Solving Systems of Equations

Example: ① $z - x + y = 9$

② $2z + x = 12 + y$

③ $-2z - 3y = x - 8$

① $z - x + y = 9$

② $2z + x - y = 12$

③ $-2z - x - 3y = -8$

Rearrange the equations so the variables are on the LEFT and the constants are on the RIGHT

Write as a matrix equation:

$$\begin{matrix} z & x & y \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} z \\ x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ -8 \end{bmatrix}$$

coefficient matrix — variable matrix — constant matrix

$$AX = B$$

inverse of A → $A^{-1}A$ $X = A^{-1}B$

identity matrix → I $X = A^{-1}B$

$$X = A^{-1}B$$

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[A]^-1*[B]
          [[7 ]
           [-3]
           [-1]]
```

$$X = \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix}$$

$\therefore z = 7, x = -3, y = -1$

Example

① $50C + 60I + 40S = 2540$

② $75C + 50S = 1850$

③ $65I + 35S = 1860$

Matrix Eq:

$$\begin{matrix} C & I & S \end{matrix}$$

$$\begin{bmatrix} 50 & 60 & 40 \\ 75 & 0 & 50 \\ 0 & 65 & 35 \end{bmatrix} \begin{bmatrix} C \\ I \\ S \end{bmatrix} = \begin{bmatrix} 2540 \\ 1850 \\ 1860 \end{bmatrix}$$

$$A \quad X = B$$

$$A^{-1}A\,X = A^{-1}B$$

$$I\,X = A^{-1}B$$

$$X = A^{-1}B$$

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[A]^-1[B]
          [[14]
           [20]
           [16]]
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$$X = \begin{bmatrix} 14 \\ 20 \\ 16 \end{bmatrix}$$

$\therefore C = 14,\ I = 20,\ S = 16$