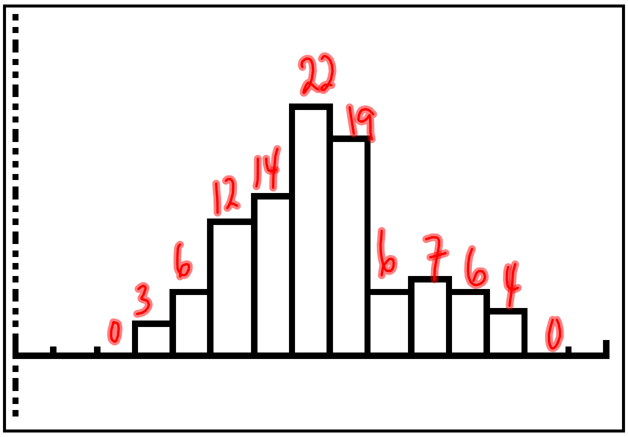
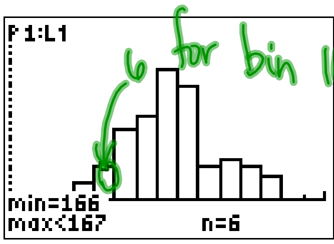
Sampling Distribution of the Sample Means





6 for bin 166-167

Use trace to find frequency.

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1-Var Stats L1
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← use 1-Var Stats to find the mean of the sample means & the st. dev of the sample means

```
1-Var Stats
 x̄=169.851
 Σx=16985.1
 Σx²=2885435.67
 Sx=2.246096277
 σx=2.234837578
↓n=100
```

x̄=169.851 ← the mean of the sample means

Sx=2.246096277 ← the st. dev of the sample means

```
12.3/√(30
         2.245662486
```

← $\frac{\sigma}{\sqrt{n}}$

σ ← the population st. dev

n ← n is the sample size

2.245662486 the same as the st. dev. of the sample means.

CENTRAL LIMIT THEOREM

A lot of samples of the same size (n) are taken from a population and the means of each sample are found. A histogram of these sample means is made and a frequency polygon is made so we can see the shape of the histogram. As well, the average of the sample means (called the mean of the sample means ($\bar{x}_{\bar{x}}$ or $\mu_{\bar{x}}$) and the standard deviation of the sample means ($\sigma_{\bar{x}}$) are calculated.

The resulting information - shape, centre, & spread - of all possible sample means from a sample of size **n** from the sampling population is called the **sampling distribution**. We now know 3 things about these sampling distributions: (The CLT only applies if the sample used is a random non-biased sample)

1. If the original population was normal, the sampling distribution will also be normal. If the original population was not normal, the sampling distibution will only be normal if n≥30.

2. The centre of the sampling distribution (the mean of the sample means) is equal to the mean of the original population.

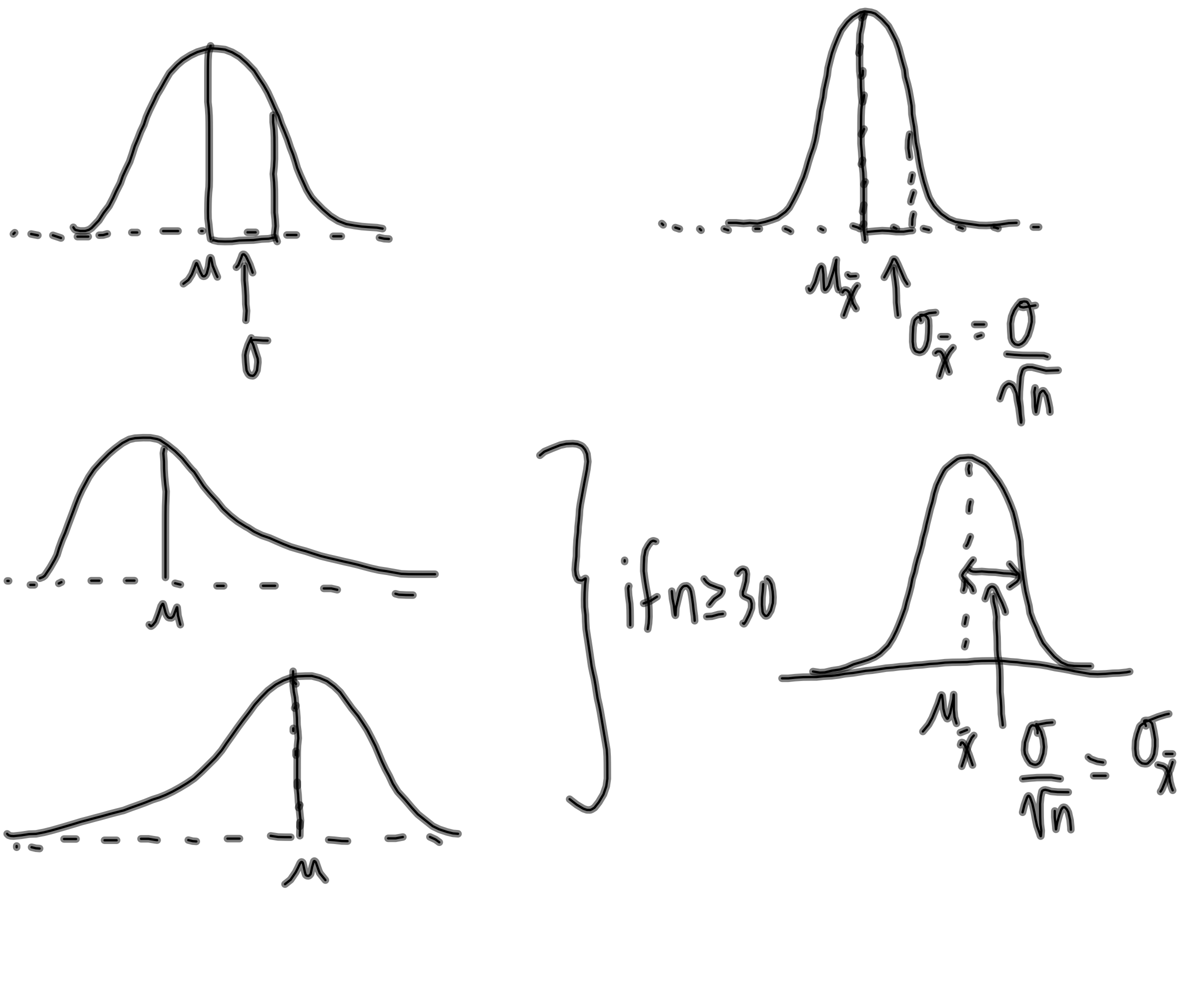
$$\bar{x}_{\bar{x}} = \mu \quad \text{or} \quad \mu_{\bar{x}} = \mu$$

3. The standard deviation of the sampling distribution ($\sigma_{\bar{x}}$) is equal to the standard deviation of the population (σ) divided by the square root of the sample size (n).

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

We can compare the distribution of the population
to the sampling distribution using the following diagrams:

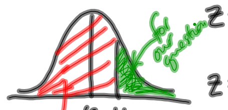
Population Sampling Distribution



Example 1

The average height of NS Gr11 students is 63" with a standard deviation of 4". The heights are normally distributed.

a) If a single grade 11 student is chosen at random, what is the probability that they will be taller than 66"?



$z = \frac{x - \mu}{\sigma}$

$z = \frac{66 - 63}{4}$

$z = 0.75$

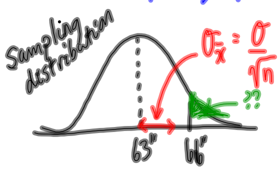
z-chart:
0.7734

or 77.34 of someone being shorter than 66"

$100 - 77.34\% = 22.66\%$

b) If 16 grade 11 students are chosen at random, what is the probability that their average ($\bar{x}$) is greater than 66"?

What does the distribution of sample means look like for a sample size of 16?



$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{16}} = \frac{4}{4} = 1$

z-score for the sample mean:

$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$ recall: $\mu = \mu_{\bar{x}}$

$z = \frac{66 - 63}{1} = 3$

Look up a z-score of 3: 0.9987

$100 - 99.87\% = 0.13\%$

The probability of the sample having a mean of greater than 66" is 0.13%