Law of Sines
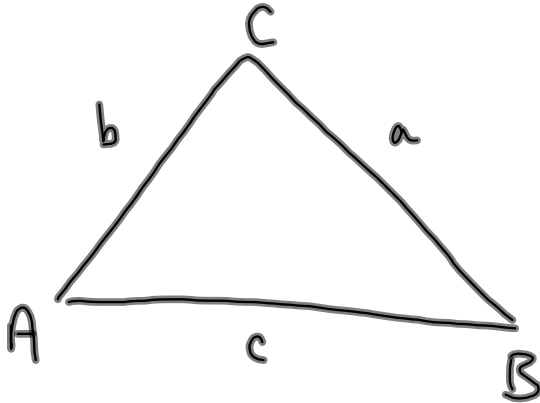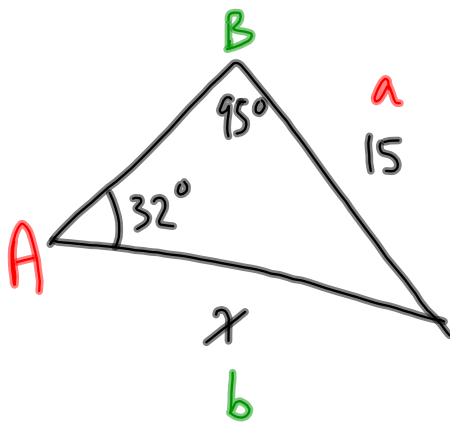

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

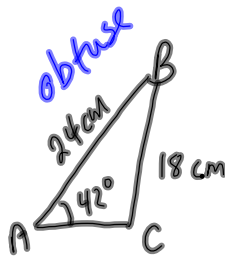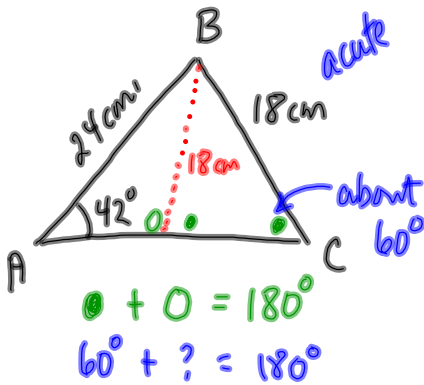$$\frac{15}{\sin 32°} \quad \cancel{=} \quad \frac{x}{\sin 95°}$$

$$\frac{x \cancel{\sin 32°}}{\cancel{\sin 32°}} = \frac{15 \sin 95°}{\sin 32°}$$

$$x = 28.2$$

Draw a triangle:

∢ A = 42°   AB = 24 cm   BC = 18 cm
             (12 cm)   (9 cm)

acute
obtuse
18cm   18 cm   about 60°
24 cm'   42° 0 •
A   C

SSA
cannot be
used to
make a
congruent
triangle

● + O = 180°
60° + ? = 180°

You can draw triangles that
are congruent IF you know:

ASA   SSS   HL
                ↑ leg
SAS   AAS   ↑
                hypotenuse

If we were asked to   answer
solve for ∢ C, then there are 2 possible

∢ C ≈ 60° ≅ 120° (supplementary
                        angles)
                   add to 180°

We must use caution when we are
given (SSA) and we are solving for an
angle using the Law of Sines. There may
two solutions.

        AMBIGUOS CASE OF LAW OF SINES

Example

A

18   14

35°   θ

B   C
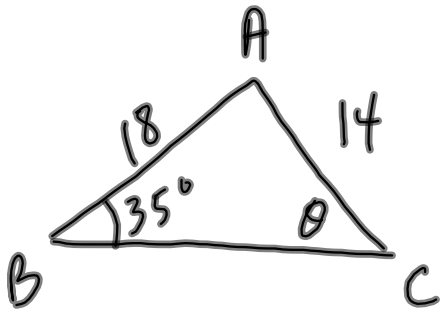
SSA ⇒ caution!

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{14}{\sin 35°} = \frac{18}{\sin θ}$$

$$\frac{14 \sin θ}{14} = \frac{18 \sin 35°}{14}$$

$$\sin θ = \frac{18 \sin 35°}{14}$$

$$θ = \sin^{-1}\left(\frac{18 \sin 35°}{14}\right)$$

$$θ = 47.5°$$

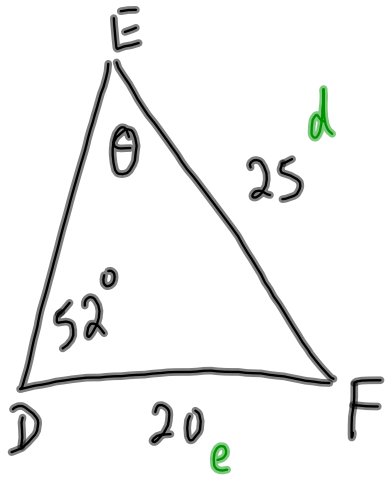There are two possible answers since there are two possible triangles that could be drawn using the given information.

or $θ = 180 - 47.5°$

$$θ = 132.5°$$

Example

$$\frac{d}{\sin D} = \frac{e}{\sin E}$$

$$\frac{25}{\sin 52°} = \frac{20}{\sin \theta}$$

SSA => be careful!

$$\frac{25 \sin \theta}{25} = \frac{20 \sin 52°}{25}$$

$$\sin \theta = \frac{20 \sin 52°}{25}$$

ONLY
SOLUTION

$$\theta = \sin^{-1}\left(\frac{20 \sin 52°}{25}\right)$$

$$\theta = 39.1°$$

We can eliminate
the second answer ⟶ or θ = 180° − 46.5°
since 140.9° + 52° > 180°   θ = 140.9°