Solving Sinusoidal Equations Algebraically

Example: $\underbrace{3\cos\big(\underbrace{2(x-1)}_{\theta}\big)^{\circ} + 5}_{y_1} = \underbrace{7}_{y_2}$

let $\theta = 2(x-1)$

$$3\cos\theta + 5 - 5 = 7 - 5$$

$$\frac{3\cos\theta}{3} = \frac{2}{3}$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\boxed{\theta_1 = 48.2^{\circ}} \leftarrow \text{The primary angle}$$

for cos: $\theta_1 + \theta_2 = 360^{\circ}$

$$\theta_2 = 360^{\circ} - 48.2^{\circ}$$

$$\boxed{\theta_2 = 311.8^{\circ}}$$

Recall: $\theta = 2(x-1)$ — $\frac{360}{per}$

$$\frac{48.2^{\circ}}{2} = \frac{2(x-1)}{2} \quad \text{and} \quad \frac{311.8^{\circ}}{2} = \frac{2(x-1)}{2}$$

$$24.1 = x-1 \qquad\qquad 155.9 = x-1$$

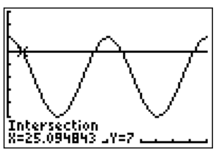
$$\underbrace{\boxed{x = 25.1} \qquad\qquad \boxed{156.9 = x}}_{\text{Starting points for solution}}$$

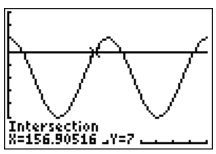
The period: $\frac{360}{per} = 2$

$$per = \frac{360}{2} = \boxed{180^{\circ}} \leftarrow \text{increment}$$

$$x = \begin{Bmatrix} 25.1 + 180k \\ 156.9 + 180k \end{Bmatrix} k \in \mathbb{Z}$$

Use your graphing calculator to check:





$y_1 = 3\cos(2(x-1)) + 5$

Example: Solve $15-10\sin(9(x-5))=20$

reflection of sine θ

$$-10\sin(9(x-5))+15=20$$

let $\theta = 9(x-5)$

$$-10\sin\theta + 15 = 20 \quad (-15 \quad -15)$$

$$\frac{-10\sin\theta}{-10} = \frac{5}{-10}$$

$$\sin\theta = -\frac{1}{2}$$

for sine: $\theta_1+\theta_2=180°$ $\qquad \theta = \sin^{-1}\left(-\frac{1}{2}\right)$

$$\theta_2 = 180° - (-30°) \qquad \boxed{\theta_1 = -30°} \leftarrow \text{primary angle}$$

$$\boxed{\theta_2 = 210°}$$

Recall: $\theta = 9(x-5)$

$$\frac{-30}{9} = \frac{9}{9}(x-5) \quad \text{and} \quad \frac{210}{9} = \frac{9}{9}(x-5)$$

$$-\frac{30}{9} = x-5 \qquad \frac{210}{9} = x-5$$

$$x = -\frac{30}{9}+5 \qquad x = \frac{210}{9}+5$$

$$\boxed{x = 1.\bar{6}} \qquad \boxed{x = 28.\bar{3}}$$

Period: $9 = \frac{360}{per}$

$$per = \frac{360}{9}$$

$$per = 40$$

$$x = \begin{Bmatrix} 1.\bar{6} + 40k \\ 28.\bar{3} + 40k \end{Bmatrix} k \in \mathbb{Z}$$

Check answer by graphing:

$$\underbrace{15-10\sin(9(x-5))}_{y_1} = \underbrace{20}_{y_2}$$