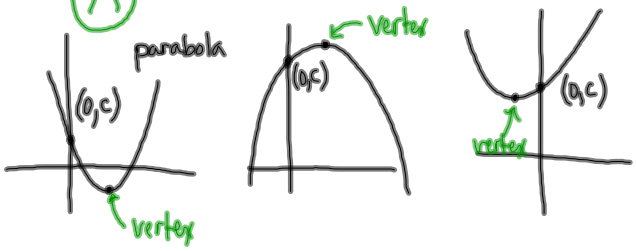
Quadratic Applications of 3x3 systems

quadratic: $y = ax^2 + bx + c$ linear: $y = mx + b$

+ opens up / − opens down

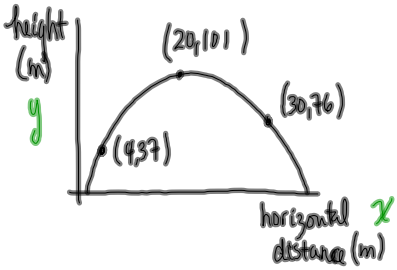
b controls the vertex

c y-intercept



Example

Suppose the path of a model plane can be described by the graph below:



1. Find the specific equation for the path of this plane.

$y = ax^2 + bx + c$

$ax^2 + bx + c = y$

point 1: $a(4^2) + b(4) + c = 37 \Rightarrow 16a + 4b + c = 37$

point 2: $a(20^2) + b(20) + c = 101 \Rightarrow 400a + 20b + c = 101$

point 3: $a(30^2) + b(30) + c = 76 \Rightarrow 900a + 30b + c = 76$

matrix equation:

$$\begin{bmatrix} 16 & 4 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 37 \\ 101 \\ 76 \end{bmatrix}$$

coefficient matrix — variable matrix — constant matrix

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[A]^-1[B]
        [[-.25]
         [10  ]
         [1   ]]
```

$A \cdot X = B$

$X = A^{-1}B$

$$X = \begin{bmatrix} -0.25 \\ 10 \\ 1 \end{bmatrix}$$

$\therefore a = -0.25, b = 10, c = 1$

SPECIFIC EQUATION → $y = ax^2 + bx + c$

$y = -0.25x^2 + 10x + 1$

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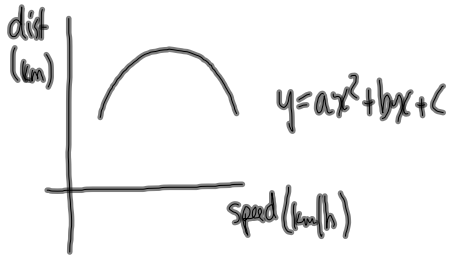
Example

The following table gives the distance driven per litre of gas at different speeds of a certain car.

	Speed (km/h)	distance (km)
① →	40	7.2
	50	10.2
③ →	60	11.4
	70	10.8
⑤ →	80	8.4



This can be modelled with a quadratic function. Find the distance you can travel on 1 L of gas if your speed is 75 km/h?

point 1: $a(40^2)+b(40)+c=7.2 \Rightarrow 1600a+40b+c=7.2$

point 3: $a(60^2)+b(60)+c=11.4 \Rightarrow 3600a+60b+c=11.4$

point 5: $a(80^2)+b(80)+c=8.4 \Rightarrow 6400a+80b+c=8.4$

$$\begin{bmatrix} 1600 & 40 & 1 \\ 3600 & 60 & 1 \\ 6400 & 80 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}=\begin{bmatrix} 7.2 \\ 11.4 \\ 8.4 \end{bmatrix}$$

```
[[-.009]
 [1.11 ]
 [-22.8]]
Ans▸Frac
[[-9/1000]
 [111/100]
 [-114/5 ]]
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$AX=B$

$X=A^{-1}B$

$$X=\begin{bmatrix} -0.009 \\ 1.11 \\ -22.8 \end{bmatrix}$$

$\therefore a=-0.009,\ b=1.11,\ c=-22.8$

$y=ax^2+bx+c$

SPECIFIC EQUATION

$y=-0.009x^2+1.11x-22.8$

if $x=75$,

$y=-0.009(75^2)+1.11(75)-22.8$

$y=9.8$ km

The car could go 9.8 km on 1 L of gas at 75 km/h

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