

# Solving Matrix Equations

Recall:  $AX = B$   
*needs to be square*

$AA^{-1}X = A^{-1}B$  (multiply both sides by  $A^{-1}$ )

$I X = A^{-1}B$  ( $I$  is the identity matrix - just like 1)

$X = A^{-1}B$

# Solving Systems of Equations using Matrices

Example:  $z - x + y = 9 \rightarrow -x + y + z = 9$   
 $2z + x = 12 + y \rightarrow x - y + 2z = 12$   
 $-2z - 3y = x - 8 \rightarrow -x - 3y - 2z = -8$   
*↑ variables constant*

- Make sure the equations line up
- Write the system as a matrix equation:

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ -8 \end{bmatrix}$$

*3x3* coefficient matrix (A)     *3x1* variable matrix (X)     *3x1* constant matrix (B)

*match*     *answer*

$AX = B$

$X = A^{-1}B$

$[A]^{-1}[B]$

$$\begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix}$$

$x = -3$   
 $y = -1$   
 $z = 7$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix}$

Example:

$$50c + 60I + 40S = 2540$$

$$75c + 50S = 1850$$

$$65I + 35S = 1860$$

$$\begin{matrix} & \text{C} & \text{I} & \text{S} \\ \begin{bmatrix} 50 & 60 & 40 \\ 75 & 0 & 50 \\ 0 & 65 & 35 \end{bmatrix} & \begin{bmatrix} c \\ I \\ S \end{bmatrix} & = & \begin{bmatrix} 2540 \\ 1850 \\ 1860 \end{bmatrix} \end{matrix}$$

$$A X = B$$

$$X = A^{-1} B$$

$$\begin{bmatrix} c \\ I \\ S \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 16 \end{bmatrix}$$

$[A]^{-1}[B]$	$\begin{bmatrix} 14 \\ 20 \\ 16 \end{bmatrix}$
■	