Solving Systems of Equations with Matrices (Graphing Calc)

$AX = B$ ← matrix equation

work:

$A^{-1}AX = A^{-1}B$

($A^{-1}A = I$)

$IX = A^{-1}B$

$$\boxed{X = A^{-1}B}$$

1a)

$x - y - z = 2$

$x + y + 3z = 7$

$9x - y - 3z = -1$

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 3 \\ 9 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}$$

A: coefficient matrix; X: variable matrix; B: constant matrix

$AX = B$

$X = A^{-1}B$

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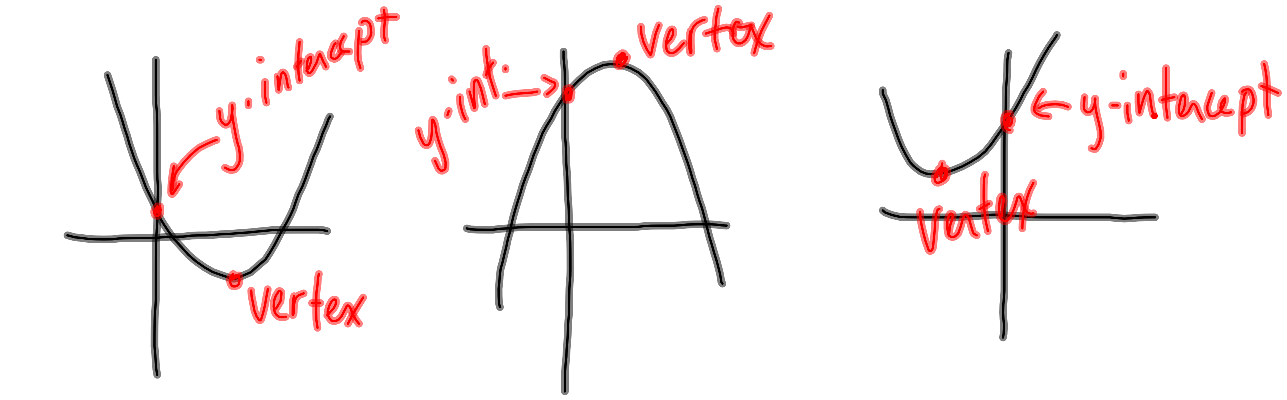
[A]-1[B]
        [[.6  ]
         [-5.3]
         [3.9 ]]
```

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.6 \\ -5.3 \\ 3.9 \end{bmatrix}$$

or $x = 0.6$, $y = -5.3$ and $z = 3.9$

Quadratic Applications of 3x3 Systems.



Quadratic:

$$y = ax^2 + bx + c$$

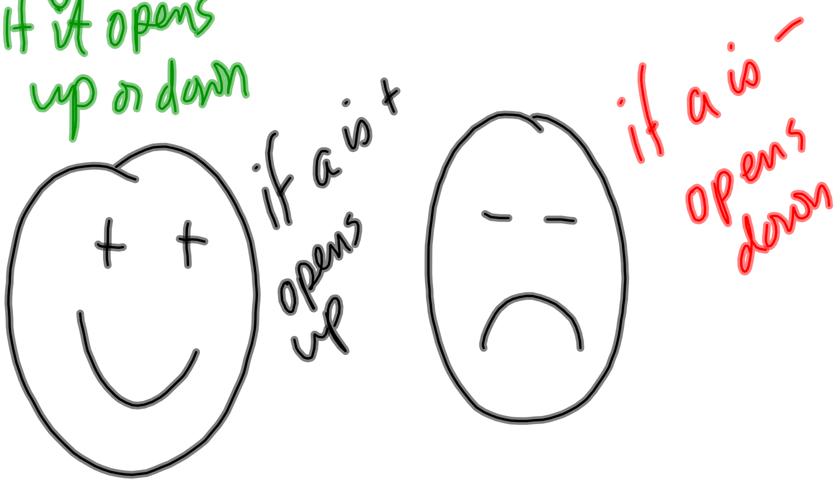
(need three points)

- a: Sign tells if it opens up or down
- b: controls vertex
- c: y-intercept

Linear:

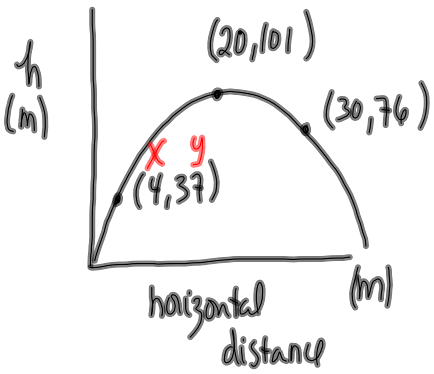
$$y = mx + b$$

(need two points)



Example

Suppose the path of a model airplane can be described by the graph below:



$ax^2 + bx + c = y$

↑ ↑ ↑

we need to find the constants ....

you know x and y for 3 points ⇒ 3 equat 3 unknown

$$ax^2 + bx + c = y$$

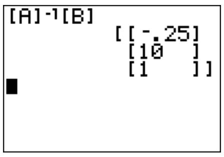
$(4,37)$ $a(4)^2 + b(4) + c = 37 \Rightarrow 16a + 4b + c = 37$

$(20,101)$ $a(20)^2 + b(20) + c = 101 \Rightarrow 400a + 20b + c = 101$

$(30,76)$ $a(30)^2 + b(30) + c = 76 \Rightarrow 900a + 30b + c = 76$

Matrix Equation

$$\underbrace{\begin{bmatrix} 16 & 4 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 37 \\ 101 \\ 76 \end{bmatrix}}_{B}$$



$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -0.25 \\ 10 \\ 1 \end{bmatrix}$$

So the equation is:

$y = ax^2 + bx + c$

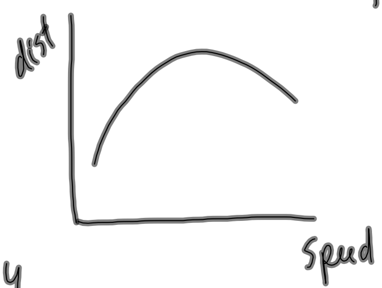
$$y = -0.25x^2 + 10x + 1$$

Example

The following table gives the distance driven per litre of gas at different speeds by a certain car:

x Speed	y distance
40 km/h	7.2 km/L
50	10.2
60	11.4
70	10.8
80	8.4

The relationship between distance and speed can be modelled by a quadratic function.



$$ax^2 + bx + c = y$$

$(40, 7.2)$ $a(40)^2 + b(40) + c = 7.2 \Rightarrow 1600a + 40b + c = 7.2$

$(60, 11.4)$ $a(60)^2 + b(60) + c = 11.4 \Rightarrow 3600a + 60b + c = 11.4$

$(80, 8.4)$ $a(80)^2 + b(80) + c = 8.4 \Rightarrow 6400a + 80b + c = 8.4$

write a matrix equation

$$\begin{bmatrix} 1600 & 40 & 1 \\ 3600 & 60 & 1 \\ 6400 & 80 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7.2 \\ 11.4 \\ 8.4 \end{bmatrix}$$

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[A]-1[B]
        [[-.009]
         [1.11  ]
         [-22.8]]
```

$$AX = B$$

$$X = A^{-1}B$$

Equation:

$$y = -0.009x^2 + 1.11x - 22.8 \qquad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -0.009 \\ 1.11 \\ -22.8 \end{bmatrix}$$

What distance ($y = ?$) (per L) could this car travel at 75 km/h (x)?

$$y = -0.009(75)^2 + 1.11(75) - 22.8$$

$$y = 9.825 \text{ km/L}$$