sat orbit}}^2} = \frac{m_{\text{sat}} 4\pi^2 r_{\text{sat orbit}}}{T^2}$

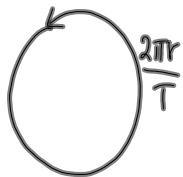
$G m_{\text{earth}} T^2 = 4\pi^2 r^3$ K_{earth}
 $r^3 = \frac{G m_{\text{earth}} T^2}{4\pi^2}$

$r^3 = \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(5.98 \times 10^{24}\text{kg})(86400\text{s})^2}{(4\pi^2)}$

radius of the satellites orbit $\rightarrow r = 4.23 \times 10^7\text{m}$

altitude: $h = 4.23 \times 10^7\text{m} - 6.38 \times 10^6\text{m}$
 $0.638 \times 10^7\text{m}$

$h = 3.59 \times 10^7\text{m}$



$v = \frac{2\pi r_{\text{sat orbit}}}{T}$

$v = \frac{2\pi(4.23 \times 10^7\text{m})}{86400\text{s}}$

To Do

- Newton's Cannon animation
- PP/591

$v = 3.07 \times 10^3 \frac{\text{m}}{\text{s}}$